24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

STRUCTURAL ACOUSTIC RADIATION OPTIMIZATION OF A TRUNCATED CONICAL SHELL BASED ON FINITE ELEMENT METHOD

Chen Hongyong, Fan Xuanhua, Li Shangming, Wang Dong

Institute of Systems Engineering, China Academy of Engineering Physics, Mianyang, Sichuan, 621999

Email:lxchenhy@caep.cn

The complex thin-wall shell structures are widely used in aircrafts, they are difficult for parametric modeling or representing the geometric characteristics by general spline functions, therefore it is difficult for acoustic optimization. In this paper a structural acoustic radiation optimization model is established based on finite element method and mesh with structural-acoustic coupling. The control of acoustic radiation power for complex structure based on mode shapes is investigated to optimization of the structural-acoustic coupling characteristics. A truncated conical shell model with its conical angle, truncated length and internal volume as designed parameters is established, and the dynamic characteristics are obtained by analyzing its mode shapes by FEM, while the acoustic characteristics are obtained by Rayleigh integral, the mode shapes are integrated to define the shape optimization function. Finally, the geometric shape with the lowest structural acoustic radiation as the optimization goal is obtained. Meanwhile, the structural acoustic radiation optimization under harmonic excitation is carried out with the mean acoustic radiation power in the structural-acoustic coupling frequency range as the optimization goal. The numerical results of the truncated conical shell of aluminium alloy structure show that the method is effective for acoustic optimization.

Keywords: Truncated conical shell, acoustic radiation, finite element method, parametric modeling, optimization,

1. Introduction

The truncated conical shell is a widely used in aircraft structures, such as main body of the reentry cabin, hood of the rocket, nozzle of the engine, et al. The dynamic response level of such structures from the exterior excitation affects directly the operation performance and life. Therefore, It is very important to investigate the vibration of and acoustic radiation from truncated conical shells in order to design new aircrafts.

The approach to analyze the dynamic characteristics of a truncated conical shell is commonly based on modal solutions. Hua and Lam[1,2] studied the frequency characteristics of the rotating conical shells by Galerkin method. Shu[3] studied the free vibration of composite laminated conical shells by means of the Generalized Differential Quadrature Method. Buchanan analyzed the free vibration of thick conical shell by Finite element method[4]. Ritz method is used to derive the equations of motion for the stiffened composite conical shell by Talebitooti et al[5], and explore the free vibration of rotating composite conical shells with the meridian stringers and circumferential rings. Leissa and So[6] also used Ritz method to obtain the natural frequencies of a truncated hollow cones of arbitrary thickness and arbitrary boundary conditions，and presented a three-dimensional elastic theory solutions to determine the free vibration frequencies and mode shapes of the structure. In those approaches, the Finite Element Methods shows better applicability for different composite shape of structure and solution efficiency.

The performance of the truncated conical shell is always optimized in order to meet the different technology requirement in engineering design based on dynamic analysis. The optimization of structure contains three parts: shape optimization, geometric optimization and topologic optimization. The shape optimization means to obtain the best performance of shpae such as minimum dynamic response, as a result of changing the surface of the structure by constructing of the parametric geometric model. Marburg and Hardtke reduced the noise level in a vehicle through defining the node coordinates in the finite element model of a rectangle panel by the polynomial equations, and obtained the best structure shape by adjusting and optimizing the parameters of the polynomial equations[7,8]. Kaneda carried out the surface shape optimization of the structure through changing the spline line on the surface of a vibrational panel, and reduced the acoustic radiation to the minimum level[9,10]. Wolfgang describes the boundary shape of the inner hold on the panel by B spline construncted by the sectional polynomial equation, in order to reduce the maximum stress[11].

