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ANALYSIS OF THE VIBRO-ACOUSTIC COUPLING FIELD IN RECTANGULAR CAVITY WITH A CLAMPED FLEXIBLE PANEL

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Although the structural form of clamped panel-cavity is widely used in practice, the vibro-acoustic coupling field under clamped boundary condition has not been studied yet so far, and previous studies are mainly concentrated on simply supported panel-cavity system. In this paper, the vibro-acoustic coupling field in rectangular cavity with a clamped flexible panel is analyzed. Firstly, an analytical model for the vibro-acoustic coupling field in clamped panel-cavity system is constructed by combining the modal theory of clamped panel with the analysis method of modal coupling. Secondly, a numerical example is analyzed and the analytical solution is verified by the coupled acoustic finite element method. Finally, the formula for calculating the modal coupling coefficient is simplified by analogy with the case of simply supported boundary condition. After simplification, the analytical model becomes simpler and more flexible, and cluster coupling characteristics between the cavity modes and the panel modes can be found. In conclusion, the analytical model proposed in this paper can be used to calculate the vibro-acoustic coupling field in clamped panel-cavity system accurately and quickly, and it provides a theoretical basis and a powerful tool for further analysis.

Keywords: vibro-acoustic coupling, clamped panel-cavity, analytical model, modal coupling coefficient

1. Introduction

Sound radiation into enclosed space is different from that in free space. It involves the coupling effect between structure and enclosure, which makes analysis and control of the vibro-acoustic coupling field becoming more complicated [1]. Since Lyon put forward the vibro-acoustic coupling problem, ongoing studies have been conducted by more and more scholars at home or abroad [2-5]. Among them, the milestone is that, Pan [3] built the vibro-acoustic coupling theory of noise transmission through a simply supported panel into a rectangular cavity by using the method of modal coupling analysis. By literature review, it can be found that the study object in previous studies is only limited to simply supported panel-cavity system. As is known to all, the structural form of clamped panel-cavity is more prevalent in actual application. However, the analytical study of the vibro-acoustic coupling field in clamped panel-cavity system has not been carried out. Though some numerical methods such as FEM [6] can be used to calculate the vibro-acoustic coupling field in panel-cavity systems whose shapes or boundary conditions are complicated, these methods apply only to specific model, so they are lack of flexibility on the analysis of vibro-acoustic coupling mechanism or on the exploration of internal laws.

Therefore, it's necessary to study the vibro-acoustic coupling field in clamped panel-cavity system. In this paper, based on the modal theory of clamped panel and the vibro-acoustic coupling theory of simply supported panel-cavity system, an analytical model for calculating the vibro-acoustic coupling field in clamped panel-cavity system is constructed, simulated and simplified.

2. Analytical model

As previous studies, we also choose a rectangular panel-cavity system as the model for this study, such that the cavity modes can be calculated. Consider a rigid walled rectangular cavity of cross section $L_x \times L_y$ and depth L_z, which is capped with a flexible rectangular panel, also of dimensions $L_x \times L_y$, as illustrated in Fig. 1. Note that the panel has clamped boundary conditions on all edges. Assuming that the external excitations contain incident plane sound wave P_{in} and point forces $f_i \left(i=1,2,\cdots,n_f\right)$, which are all applied to the panel surface S_F. For convenience, the time dependence term $\exp(j\omega t)$ has been suppressed in all following expressions.

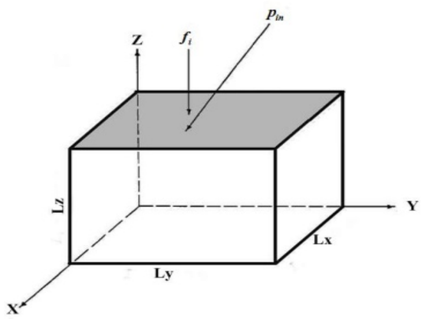


Figure 1: Rectangular panel-cavity coupled system.

