Proceedings of The Institute of Acoustics

PERTURBATION ALGORITHMS ON THE DAVIES BEAMFORMER

C. C. KO

Department of Electrical and Electronic Engineering
Loughborough University of Technology

1. Introduction

The Davies Beamformer [1] has attracted considerable interest [2] in the area of adaptive array recently, mainly because it is possible to steer independent nulls in the directional pattern by simple variation of the setting of sets of phase shifters. Thus, an adaptive array based on this beamformer may be expected to have better performance in tracking moving jammers than the conventional real-and-quadrature-weights system. Furthermore, the number of variables that need to be controlled is halved, although the total number of phase shifters required is increased.

Since it is not clear how a feedback algorithm, eg. the steepest descent, can be applied to the beamformer, perturbation algorithms have been considered instead. Another reason for using perturbation algorithms is the system simplicity allowed by such algorithms.

The paper begins with a general discussion of the beamformer. The 'basic updating step', which is the building block of all the perturbation algorithms, is then discussed and followed by a description of the various algorithms. Some simulation results are then given and these verified the theoretical results derived. When compared with perturbation algorithms based on the real-and-quadrature-weights system, the algorithm described are shown to be considerably faster.

2. General Discussion of the Davies Beamformer

The narrowband Davies Beamformer treated is shown in Fig 1. For broadband application, the phase shifters should be replaced by a variable length delay line. The equivalent weights, w_i, i=1, ..., M+1 for the beamformer can be found from the equation for the directional pattern of the array,

$$D(z) = \sum_{i=1}^{M+1} w_i z^{i-1} = \prod_{i=1}^{M} (z - e^{j\alpha_i}) \tag{1}$$

where $z = \exp(j \frac{2\pi d}{\lambda} \sin\theta)$

Without loss of generality, let the noise environment be represented by M jammers with powers $\overline{|s_m|^2}$, m=1, ..., M at directions θ_m with respect to the normal of the array. The output power can then be shown to be

$$\overline{|y|^2} = \sum_{M-1}^{M} \overline{|s_m|^2} \prod_{i=1}^{M} 4 \sin^2 \frac{\gamma_m - \alpha_i}{2} + \overline{|n_o|^2} \sum_{i=1}^{M+1} |w_i| \tag{2}$$

where $\gamma_m = \frac{2\pi d}{\lambda} \sin\theta_m$, $\overline{|n_o|^2}$ = receiver noise power

If all the jammers have powers much greater than receiver noise (the case when this is not so will be discussed later), the receiver noise component in (2) can be assumed, for mathematical simplicity, to be constant and equal to

$$\overline{|n_o|^2} \sum_{i=1}^{M+1} |w_i|^2 = \overline{|y_{opt}|^2} = \text{optimal output power after convergence} \qquad (3)$$

From this assumption, it is obvious that the phase settings are optimal when equal to the set of γ_m, m=1, ..., M. For convenience, it is further assumed that α_i converges to γ_i, i=1, ..., M, without loss of generality.

3. The Basic Updating Step

Essentially, all perturbation algorithms involve a series of small updating steps as building blocks. Stated generally, the updating of a certain control variable is accomplished by finding the corresponding output power gradient by perturbing the variable concerned. The variable is then updated by an amount proportional to and with direction opposite to the gradient estimated. Thus, various algorithms are different because

(i) They use different sequences of updating steps. For the steepest descent algorithm, the basic updating step is applied sequentially, one at a time, to each control variable in turn. Contrary, in the relaxation algorithm, a fixed number of basic updating steps is applied to one control variable until convergence is roughly achieved. This is then repeated for another phase setting and so on. Clearly, various other sequences are possible.

(ii) The method of estimating the perturbed powers are different. In a 'one-receiver' system where only one output power measurement can be made per sampling instant, the perturbed powers have to be measured using independent samples. Alternatively, in a 'two-receiver[3]' system, the perturbed powers can be measured at the same instant.

(iii) The feedback factors and perturbation sizes associated with the basic updating steps are different.