The structure shape optimization is to define the best structure shape for some performance that satisfies the requirement of restriction, according to the given restriction condition for design. The key of this work is to construct the shape parametric model and choose the design parameter during the access of optimization, the control points are moved and adjusted to change the shape of the structure. Jeawon and Semyung[12] studied the acoustic sensitivity of a two-dimensional disk by applying the continuous shape sensitivity analysis on the acoustic control of the thin shell structure. Christian and Jean[13] investigated the inner acoustic response function by optimizing the long-width ratio of the structure. Annicchiarico and Cerrolaza[14] derived the shape optimization function based on B-spline and combined the GA(Genetic Algorithm) and FEM to optimize the stress level of the plane and plane connecting rod structure. Vinot[15] created the relationship between the cross section and moment of inertia of the beam by setting the keypoint of the beam cross section as the coordinates in FEM model of the thin shell beam. Steffen[16-17] proposed the shape optimization model based on FEM meshes, the nodes of the mesh are connected by polynomial equations, and the coefficients of the polynomial equations are defined as design parameters, the mean square root of the transfer function on the vehicle floor board reduces 2dB. Most of the optimization work are based on B-spline or curved surface, and defined the boundary of the structure by polynomial function or key geometric parameters, set the coordinates of the control points as design parameter, which can not work effectively for complex structures as a result of the bigger design parameter or the difficulty on parametric model. One effective way to solve this problem is to integrate the theory of optimization and FEM.

In this paper a structural acoustic radiation optimization model is established based on finite element method and mesh with structural-acoustic coupling. The control of acoustic radiation power for complex structure based on mode shapes is investigated to optimization of the structural-acoustic coupling characteristics. A truncated conical shell model with its conical angle, truncated length and internal volume as designed parameters is established, and the dynamic characteristics are obtained by analyzing its mode shapes by FEM,, the mode shapes are integrated to define the shape optimization function. The structural acoustic radiation optimization under harmonic excitation is carried out with the mean acoustic radiation power in the structural-acoustic coupling frequency range as the optimization goal.

2. Structural acoustic radiation

The dynamic equation of the structure under the excitation of external force is

$$\mathbf{M}\ddot{\boldsymbol{u}}(t)+\mathbf{C}\dot{\boldsymbol{u}}(t)+\mathbf{K}\boldsymbol{u}(t)=\boldsymbol{f}(t) \qquad (1)$$

where $\mathbf{M}$ presents for the mass matrix, $\mathbf{C}$ is the matrix of damp, $\mathbf{K}$ is the matrix of stiffness; $\boldsymbol{u}(t)$ represents for the vector of displacement, $\boldsymbol{f}(t)$ is the excitation vector of external force.

Take the Fourier transform on both sides of the Eq.(1)

$$\left(-\omega^2\mathbf{M}+i\omega\mathbf{C}+\mathbf{K}\right)\boldsymbol{U}(\omega)=\boldsymbol{F}(\omega) \qquad (2)$$

where $i=\sqrt{-1}$, ω is the natural frequency, $\boldsymbol{U}(\omega)$ is the FFT(Fast Fourier Transfer) form of $\boldsymbol{u}(t)$, $\boldsymbol{F}(\omega)$ is $\boldsymbol{f}(t)$ by FFT.

After solving the displacement response of the structure, it could be transferred to the normal velocity on the surface of the structure

$$\boldsymbol{v}_n=i\omega\boldsymbol{T}\boldsymbol{U}=\frac{i\omega\boldsymbol{T}\boldsymbol{F}}{\left(-\omega^2\mathbf{M}+i\omega\mathbf{C}+\mathbf{K}\right)} \qquad (3)$$

where $\boldsymbol{v}_n$ is the vector of the normal velocity on the surface of the structure, $\boldsymbol{U}$ is the vector of the displacement, $\boldsymbol{T}$ is the transform matrix, only relates to the surface shape of the structure.

The acoustic pressure of the o point on the surface of the shell under harmonic excitation can be obtained by Rayleigh integration

$$p(o) = i\omega\rho\int_S G(\mathrm{o}, q) v_n(q) ds \qquad (4)$$

where $p(o)$ represents for the acoustic pressure of the o point, ρ is the density of the fluid domain, S is the surface domain of the structure surface, q is an arbitrarily point on the structure surface, $v_n(q)$ is the normal velocity of the q point, $G(o,q)$ is the Green function of the semi-infinite free domain, which can be expressed as

$$G(o,q) = \exp(-ikr)/2\pi r \qquad (5)$$

where $k = \omega/c$ is the wave number, c is the speed of sound, $r = |q - o|$ is the distance between o and q point.

