RESONANCE SCATTERING FROM FISH SCHOOLS

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1. INTRODUCTION

Acoustic scattering from individual fish and fish schools is an important issue in fisheries research and in naval defense applications. In fisheries research, acoustics is the major tool for assessing the abundance of many fish stocks. In naval applications, dispersed fish can cause reverberation that masks echoes from targets of interest, while schools of fish can cause echoes that may be mistaken as being from such targets. Thus, there are both commercial and military needs for a complete understanding of the unique acoustical nature of scattering from fish.

Generally, scattering from an individual fish produces a relatively weak signal except at swimbladder resonance frequencies. The resonances are due to excitation of vibrational motion of the bladders in much the same way that bubbles resonate, but with a significant damping effect due to viscosity. When this occurs, return signals may be enhanced compared to specular scatter, particularly at the monopole resonance (sometimes called the "breathing" mode) which is the predominant feature of low frequency scattering from fish.

Since the monopole resonance behavior of fish swimbladders bears a close physical similarity to the scattering properties of air bubbles in water, "bubble-like" models have been a popular method for describing resonant scattering from swimbladder bearing fish.[1,2] Likewise, the fish school problem is clearly related to that of scattering from a cloud of bubbles, and the results described in this paper have application to both issues.

The purpose of the work described here is to determine levels of acoustical scattering from schools of fish at frequencies near the swimbladder resonance. Fish of a similar size (and therefore of uniform swimbladder dimensions) tend to pack themselves tightly within a school.[3] At resonance frequencies this causes multiple scattering processes between the fish, and interference between their individual scattered wave fields, to become significant and complex. The configurational averaging approach of Foldy,[4] traditionally used for low frequency bubble scattering, functions incorrectly for high densities and resonance frequencies. We present an alternative formalism which accurately describes scattering from ensembles of resonators in close proximity to each other. A novel aspect of this work is that it : (a) allows an appropriate model for scattering from one object to be introduced ; (b) describes scatter from one object to adjacent ones; (c) includes higher order scatter correlations; and (d) exactly accounts for scattering from the aggregate field.

2. BACKGROUND

The great difference between the acoustic impedances of air and water causes air bubbles to be highly effective scatterers of sound. The acoustical properties of bubbles were first treated by Minnaert[5] who showed that for an ideal spherical bubble (where the processes are considered adiabatic and other lossy mechanisms, such as water viscosity, are neglected) the frequency ω_0 of the monopole resonance is given by

$$\frac{\omega_0 a}{c} = k_0 a = \frac{1}{c}\sqrt{\frac{3\gamma P_A}{\rho}} \quad . \tag{1}$$

Here, k_0 is the propagation wavenumber at resonance, a is the bubble radius, γ is the ratio of gas specific heats; and P_A, ρ and c are the ambient pressure, density and sound speed respectively of the surrounding liquid. Clay and Medwin[6] give a full description of this method. They point out that for an air bubble at atmospheric pressure in water (c = 1500 m/s) the value of $k_0 a$ is 0.0136.

A swimbladder is essentially just an air bubble within a fish; and this fact led Marshall[7] to examine the possibility that scattering from fish with air-filled swimbladders was the cause of high levels of acoustic volume reverberation within deep scattering layers. The primary mechanism was considered to be swimbladders vibrating in the volume pulsation mode when ensonified at the appropriate monopole resonance frequency. It is generally accepted that resonant scattering by swimbladder-bearing fish is the major cause of volume reverberation in the ocean at frequencies up to at least 20 kHz.[8]

The resonance behavior of a swimbladder is modified from that of an air bubble by the presence of the fish body around it. Acoustically, fish flesh is like soft rubber and can be closely approximated by a viscous fluid. Love[2] has treated fish flesh in this way to develop a spherical swimbladder model which has been successfully used to correlate fisheries data with acoustic measurements of volume reverberation.[9]

The acoustical properties of multiple air bubbles in water have also received a lot of attention but are somewhat less well understood than those of single bubbles. The passage of sound through water may be significantly modified by the presence of clouds containing many closely spaced bubbles; and the behavior of bubbly liquids has consequently assumed great importance in studies of acoustical propagation, attenuation, scattering and reverberation in sea water.[10,11] The classical work in this area was done by Foldy in his development of the "effective" medium model.[4] This model allows a bubble cloud to be described as a single scattering object with collective resonance characteristics,[12] but does not correctly predict acoustic properties for high volume fraction clouds and narrow bubble size distributions at frequencies close to the bubble resonance where there is a dramatic increase in the individual scattering cross section. Commander and Prosperetti[13] have recently published a review of the current theory available for modeling acoustic propagation through bubbly liquids. They compare theoretical predictions with experimental data and discuss the reasons for the limitations of the Foldy model (and other approaches equivalent to it). For the same reasons, because of close packing and narrow size distributions of fish in a school, the Foldy model does not provide a generally satisfactory method for describing the acoustical characteristics of fish schools at frequencies near swimbladder resonance.