For the Davies Beamformer, the basic updating step for updating the ith phase setting can be written as

$$\alpha_i(k+1) = \alpha_i(k) - \frac{c_i}{\sin \delta\alpha_i} \{|y^+(k)|^2 - |y^-(k)|^2\} \qquad (4)$$

where k denotes the kth updating, $|y^\pm(k)|^2$ are estimates of the average perturbed powers when $\alpha_i(k)$ is perturbed to $\alpha_i(k) + \delta\alpha_i$ and $\alpha_i(k) - \delta\alpha_i$ respectively, c_i is the feedback factor for the updating step and $\delta\alpha_i$ is the perturbation size. From (4), the asymptotic time constant, τ_i, for the ith phase setting can be shown to be

$$\tau_i = 1/4c_i \overline{|s_i|^2} \, |d_i|^2 \quad \text{updating steps} \qquad (5)$$

where $$d_i = \prod_{\substack{m=1\\ \neq i}}^{M} (e^{j\gamma_i} - e^{j\alpha_m(k)}) \simeq \prod_{\substack{m=1\\ \neq i}}^{M} (e^{j\gamma_i} - e^{j\gamma_m}) \text{ near equilibrium}$$

The one- and two-receiver systems associated with the Davies Beamformer are as shown in Fig 1 and 2. One drawback about the much simpler system of Fig 2b as well as some of the faster algorithms to be discussed is that it is necessary to interchange the ith and Mth phase settings before the perturbed powers can be measured. Assuming that the frequency of adjustment of the phase settings is the same and S samples are used for $|y^+(k)|^2$ or $|y^-(k)|^2$, the misadjustments (defined as the ratio of total excess output power due to the variances and perturbations of the phase settings to optimal output power) for the one- and two-receiver systems are found to be

$$MS1 = \frac{\overline{|y_{opt}|^2}}{S} \sum_{i=1}^{M} \frac{c_i}{4\delta\alpha_i^2} + \frac{1}{M\overline{|y_{opt}|^2}} \sum_{i=1}^{M} \overline{|s_i|^2}|d_i|^2\delta\alpha_i^2 \qquad (6a)$$

$$MS2 = \sum_{i=1}^{M} c_i \overline{|s_i|^2}|d_i|^2 / \{S - \sum_{i=1}^{M} c_i \overline{|s_i|^2}|d_i|^2\} \qquad (6b)$$

respectively. Detailed derivation of the results can be found in [4].

4. The Algorithms

Five algorithms are investigated and will now be described.

(i) Algorithm SDC1 (Steepest Descent with constant feedback factor on the one-receiver system): The feedback factors and perturbation sizes for all the phase settings of Fig 1 are constant and equal to c and $\delta\alpha$ respectively. The basic updating step is applied sequentially to each phase setting, one step at a time, in turn. In cases of severe jamming where $\overline{|s_1|^2} >> \overline{|s_2|^2} \ldots >> \overline{|s_m|^2}$ and the $|d_i|^2$'s do not vary over a wide range, c and $\delta\alpha$ can be determined from (6a) as

$$\delta\alpha = \frac{MS1}{2} \cdot \frac{\overline{|y_{opt}|^2}}{\overline{|s_1|^2}\ |d_1|^2} \qquad (7a)$$

$$c = \frac{(MS1)^2 S}{\overline{|s_1|^2}\ |d_1|^2} \qquad (7b)$$

where $\overline{|s_1|^2}$ in (7a) can be approximated by $\overline{|v_1|^2}$ (see Fig 1), $\overline{|y_{opt}|^2}$ can be measured and $|d_1|^2$ calculated from the phase settings when convergence is roughly achieved. From (5) and (7), the asymptotic convergent time constant, τ_p, of the output power curve as determined by the weakest jammer is

$$\tau_p = \frac{M\overline{|s_1|^2}\ |d_1|^2}{4(MS1)^2\ \overline{|s_m|^2}|d_m|^2} \quad \text{data samples} \qquad (8)$$

Thus, the asymptotic time constant of the output power curve is proportional to the number of element, the jammer power ratio and inversely proportional to the square of misadjustment. Note that equation (8) is very similar to the results derived for the steepest descent one-receiver algorithm on the real-and-quadrature- weight system in [5]. The two algorithms actually have roughly the same convergence time constants for the same misadjustment.