Dispersing the structure surface by meshes, the vibration velocity and the acoustic pressure on the surface of the element is uniform when the dimensions of the element is small enough. Then, the acoustic pressure on the surface of the element is

$$\boldsymbol{p} = \boldsymbol{Z}\boldsymbol{v}_n \qquad (6)$$

where $\boldsymbol{p}$ is the vector of the acoustic pressure on the structure surface, $\boldsymbol{Z}$ is the acoustic impedance matrix.

For a structure with arbitrarily shape, the expression to calculate the acoustic radiation of the structure vibration[18-19] is

$$W = \frac{1}{2}\int_S \mathrm{Re}\left(\boldsymbol{p}\boldsymbol{v}_n^*\right)\mathrm{d}s = \frac{1}{2}\boldsymbol{v}_n^H\left(\int_S \mathrm{Re}\left(\boldsymbol{Z}\right)\mathrm{d}s\right)\boldsymbol{v}_n = \frac{1}{2}\boldsymbol{v}_n^H\boldsymbol{R}\boldsymbol{v}_n \qquad (7)$$

where W is the acoustic radiation power of the structure, $\boldsymbol{v}_n^H$ is the complex conjugate maxtrix $\boldsymbol{v}_n$, $\boldsymbol{R}$ acoustic impedance matrix of the structure, only relates to the surface shape, the frequency domain of the analysis and the characteristics of the fluid domain.

It is shown in Eq.(7) that the acoustic radiation of the structure can be controlled by adjusting the $\boldsymbol{v}_n$ and $\boldsymbol{R}$, which are both related to the surface shape. Therefore, it could be an effective way to reduces the level of acoustic radiation power by optimizing the surface shape. In a FEM model, the surface shape is directly related to the coordinates of the nodes. Meanwhile, the acoustic radiation power can be expressed as the function of the coordinates of the nodes, as a result, optimizing the coordinates of the nodes of the FEM model can reduce the acoustic radiation power.

3. Optimization of acoustic radiation

The modal is the natural characteristics of the structure, the modal shape reflects the stiffness distribution. The large deformation of one modal shape usually represents for low stiffness and high energy of strain, and small deformation usually represents for high stiffness and low energy of strain. The best shape should match the model shape of the structure in order to obtain the best distribution of stiffness at he defined frequency range. The matched shape reaches the pre-deformation of the structure at the defined modal in order to reduce the dynamic response at the defined natural modal frequency. The modal shape describes the relative amplitude of the position of the structure nodes. In order to make up the reflection relationship between the modal shapes and the coordinates, the dynamic response amplitude of one node is set as the variation of the physical coordinates of the node on this orientation. In each modal frequencies, the components of the modal shape is not unique, and is permitted to be different with a constant coefficient. Then, each modal shape reflects a similar structure shape, the absolute value of the modal shape amplitude is different, and the absolute variation of the nodal coordinate is different, but the relative position of each nodes related to the structure shape reflected from the modal shape is not changed. Therefore, the linear superposition of a group of the model shape amplitude could be used as the optimization function of the design modification domain of the structure, the participation factor of each modal shape could be the optimization design parameter, different design parameter can reflect different shape of the structure.

In order to describe the new coordinates of the nodes after modification of the shape, and to obtain the components of the modal shape, every modal shape is normalized. The maximum amplitude of each node of every normalized modal shape is defined to be one unit of displacement in the physical coordinate. Only one shape of structure corresponds to the giving participation factor of the modal shape group. The structure shape is described by three coordinates of each node. Define the positions of each nodes by vector, set $\boldsymbol{x}_0$ and $\boldsymbol{x}_1$ to be the coordinate at the x orientation of the original and new shape, respectively. Meanwhile, $\boldsymbol{y}_0$ and $\boldsymbol{y}_1$ represent for the coordinate at the y orientation, $\boldsymbol{z}_0$ and $\boldsymbol{z}_1$ represent for the coordinate at the z orientation. The coordinates of the parametric shape model can be expressed as

$$\begin{cases} x_1 = x_1 + C_1\left|\varphi_1^x\right| + C_2\left|\varphi_2^x\right| + C_3\left|\varphi_3^x\right| + \cdots + C_n\left|\varphi_n^x\right| \\ y_1 = y_1 + C_1\left|\varphi_1^y\right| + C_2\left|\varphi_2^y\right| + C_3\left|\varphi_3^y\right| + \cdots + C_n\left|\varphi_n^y\right| \\ z_1 = z_1 + C_1\left|\varphi_1^z\right| + C_2\left|\varphi_2^z\right| + C_3\left|\varphi_3^z\right| + \cdots + C_n\left|\varphi_n^z\right| \end{cases} \tag{8}$$

where, C_i is the participation factor of the i^{th} modal shape, $\left|\varphi_i^x\right|, \left|\varphi_i^y\right|, \left|\varphi_i^z\right|$ represent for the amplitude at x、y、z orientation of the i^{th} modal shape, respectively.