In this paper we use a generalized coupled differential equation method to describe acoustic scattering from schools of closely spaced fish, using the Love spherical swimbladder model[2] as the kernel for each scattering center in the ensemble. This approach has already been used successfully to study the modal oscillations of small numbers of closely spaced bubbles and superresonance phenomena.[14] Its extension, using matrix methods, to describe acoustic scattering from large ensembles, such as fish schools, is quite straightforward.

3. THEORY

Devin[15] extended Minnaert's approach to investigate viscous and thermal (as well as radiative) damping processes of air bubbles in water. He gives an equation of motion for the monopole resonance of a bubble as follows

$$m\ddot{v} + b\dot{v} + \kappa v = -P e^{i\omega t} , \qquad (2)$$

where the variable v is the differential volume (i.e., the difference between the instantaneous and equilibrium bubble volumes). The coefficient m (= $\rho / 4\pi a$) is termed the inertial "mass" of the bubble; and κ (= $3\gamma P_A / 4\pi a^3$) is the "adiabatic stiffness". The coefficient b describes the damping of the bubble motion, while P and ω represent the amplitude and frequency respectively of the external pressure field applied to the bubble (P is preceded by a minus sign since a decrease in pressure results in an increase in the bubble volume). If a harmonic steady state solution of (2) of the form $v = \bar{v} e^{i\omega t}$ is assumed, substitution gives the resonance response

$$\bar{v} = \frac{-P}{\kappa - \omega^2 m + i\omega b} = \frac{-(P/m\omega^2)}{\left[\frac{\omega_0^2}{\omega^2} - 1\right] + i\frac{b}{m\omega}} , \qquad (3)$$

where $\omega_0 = (\kappa / m)^{1/2}$ is Minnaert's resonance frequency. Expression (3) describes a Lorentzian resonance response. The imaginary component (b/mω) in the denominator can be identified with a damping constant δ for the bubble, consisting of radiative, viscous and thermal terms, i.e.,

$$\frac{b}{m\omega} = \delta = \delta_r + \delta_v + \delta_t . \qquad (4)$$

At resonance the damping constant is equivalent to the reciprocal of the "quality factor" Q, so that

$$\delta_R = \delta_{Rr} + \delta_{Rv} + \delta_{Rt} = \frac{1}{Q} . \qquad (5)$$

In the theory of resonant acoustic scattering by fish swimbladders developed by Love, the quantity δ is replaced by a factor ($\omega_0/\omega H$). The frequency dependent parameter H (which is equal to Q at resonance) also consists of three components, which are combined as follows

$$\frac{1}{H} = \frac{1}{H_r} + \frac{1}{H_v} + \frac{1}{H_t} . \qquad (6)$$

In the case of fishes, the damping due to thermal conductivity effects is generally negligible compared to radiative and viscous damping. The values of H_r and H_v are given by

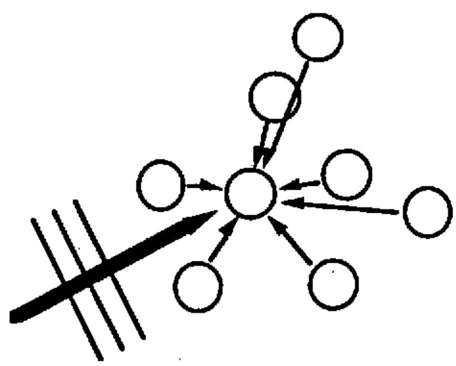
$$H_r = \frac{\omega_0 c}{\omega^2 a} ; \quad H_v = \frac{\omega_0 \rho a^2}{2\xi} , \qquad (7)$$

where ξ is the viscosity of fish flesh surrounding the bladder. The difference between water and fish flesh densities is very small and may be neglected.