By the method of modal analysis, the vector of normal velocity on the panel $\boldsymbol{v}(\boldsymbol{\sigma},\omega)$ and the vector of sound pressure in the cavity $\boldsymbol{p}(\boldsymbol{r},\omega)$ can be expressed by their normal mode expansions as

$$\boldsymbol{v}(\boldsymbol{\sigma},\omega)=\boldsymbol{\Psi}^T\boldsymbol{V}_M , \quad (1)$$

$$\boldsymbol{p}(\boldsymbol{r},\omega)=\boldsymbol{\Phi}^T\boldsymbol{P}_N , \quad (2)$$

where $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$ are respectively mode shape matrices of the panel and the cavity. $\boldsymbol{V}_M$ and $\boldsymbol{P}_N$ are vectors representing the modal amplitudes of the panel velocity and the modal amplitudes of the sound pressure in the cavity respectively. $\boldsymbol{\sigma}$ and $\boldsymbol{r}$ are, respectively, the position vectors for points on the flexible panel and in the cavity.

For the clamped panel-cavity system discussed here, the mode shape matrix of the rectangular cavity is easy to obtain, as given in detail in Ref. 3. However, it is much more difficult to obtain the mode shape matrix of the clamped flexible panel. This is the main reason that no work has been done for the clamped boundary condition in previous vibro-acoustic coupling studies. But fortunately, Jorge P. Arenas has calculated mode shape functions and natural frequencies of a clamped panel by using the virtual work principle [7]. It provides a possibility for the analytical study of the vibro-acoustic coupling field in clamped panel-cavity system.

According to Ref. 7, the mode shape functions of panel can be decomposed in a product form

$$\psi_{mn}(x,y)=X_m(x)\cdot Y_n(y) , \quad (3)$$

where the terms $X_m(x)$ and $Y_n(y)$ can be arbitrarily chosen as long as they are quasi-orthogonal and both of them satisfy the boundary condition. For a clamped flexible panel, they are defined as

$$X_m(x)=\cosh\left(\frac{\lambda_m x}{L_x}\right)-\cos\left(\frac{\lambda_m x}{L_x}\right)-\beta_m\left[\sinh\left(\frac{\lambda_m x}{L_x}\right)-\sin\left(\frac{\lambda_m x}{L_x}\right)\right], \tag{4}$$

$$Y_n(y)=\cosh\left(\frac{\lambda_n y}{L_y}\right)-\cos\left(\frac{\lambda_n y}{L_y}\right)-\beta_n\left[\sinh\left(\frac{\lambda_n y}{L_y}\right)-\sin\left(\frac{\lambda_n y}{L_y}\right)\right], \tag{5}$$

where $\beta_i=\frac{\cosh(\lambda_i)-\cos(\lambda_i)}{\sinh(\lambda_i)-\sin(\lambda_i)}$, $i=m\ or\ n$. λ_m and λ_n are the roots for the equation $\cosh(\lambda)\cos(\lambda)=1$, (m,n) is the panel modal indices. It is noticed that for large values of the integer i then $\lambda_i\approx\frac{(2i+1)\pi}{2}$. Furthermore, the natural frequencies are given by

$$\omega_{mn}=\sqrt{\frac{D}{m_s}}\cdot\sqrt{\frac{I_1I_2+2I_3I_4+I_5I_6}{I_2I_6}}, \tag{6}$$

where

$$\begin{array}{lll} I_1=\int_0^{L_x}\frac{\partial^4 X_m(x)}{\partial x^4}X_m(x)\,dx & I_2=\int_0^{L_y}(Y_n(y))^2\,dy & I_3=\int_0^{L_x}\frac{\partial^2 X_m(x)}{\partial x^2}X_m(x)\,dx \\ I_4=\int_0^{L_y}\frac{\partial^2 Y_n(y)}{\partial y^2}Y_n(y)\,dy & I_5=\int_0^{L_y}\frac{\partial^4 Y_n(y)}{\partial y^4}Y_n(y)\,dy & I_6=\int_0^{L_x}(X_m(x))^2\,dx \end{array}. \tag{7}$$