The variation of the coordinates of the nodes at x、y、z orientation can be expressed as

$$\begin{cases} \Delta x = x_1 - x_0 = C_1\left|\varphi_1^x\right| + C_2\left|\varphi_2^x\right| + C_3\left|\varphi_3^x\right| + \cdots + C_n\left|\varphi_n^x\right| \\ \Delta y = y_1 - y_0 = C_1\left|\varphi_1^y\right| + C_2\left|\varphi_2^y\right| + C_3\left|\varphi_3^y\right| + \cdots + C_n\left|\varphi_n^y\right| \\ \Delta z = z_1 - z_0 = C_1\left|\varphi_1^z\right| + C_2\left|\varphi_2^z\right| + C_3\left|\varphi_3^z\right| + \cdots + C_n\left|\varphi_n^z\right| \end{cases} \tag{9}$$

In order to represent for the acoustic radiation power in the analyzing frequency domain, set the mean value of the acoustic radiation power as the design response, the optimization object function can be expressed as

$$F = \frac{1}{f_{\max} - f_{\min}} \int_{f_{\min}}^{f_{\max}} W(f)\,\mathrm{d}f \tag{10}$$

where $f_{\min}$ is the lower boundary of the frequency range, $f_{\max}$ is the upper boundary of the frequency range, F represents for the mean value of the acoustic radiation power.

In this work we choose the first 3~10 modal shapes of the original structure shape to be the participation modal shape of the shape parametric model, and set the participation factor as the design parameter, then the coordinates of each node of the structural shape parametric model could be transferred by the function of the first modal shape. The object of the optimization is the lowest acoustic radiation power of the structure, the restriction of design is the modal frequencies, the optimization model of the acoustic radiation can be expressed as

$$\min F \quad \text{s.t. } \mathrm{g}_i \le 0 \quad i = 1, 2, \cdots, m; \quad C_{\min} \le C_1, C_2, \cdots, C_n \le C_{\max} \quad , \quad n = 3{,}10 \tag{12}$$

where g_i is the restriction function, m is the number of g_i. $C_{\min}$ is the lower boundary of the design parameter, $C_{\max}$ is the upper boundary of the design parameter. This is a typical nonlinear minimum problem and could be solved by twice planning arithmetic. The parameter shape optimization and coordinates updating by FEM is integrated with the optimization arithmetic based on Matlab in order to develop a practicable approach for engineering structure.

4. Numerical examples

Take a typical truncated conical shell model as an example, a parametrical shape model is constructed with aluminium alloy as material, shown in Fig.1. where r_1 and r_2 represents for the radius of the upper and lower end face, h_0 is the height of the shell, t_0 is the thickness of the shell.

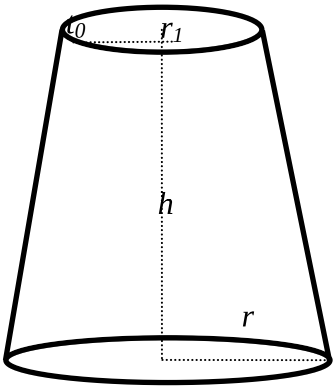
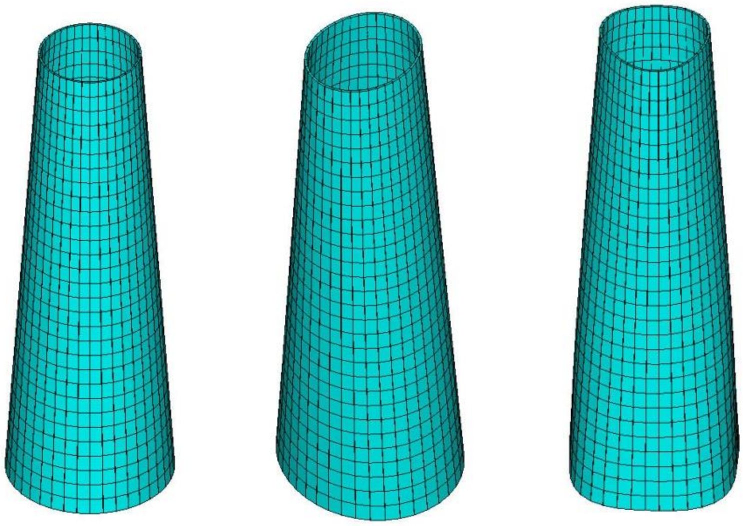