(ii) algorithm SDO1 (Steepest Descent with optimal feedback factor on the one-receiver system): Algorithms with optimal feedback factors are inspired by the fact that after convergence, the power $\overline{|v_M|^2}$ (see Fig 1) is roughly equal to $\overline{|s_M|^2}|d_M|^2$. This algorithm is similar to SDC1 except that the ith and mth phase settings are interchanged to obtain $\overline{|s_i|^2}|d_i|^2$ before updating for the ith phase setting. With this measurement, c_i and $\delta\alpha_i$ are then normalized as

$$\delta\alpha_i^{\,2} = \frac{MS1}{2} \cdot \frac{\overline{|y_{opt}|^2}}{\overline{|s_i|^2}|d_i|^2} \qquad (9a)$$

$$c_i = \frac{(MS1)^2 S}{M\overline{|s_i|^2}|d_i|^2} \qquad (9b)$$

The asymptotic time constant for the ith phase setting is thus given, from (5) and (9), by

$$\tau_i = M^2/2(MS1)^2 \text{ data samples} \qquad (10)$$

Thus, the time constants for the phase settings have been equalized and the time constant for the output power curve becomes

$$\tau_p = M^2/4(MS1)^2 \text{ data samples} \qquad (11)$$

Comparing (11) with (8), it is obvious that under severe jamming conditions where the jammer power ratio is large, this algorithm can be very much faster than SDC1.

(iii) Algorithm RL1 (Relaxation on the one-receiver system): Because the phase settings have the same convergence time constants in SDO1, it is possible to apply a fixed number, N, of updating steps to one phase setting until it is roughly 'relaxed' before repeating the updatings for the next phase setting. Apart from this difference, this algorithm is the same as SDO1 and (9) - (11) are valid. Comparing with SDO1, however, this algorithm has the advantage that the number of phase setting interchanges is decreased by a factor of N. In this paper, N is chosen, for convenience, so that, according to (5), the phase setting converges to within 5% of its final value or

$$N = M\ln 20/4(MS1)^2 S \text{ updating steps} \qquad (12)$$

(iv) Algorithm SDO2 (Steepest Descent with optimal geedback factor on the two-receiver system): This algorithm is the two-receiver counterpart of SDO1. Note that if the more complicated two-receiver system of Fig 2a is used, $\overline{|s_i|^2}\,|d_i|^2$ can be measured in the subsidary beamformer instead and it is not necessary to interchange phase settings in the primary beamformer. From (6b) and (5), c_i, τ_i and τ_p are given by

$$c_i = \frac{S(MS2)}{M(1+MS2)\overline{|s_i|^2}|d_i|^2} \qquad (13)$$

$$\tau_i = M^2(1+MS2)/4(MS2) \text{ data samples} \qquad (14)$$

$$\tau_p = M^2(1+MS2)/8(MS2) \text{ data samples} \qquad (15)$$

(v) Algorithm RL2 (Relaxation on the two-receiver system): This is exactly the same as SDO2, except that N updating steps are applied to one phase setting before adjustment of the next phase setting. Similar to RL1, (13)-(15) still apply and N is chosen to be

$$N = M(1+MS2)\ln 20/4(MS2)S \text{ updating steps} \qquad (16)$$

Note that for the same misadjustment, the ratio of the time constants (11) and (15) for the one- and two-receiver systems respectively is 2/misadjustment. Thus, the two-receiver system is faster than the one-receiver system by factor of tens.