Next, using Green's functions to describe the sound field in the cavity and the vibration on the panel, and using the orthogonal properties of the modal shape functions, we obtain

$$\boldsymbol{P_N}=\boldsymbol{Z_A}\boldsymbol{V_M}, \tag{8}$$

$$\boldsymbol{V_M}=\boldsymbol{Z_P}^{-1}\boldsymbol{P_M^{ext}}, \tag{9}$$

where $\boldsymbol{Z_A}$ is the internal modal radiation impedance matrix of the panel, and $\boldsymbol{Z_P}$ is the panel input modal impedance matrix. Here, $\boldsymbol{Z_A}$ and $\boldsymbol{Z_P}$ can be written as

$$\boldsymbol{Z_A}=\rho_0c_0\begin{pmatrix} B_{1,1}/\chi_1^A & \cdots & B_{1,M}/\chi_1^A \\ \vdots & \ddots & \\ B_{N,1}/\chi_N^A & & B_{N,M}/\chi_N^A \end{pmatrix}, \tag{10}$$

$$\boldsymbol{Z_P}=\rho_0c_0\begin{pmatrix} \chi_1^P+\sum_N\frac{B_{N,1}B_{N,1}}{\chi_N^A} & \cdots & \sum_N\frac{B_{N,1}B_{N,M}}{\chi_N^A} \\ \vdots & \ddots & \\ \sum_N\frac{B_{N,M}B_{N,1}}{\chi_N^A} & & \chi_M^P+\sum_N\frac{B_{N,M}B_{N,M}}{\chi_N^A} \end{pmatrix}. \tag{11}$$

In above matrices, χ_M^P and χ_N^A are the modification terms because of the presence of modal damping, as described in detail in Ref. 3. ρ_0 and c_0 are, respectively, air density and speed of sound in air. $B_{N,M}$ is the modal coupling coefficient between the Mth panel mode and the Nth cavity mode,

$$B_{N,M}=\frac{1}{S_F}\int_{S_F}\boldsymbol{\psi_M}(\boldsymbol{\sigma})\boldsymbol{\varphi_N}(\boldsymbol{\sigma})\,d\boldsymbol{\sigma}. \tag{12}$$

In Eq. (9), $\boldsymbol{P_M^{ext}}$ is the generalized force vector, which is determined by the external excitations. Here, $\boldsymbol{P_M^{ext}}$ can be separated into two parts, i.e., $\boldsymbol{P_M^P}$ due to incident plane sound wave P_{in} and $\boldsymbol{P_M^S}$ due to point forces $f_i\,(i=1,2,\cdots,n_f)$, that is

$$\boldsymbol{P_M^{ext}}=\boldsymbol{P_M^P}+\boldsymbol{P_M^S}. \tag{13}$$

If the contribution of the sound pressure radiated by the vibrating panel to the panel velocity is neglected, the vector $\boldsymbol{P}_M^P$ can be expressed as

$$\boldsymbol{P}_M^P = -\begin{pmatrix} P_1^P \\ \vdots \\ P_M^P \end{pmatrix}, \tag{14}$$

where

$$P_i^P = \frac{2}{S_F} \int_{S_F} \boldsymbol{\psi_i}(\boldsymbol{\sigma}) P_{in}(\boldsymbol{\sigma})\, d\boldsymbol{\sigma} \, . \tag{15}$$

Meanwhile, the vector $\boldsymbol{P}_M^S$ can be represented by the point force location matrix $\boldsymbol{\Psi'}$ and the point force vector $\boldsymbol{F}$ with units of pressure,

$$\boldsymbol{P}_M^S = \boldsymbol{\Psi' F} = \frac{1}{S_F} \begin{pmatrix} \boldsymbol{\psi_1}(\boldsymbol{\sigma_1}) & \cdots & \boldsymbol{\psi_1}\left(\boldsymbol{\sigma_{n_f}}\right) \\ \vdots & \ddots & \\ \boldsymbol{\psi_M}(\boldsymbol{\sigma_1}) & & \boldsymbol{\psi_M}\left(\boldsymbol{\sigma_{n_f}}\right) \end{pmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_{n_f} \end{pmatrix}. \tag{16}$$