Fig. 1 Sketch of a parametric truncated conical shell model

Take the minimum acoustic radiation power at 100Hz~2000Hz frequency range as the goal of optimization. For different parametrical shapes of structure, the optimized surface shapes are obtain according to the integration of updating the FEM model by node coordinates and the optimization arithmetic, shown in Fig.2~Fig.6. The numerical results of surfaces shape and acoustic radiation are compared for different slenderness ratio, thickness of shell and the angle of cone.



a) Original shape b) Optimized by 3 modal shapes c) Optimized by 10 modal shapes

Fig.2 Shape comparison of a truncated conical shell between original and optimized with r_1=0.12m,r_2=0.2m,h_0=1.2m,t_0=0.005m

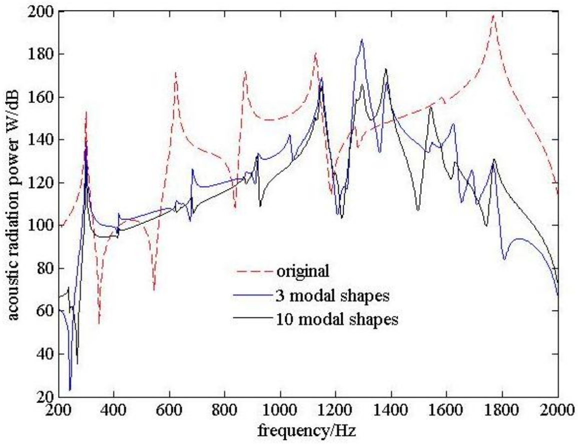
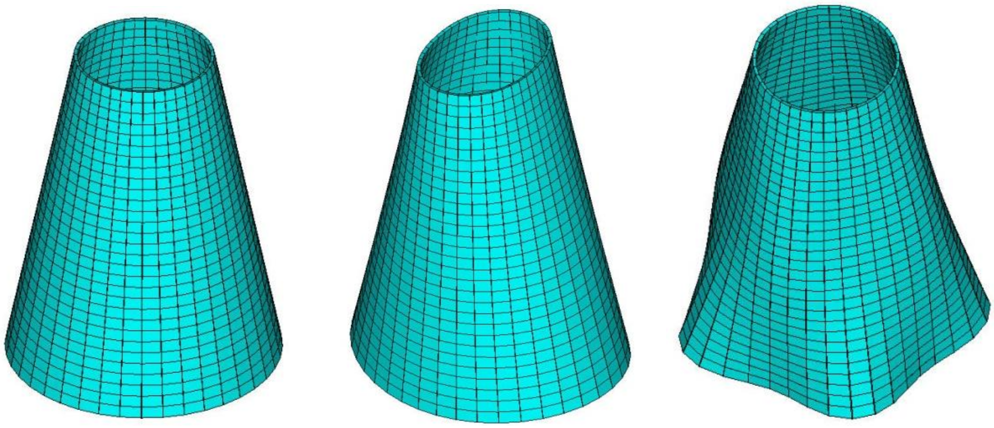


Fig.3 acoustic radiation power between original and optimized shapes, r_1=0.12m,r_2=0.2m,h_0=1.2m,t_0=0.005m



a) Original shape b) Optimized by 3 modal shapes c) Optimized by 10 modal shapes

Fig.4 Shape comparison of a truncated conical shell between original and optimized with r_1=0.1m,r_2=0.2m,h_0=0.5m,t_0=0.005m