When the swimbladder is ensonified by an external field, it scatters sound. The acoustic field reradiated by the bladder is predominantly monopolar, so that the pressure field at radial distance r due to scattering is given by[6]

$$p(r) = \frac{\rho e^{-ikr}}{4\pi r}\ddot{v} \qquad (8)$$

Let us now consider an external field driving an ensemble of N interacting swimbladders. The total field incident on any one of the bladders is the sum of the external field and the scattered fields from all of the others.



The response of the whole ensemble may be represented by a set of coupled differential equations as follows:

$$m_1\ddot{v}_1 + b_1\dot{v}_1 + \kappa_1 v_1 = -P_1 e^{i(\omega t+\phi_1)} - \sum_{j\neq 1}^{N} \frac{\rho e^{-ikr_{j1}}}{4\pi r_{j1}}\ddot{v}_j$$

$$\cdots\cdots\cdots\cdots$$

$$m_n\ddot{v}_n + b_n\dot{v}_n + \kappa_n v_n = -P_n e^{i(\omega t+\phi_n)} - \sum_{j\neq n}^{N} \frac{\rho e^{-ikr_{jn}}}{4\pi r_{jn}}\ddot{v}_j \quad ; \qquad (9)$$

$$\cdots\cdots\cdots\cdots$$

$$m_N\ddot{v}_N + b_N\dot{v}_N + \kappa_N v_N = -P_N e^{i(\omega t+\phi_N)} - \sum_{j=1}^{N-1} \frac{\rho e^{-ikr_{jN}}}{4\pi r_{jN}}\ddot{v}_j$$

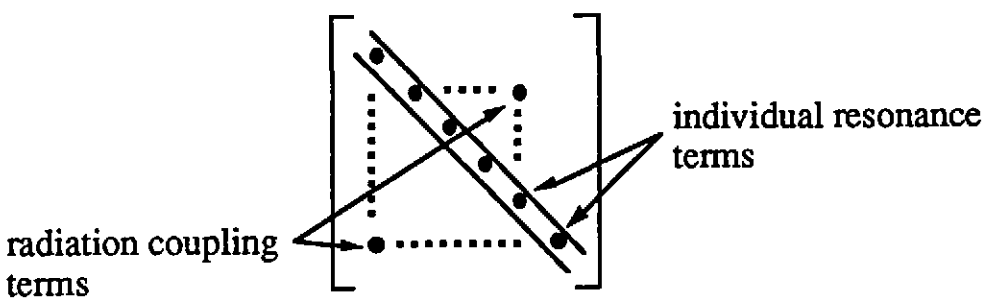
where P_n and ϕ_n are the amplitude and phase respectively of the external field experienced by the nth bladder, and r_{jn} is the radial distance from the center of the nth bladder to the center of the jth bladder. The quantities b_n etc. in these equations will incorporate the damping factors from the Love swimbladder model, as indicated by equations (4)-(7).

Looking again for harmonic steady state solutions, by substituting $v_n = \bar{v}_n e^{i\omega t}$ etc. in (9), a matrix equation is obtained which may be written $\mathbf{Mv} = \mathbf{p}$; where $\mathbf{v} = \{ \bar{v}_1,...,\bar{v}_n ,...,\bar{v}_N \}$ and $\mathbf{p} = \{ -P_1 e^{i\phi_1},..., -P_n e^{i\phi_n} ,..., -P_N e^{i\phi_N} \}$ are column vectors containing the steady state volume oscillation amplitudes and external fields respectively for the individual bladders, and $\mathbf{M}$ is an NxN matrix with elements:

$$M_{nn} = \kappa_n - \omega^2 m_n + i\omega b_n \ ;$$

$$M_{nj} = \frac{-\omega^2 \rho\, e^{-ikr_{jn}}}{4\pi r_{jn}} \quad (n \neq j) \ . \tag{10}$$

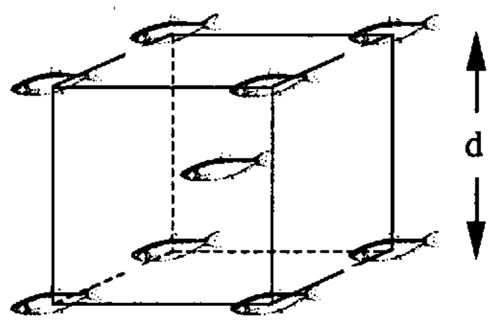
The diagonal terms describe the resonance behavior of the individual bladders, as if they were uncoupled from each other. The off-diagonal terms incorporate the coupling between the bladders due to acoustic scattering. The solution of the matrix equation (i.e., $\mathbf{v} = \mathbf{M}^{-1}\mathbf{p}$) enables the description of steady state scattering from the whole ensemble of swimbladders as a function of the external field amplitude and frequency. The matrix inversion process automatically includes all orders of multiple scattering. Once the solutions $\bar{v}_n$ are found for the individual swimbladders, the scattered pressure field (and hence target strength) for the whole school may be readily obtained using coherent summation.