From above equations, the responses of the vibro-acoustic coupling field in a clamped panel-cavity system, i.e., the normal velocity on the panel and the sound pressure in the cavity, can be written as

$$\boldsymbol{v} = \boldsymbol{\Psi}^T \boldsymbol{Z_P}^{-1} \left(\boldsymbol{P}_M^p + \boldsymbol{\Psi' F}\right), \tag{17}$$

$$\boldsymbol{p} = \boldsymbol{\Phi}^T \boldsymbol{Z_A Z_P}^{-1} \left(\boldsymbol{P}_M^p + \boldsymbol{\Psi' F}\right). \tag{18}$$

Using Eq. (18), and integrating over the whole cavity volume V, the acoustic potential energy is obtained as

$$E_p = \frac{2}{4\rho_0 c_0^2} \int_V |\boldsymbol{p}(\boldsymbol{r}, w)|^2 d\boldsymbol{r} \, . \tag{19}$$

3. Numerical simulation

To simulate and verify the analytical model established in Section 2, we specialize the problem. Assuming that the medium in the cavity is air and the flexible panel is a thin aluminum panel. Model dimensions and material parameters are shown in Table 1 and Table 2. For convenience of simulation but without loss of generality, a single point force inertia actuator is used as the disturbance and is located at the center of the flexible panel. The amplitude of the point force is 10N, and the frequency range of interest is from 11Hz to 180Hz.

Based on the modal theory of a clamped panel defined earlier (Eqs. (3-6)), the structural modes and the resonance frequencies of the clamped panel can be calculated firstly. The first six order panel modes are shown in Fig. 2.

Table 1: Model dimensions

	Value	Unit
Cavity length L_x	0.868	m
Cavity width L_y	1.150	m
Cavity depth L_z	1.000	m
Panel thickness h	0.006	m

Table 2: Material parameters

	Value		Unit
	Aluminum	Air	
Poisson's ratio	0.33	—	—
Young's modulus	70	—	GPa
Sound speed	5150	343	m/s
Density	2700	1.21	kg/m^3

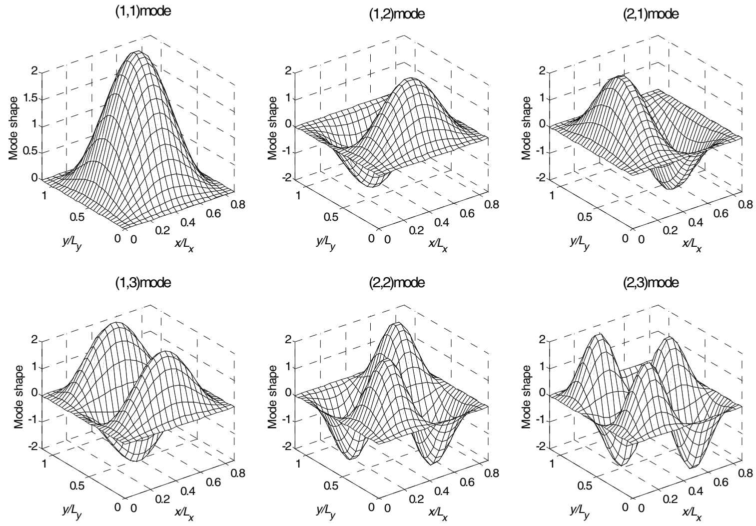
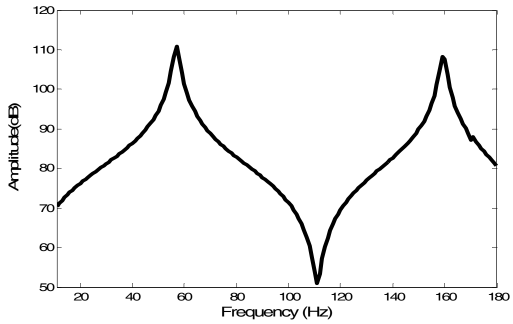
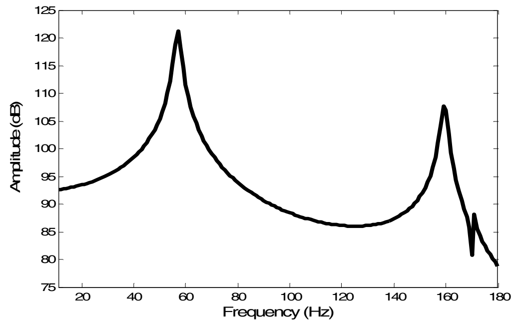