5. Comments on situations where the number of degrees of freedom of the array is not fully utilized

The theoretical results derived so far have assumed that all the jammers' powers are greater than receiver noise. The case when this is not so is highly complex. Consider, for simplicity, a three-element one-jammer situation. In this case, one of the optimal phase settings corresponds to steering a null in the direction of the jammer while the other is determined so that the output power contribution from receiver noise is minimized. It is found that the former phase setting converges very much faster than the latter one. Furthermore, the additional output power reduction due to the latter phase setting is minimal. Thus, to avoid wasting valuable processing time on this phase setting, the power rejected by each level of phase setting can be measure and adjustment is carried out only on those phase settings which give a rejection of power greater than a certain threshold. The rest of the phase settings will simply be set to a convenient value (which may be constant or depend on the phase setting adjusted). If 'intelligent' schemes like the one described are used, the assumption on jammers and reciever noise power will be valid to some extent and the theoretical results derived will apply with perhaps some slight modification. Such schemes are not investigated in this paper which treats the more basic consequences of applying perturbation algorithms to the Davies Beamformer.

6. Simulation Results

As an example, Fig 3-5 show the convergence behaviour of different perturbation algorithms on a three element array under the same noise environment of -20 dB receiver noise, with two jammers of powers 0 and -15 dB at directions 20° and -10° respectively. The misadjustment for all three cases are roughly 10%. The smooth curves are the ensemble average curves, while the randomly fluctuated ones are due to typical samples. The curves of Fig 3 and 4 are due to algorithms SDO1 and RL2 of section 4 respectively, while a two-receiver perturbation (relaxation) algorithm [3] has been used on the conventional real-and- quadrature-weights array for Fig 5. The efficiency of the Davies Beamformer is clearly demonstrated by comparison. Note that only one time constant is evident in Fig 3a and 4, whereas two very different time constants can be seen in Fig 5.

Next, for all combinations of jammers' angle of arrival on a three-element two-jammer situation, tables 1 and 2 show the number of data samples required for the SDO2 algorithm to converge to a threshold output power of -22dB when the receiver noise is at -30 dB. Both jammers have powers of -3dB for table 1, while jammer 1 and 2 have powers 0 and -15dB respectively for table 2. The independence of the convergence behaviour with jammers' directions and powers is clearly illustrated. Except for algorithm SDC1, this independence has been found with the other algorithms of section 4.

7. Conclusions

The basic consequences of applying various perturbation algorithms to the Davies Beamformer have been investigated. If the feedback factors used in updating the phase settings are 'optimized', it is found that the beamformer has more uniform and much faster convergence behaviour than the conventional real-and-quadrature-weights system. However, various practical aspects associated with the beamformer,eg phase setting errors and unequal element powers, have to be investigated as well before any real implementation. These practical problems are obviously avoided in an all digital software approach. In this context, the algorithms offer a compromise between the poor-performance low computation-rate steepest descent algorithm and the various high-performance high computation-rate accelerated algorithms.

8. Acknowledgement

The author would like to thank Dr J E Hudson for valuable discussions and criticisms.

9. References

(1) Davies D E N,'Independent angular steering of each zero of the directional pattern of a linear array'. IEEE Tran AP-15, 1967, pp 296-298.

(2) Hicks D S, 'Adaptive null steering antennas'. IEE colloquium on adaptive antennas, 20, Feb 1980.

(3) Hudson J E,'A perturbation algorithm for adaptive antennas with rapid analogue loop'. Unpublished report, Department of Electrical and Electronic Engineering, Loughborough University of Technology.

(4) Ko C C,'Perturbation algorithms on the Davies Beamformer'. Departmental Memorandum No 133, Department of Electrical and Electronic Engineering, Loughborough University of Technology.

(5) Widrow B and McCool J M, 'Comparison of adaptive algorithms based on the methods of steepest descent and random search'. IEEE Tran. AP-24, 1976, pp 615-637.