4. RESULTS

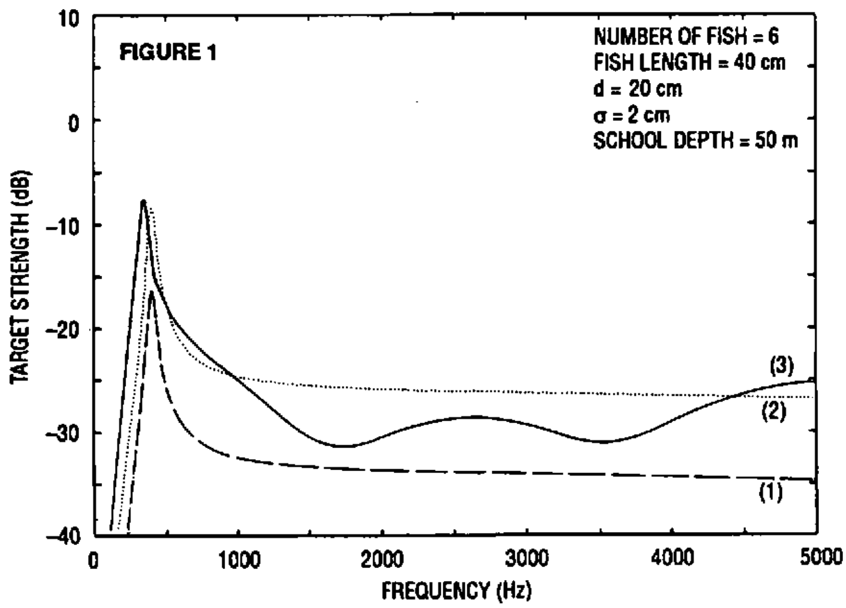
To illustrate the application of the scattering formalism described in the previous section, we consider several examples consisting of small to moderately sized schools of fish. We simulate the schools by grouping together individual fish in a way that simply approximates the formations which fish typically adopt when swimming closely together. The schools consist essentially of a number of basic cellular units. Each school unit is a cube with a fish in each corner and one at the center, all fish having the same heading. The overall ensemble, made from these units, is given a loosely ellipsoidal shape, to again approach the form of actual schools. A school is constructed by starting with a first "central" fish, and then adding fish sequentially to the corners and centers of the school units, so as to pack the school from the center outwards. The packing density is parameterized by the mean distance d between any two closest neighbors placed in the corners of the cube. Of course, as the fish swim along, the distances between these neighbors will vary from the value of d. The direction in which the school is swimming may also change in a quite unpredictable manner. We attempt to account for these variations by averaging the target strength of the school over a series of "snapshot" simulations. In each snapshot the individual fish locations are varied randomly from their mean positions with a normal distribution of standard deviation σ. The fish school is always ensonified horizontally, but the azimuthal angle of ensonification is varied randomly between 0 deg and 360 deg for each snapshot in order to average over changes of direction.