Figure 2: The first six order panel modes of the clamped panel.

Then, using the analytical model established in Section 2, the responses of the vibro-acoustic coupling field in the clamped panel-cavity system, i.e., the normal velocity on the panel and the sound pressure in the cavity, can be calculated. Here, to save space, we just called out the frequency response of the normal velocity at the center of the panel and the frequency response of the sound pressure at the center of the cavity, as illustrated in Fig. 3.



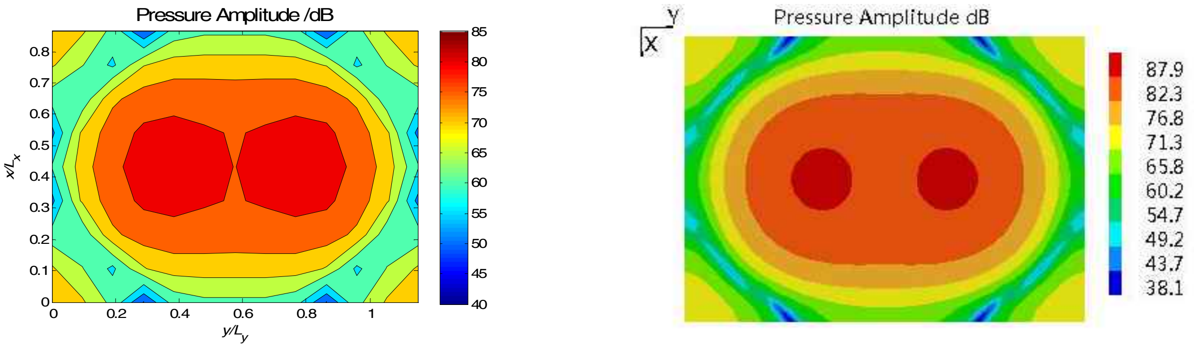
(a) Normal velocity at the center of the panel



(b) Sound pressure at the center of the cavity

Figure 3: Frequency responses of the vibro-acoustic coupling field.

Next, in order to verify the accuracy of the analytical solution calculated by the analytical model, we also calculated the responses of the vibro-acoustic coupling field using the software of LMS Virtual. Lab, i.e. , using the coupled acoustic finite element method (CAFEM, for short). Here, we named the calculated results as numerical solution, and just evaluated the spatial distribution of sound pressure in the cavity at a particular frequency, for example, 96 Hz. For convenience of observation, only the sound pressure distributed on the internal surface of the flexible panel was selected to compare, as shown in Fig. 4. It can be seen that the analytical solution and the numerical solution are in good agreement. So it illustrates that the analytical solution calculated by the analytical model has high accuracy. In other words, using the analytical model to calculate the responses of the vibro-acoustic coupling field in the clamped panel-cavity system is feasible.



(a) Analytical solution (b) Numerical solution

Figure 4: The sound pressure distribution on the internal surface of the panel at 96Hz.