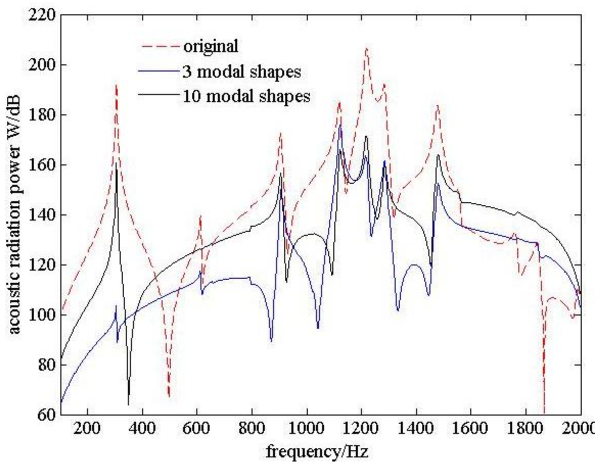
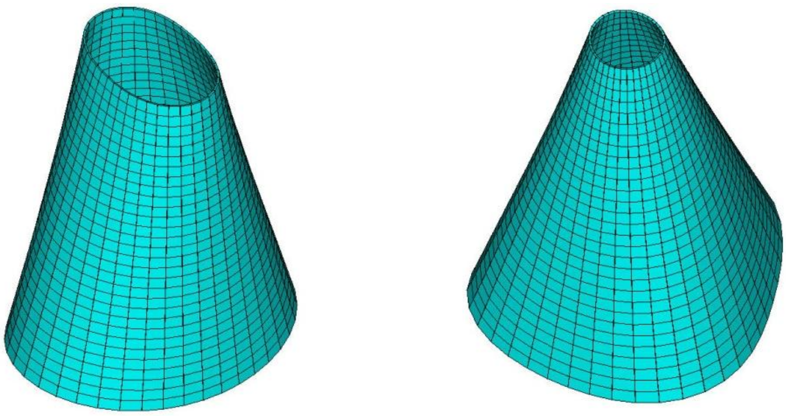


Fig.5 acoustic radiation power between original and optimized shapes, r_1=0.1m,r_2=0.2m,h_0=0.5m,t_0=0.005m



a) r_1=0.1m,r_2=0.2m,h_0=0.5m,t_0=0.003m b) r_1=0.05m,r_2=0.2m,h_0=0.5m,t_0=0.003m

Fig.6 Optimized shape comparison of a truncated conical shell with by 3 modal shapes

It is shown in above figures that the optimized shape of the truncated conical shell with different slenderness ratio and parametrically modeled by 3 modal shapes is the general tendency of being flat at the two normal orientations. With the reduces of the slenderness ratio, the optimized shape is affected much by the large end face according to its reduction of stiffness and increasing energy of strain. The acoustic radiation power reduces at most frequency range after optimization of the participation factor of the modal shape. When the number of modal shapes is higher, for example, from 3 to 10, the optimized value of acoustic radiation value becomes stable.

The tendency of the optimized shape changes little when the thickness of the shell becomes thinner, it can be concluded that the thickness affects little on the shape optimization. When r_1/ r_2 goes lower (the angle of the cone becomes larger), the optimized shape shows little change at the upper end face while the lower end face becomes polygon shape or cyclic spline as a result of the modal shapes.

5. Conclusions

The parametrical model of a truncated conical shell based on the FEM mesh is established in this work. According the integration of FEM and optimization theory, the optimized geometric surface shape with the lowest acoustic radiation power as the optimization goal is obtained based on the participation factor optimization of structural modal shapes. The numerical results of a aluminium alloy truncated conical shell shows that :(1) The acoustic radiation power reduces after optimization work, the value of optimized acoustic radiation power becomes stable while the number of participation modal shapes raises; When the number of participation modal shapes is higher, the effect from higher modal shapes becomes greater on the optimization shape. (2) For the parameter of the FEM model, the slenderness ratio affects the optimized shape much more than the thickness of the shell. The angle of the cone affects the shapes on the lower end face mainly.

Acknowledgement

This work would thank to the financial support from the National Science Foundation of China(Grant No. 11472256), the Presidential Foundation of CAEP(Grant No. YZ2015011) and the Science Development Foundation of CAEP(2015B0201024).

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