SCHOOL UNIT



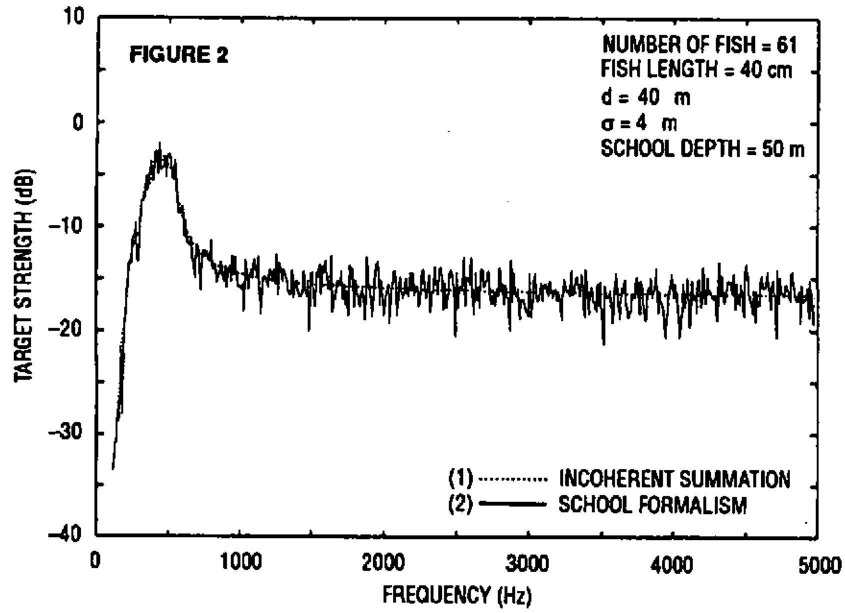
For our first example we consider a small ensemble of only 6 fish, each of length L = 40cm. In past applications of the Love spherical swimbladder model to interpret experimental data, it has been found that using a bladder radius prescribed by a = 0.05L has given successful results. Accordingly, for the fish in this school, we adopt a swimbladder radius of a = 2cm. The fish flesh viscosity is given the value ξ = 500 poise, which appears to be a reasonable value from the studies of fish tissue data that are available. The value of d is 20cm and σ is 2cm. It would be unusual for fish to form a school with the neighbor separation d less than a body length, but we have considered this case nevertheless because we wanted to see the effects of multiple scattering in a densely packed school. The mean water depth of the school is 50m. The target strength of the school, which is averaged over ten randomized simulations of fish positions and school orientation, is calculated between 0Hz and 5kHz. In figure 1 we see three curves. Curve (1) shows the target strength of one single fish placed at the center point of the school, calculated by using the Love model. A clear peak is seen at about 400Hz. Curve (2) shows the target strength for the school of 6 fish, but calculated after simply adding together the scattering cross sections for the individual fish (obtained from the Love model), and thereby neglecting multiple scattering and phase difference effects between the scattered fields. This amounts to incoherent summation of the scattered wave fields from the different fish in the school. The values of curve (2) are seen to be approximately equivalent to those of curve (1) plus $10\log_{10}(6)$ (i.e., ≈ 7.8dB), which would be the expected enhancement resulting from incoherent summation. Curve (3) shows the target strength calculated using the school scattering formalism described in the previous section. We see a number of notable features. First, while the school response has a marked resonance peak, we see that this has been shifted to a lower frequency from that of an individual fish. This effect has been reported previously, for other systems of resonators, by Twersky[16] and Weston.[17] Second, the peak value of curve (3) is greater than that of curve (2) by about 1.5dB, and greater than that of curve (1) by about 9.3dB. However, since the overall dimensions of this school are much less than the acoustic wavelength at the resonance frequency, we should expect fully coherent summation of the scattered wave fields to occur in this case. This should enhance the peak school target strength by about $20\log_{10}(6)$ (i.e., ≈ 15.6dB) over that of a single fish, if multiple scattering effects were unimportant. The fact that this enhancement is not seen indicates that there is a reduction in the target strength of the individual fish in the school due to the suppressing effect of multiple scattering on closely spaced identical resonators (this was also noted by Twersky and Weston).

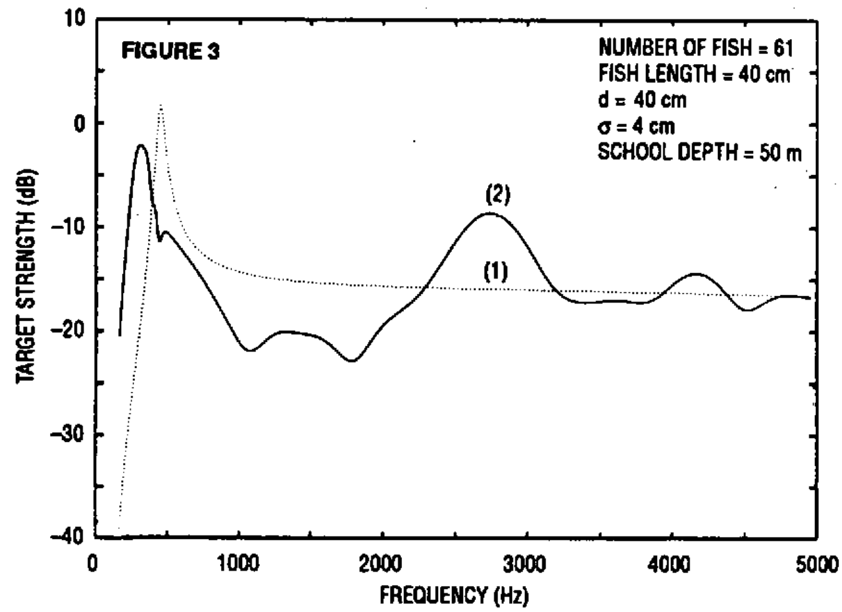
The third feature of curve (3) is that it falls below curve (2) in the middle section of the frequency range. This is due to destructive interference of the scattered wave fields in this frequency region. At the high frequency end curve (3) begins to rise above curve (2) again, as the wavefronts begin to interfere constructively.