4. Model simplification

As Ref. 3 points out, for the simply supported panel-cavity system, the integral formula of the modal coupling coefficient between the Mth panel mode (m,n) and the Nth cavity mode (l,u,v), i.e., Eq. (12), can be simplified into the following form

$$B_{N,M}=\begin{cases}(-1)^{v}\dfrac{mn\left[(-1)^{l+m}-1\right]\left[(-1)^{u+n}-1\right]}{\pi^{2}\left(l^{2}-m^{2}\right)\left(u^{2}-n^{2}\right)} \\ 0, \quad l=m \text{ and/or } u=n\end{cases}. \tag{20}$$

So now comes the question, is there the same simplification for the clamped panel-cavity coupling system?

In this section, in order to investigate this problem, we calculate the responses of vibro-acoustic coupling field using Eq. (12) and Eq. (20) respectively. Illustrated in Fig. 5 is the comparison of the acoustic potential energy in both cases, which is obtained using Eq. (19). It can be seen that, there may be some small differences on the values in both cases, but the similarities on the trend are much greater. Hence, we can come to a conclusion that, for the clamped panel-cavity system, the formula of the modal coupling coefficient can also be simplified into the form of Eq. (20), especially in theoretical studies, for example, exploring vibro-acoustic coupling mechanism, analyzing the vibro-acoustic coupling characteristics, and so on.

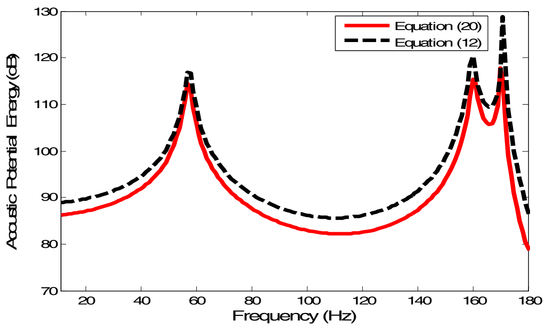


Figure 5: The comparison of the acoustic potential energy.

Comparing Eq. (20) with Eq. (12), one can deduce that the simplification has the following potential advantages.

(1) As the simplification eliminates the complicated integral term in the calculation of the modal coupling coefficients, the calculated quantity is dramatically reduced. So after simplification, the analytical model becomes simpler and more flexible.

(2) Just like the case of simply supported boundary condition, based on Eq. (20), we can find that the coupling of the cavity modes and the panel modes in the clamped panel-cavity coupling system

is also very selective (i.e., cluster coupling characteristics). Table 3 shows the possible coupling modal pairs.

Table 3: Selection rule for the clamped panel-cavity modal interaction
(o, odd number; e, even number; *v*, arbitrary cavity modes index in Z direction) (√: $B_{N,M} \neq 0$)

Panel modes	Cavity modes			
	(o, o, *v*)	(e, e, *v*)	(o, e, *v*)	(e, o, *v*)
(o, o)		√		
(e, e)	√			
(o, e)				√
(e, o)			√	

5. Conclusions

In this paper, a rectangular cavity with a clamped flexible panel was selected as the object of study, and an analytical study of the vibro-acoustic coupling field in clamped panel-cavity system was carried out. Firstly, combining the modal theory of clamped panel with the vibro-acoustic coupling theory of simply supported panel-cavity system, an analytical model for calculating the vibro-acoustic coupling field in clamped panel-cavity system was constructed. Then, a numerical simulation is carried out and the correctness of the analytical model was verified by CAFEM. Finally, similar to the case of simply supported boundary condition, the formula for calculating the modal coupling coefficient was simplified. After simplification, the analytical model became simpler and more flexible, and cluster coupling characteristics between the cavity modes and the panel modes were found. In conclusion, the analytical model constructed and simplified in this paper can be used to calculate the vibro-acoustic coupling field in clamped panel-cavity system accurately and quickly, and the work in this paper provides a theoretical basis and a powerful tool for further analysis.

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