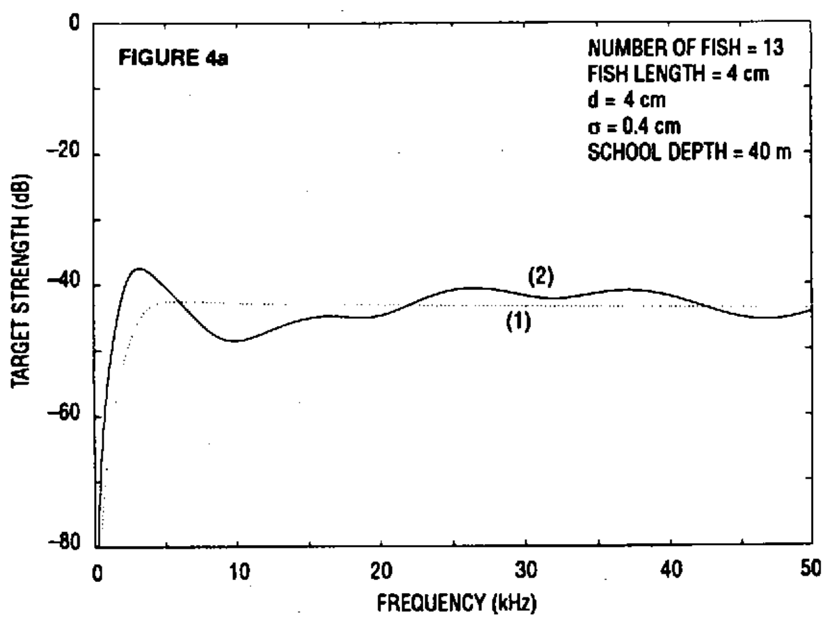
The second example we consider is a larger school of 61 fish with identical individual characteristics to those of the previous example. The mean school depth is again 50m, but now d = 40m (i.e., the fish are widely dispersed) and σ = 4m. The fish are so far apart that multiple scattering effects should play only a minor role in determining the target strength of the school. Also, σ is large enough to ensure that the individual scattered fields should average incoherently over the ten simulations performed. The scattering formalism should therefore, in this case, reproduce the result obtained by adding the cross sections together. Figure 2 shows that this does, indeed, occur. Two curves are displayed. Curve (1) is that obtained from incoherent summation of the individual fish scattering cross sections, as explained in the previous example. Curve (2) is the prediction of the new scattering formalism. Apart from jitter, which is produced by constructive and destructive interference effects at successive frequencies, we see that the target strength predicted by the formalism essentially overlays and reproduces the incoherent summation result.

The third example again considers a school of 61 fish (each with the same individual characteristics as in the previous cases) at a mean depth of 50m. Now, however, the overall school size is much reduced. The value of d is 40cm and σ is 4cm. In figure 3 we see significant differences between the school target strength predicted by the formalism (i.e., curve (1)) and that which results from incoherent summation (curve (2)). This is due to both multiple scattering and interference effects. At the low frequency end the formalism predicts a clear resonance peak. It is downshifted in frequency, but now multiple scattering effects hinder the resonances of the individual bladders so much that the peak target strength value is several dB below the incoherent summation peak. At higher frequencies the scattered fields from individual fish sometimes interfere constructively and sometimes destructively, leading to school target strength values which may be several dB above or below the incoherent summation values.



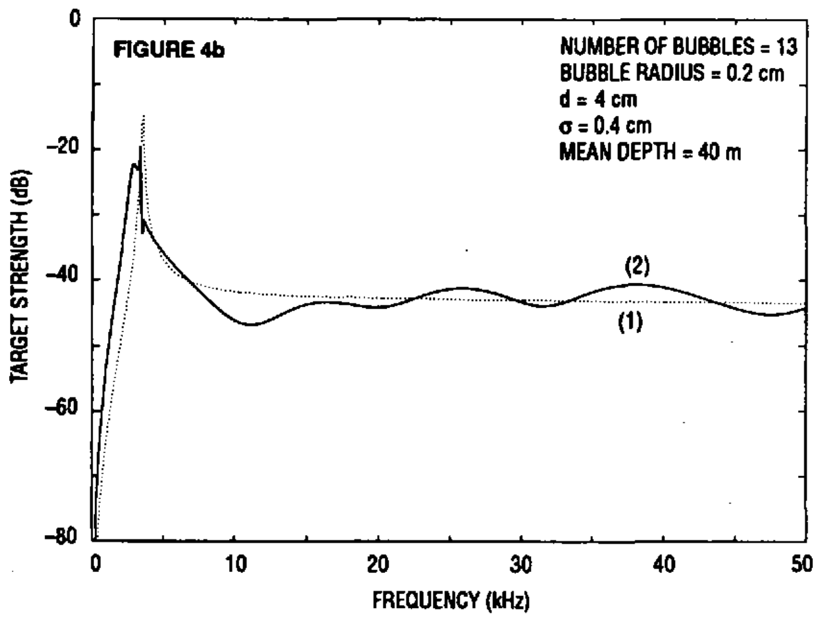


In the final example, we examine the effect of viscosity on the school target strength. In sections 1 and 2 we explained that the monopole resonances of bubbles and fish swimbladders are essentially identical physical phenomena, apart from the high viscous damping of fish flesh around the bladder. This damping is incorporated into the Love swimbladder model via the quantity H_V, which is defined in equation (7). If ξ is made very small in this expression, then the Love model will reproduce the scattering characteristics of an air bubble in water. We can compare the target strength of a school of fish with that of an identically sized cloud of bubbles of the same radius by simply varying the value of ξ in the scattering formalism. In figure 4a we plot the target strength of a school of 13 fish of length 4cm (i.e., swimbladder radius 0.2cm) with mean spacing d = 4cm and $\sigma = 0.4$cm. The fish flesh viscosity is $\xi = 500$ poise, and the mean depth of the school is 40m. Ten simulations are performed. Curve (1) shows the results given by incoherently adding the scattering cross sections together. There is no discernible resonance peak in this curve. This is because viscous damping greatly predominates over radiative damping for a bladder of this small size: it severely reduces the peak height and broadens the resonance. Curve (2) shows the prediction of the scattering formalism. It is clear that the school target strength deviates significantly from the incoherent summation result. However, the oscillations seen are not due to the specific characteristics of the swimbladders themselves, but to constructive and destructive interference effects which vary with frequency as sound scatters from the various individual bladders. In figure 4b we plot the target strength of a small group of 13 bubbles of radius 0.2cm, with the same mean spacing and series of randomized positions as the school of fish just considered. The two calculations are identical in every way apart from the viscosity, which is now reduced to $\xi = 1$ poise and thereby approaches the properties of an air bubble. Curve (1) shows the results given by incoherently adding the scattering cross sections together. There is now a distinct resonance peak at a frequency of about 4 kHz, which did not appear in the previous curves for the fish school calculation. Curve (2) in figure 4b shows the prediction of the scattering formalism. As with the corresponding curve in figure 4a, there are frequency dependent oscillations in the target strength due to constructive and destructive interference effects between the various bubbles. However, there is also a clear resonance feature superimposed on these oscillations at a frequency slightly less than 4 kHz. The peak has been reduced in height and downshifted in frequency by multiple scattering effects, as we have seen in previous examples.



5. CONCLUSIONS

A new formalism for describing scattering from fish schools has been developed. The method allows an appropriate model (the "kernel") for scattering for an individual fish in the school to be introduced. It includes multiple scattering effects between the fish, and calculates the scattering of the aggregate field by coherent summation. Application to ensembles of closely spaced fish



predicts shifts in the peak resonance frequency and reductions in target strength which correspond to the results of previous investigators. Frequency variations in the target strength due to interference effects are also observed. Application to widely dispersed ensembles reproduces the results of incoherent scattering. By reducing the viscous damping in the scattering kernel the formalism can also describe scattering from bubble clouds.

6. ACKNOWLEDGMENTS

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