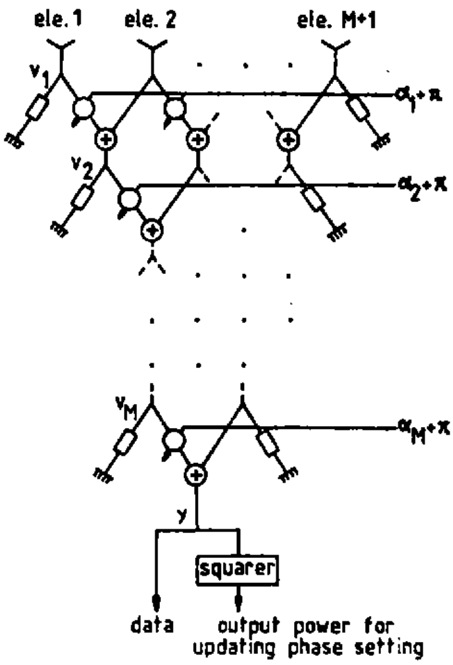

Fig 1 The Davies Beamformer.

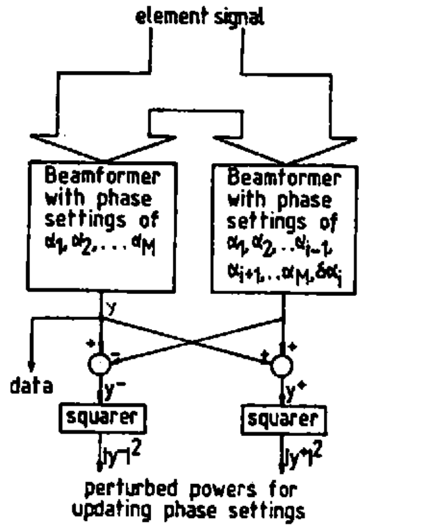

Fig 2a The two-receiver system for the Davies Beamformer.

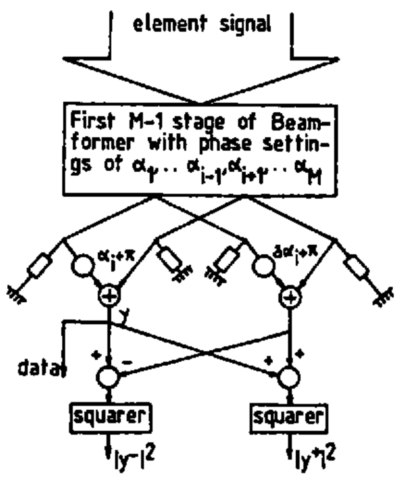

Fig 2b The simplified two-receiver system for the Davies Beamformer.

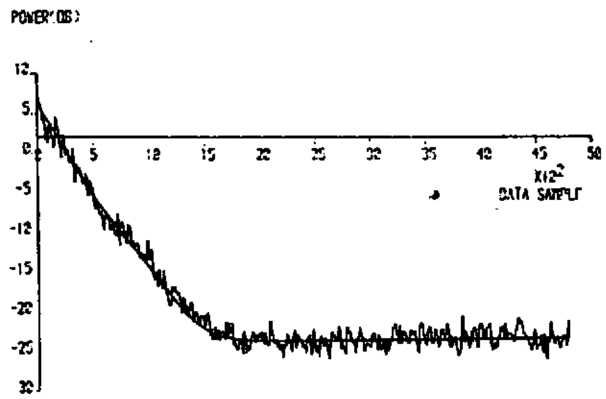

Fig 3a A typical output power convergence curve for the one-receiver Davies Beamformer (algorithm SD01).

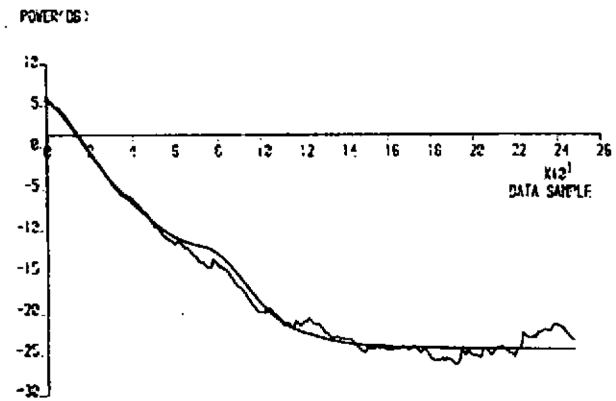

Fig 4 A typical output power convergence curve for the two-receiver Davies Beamformer (algorithm RL2).

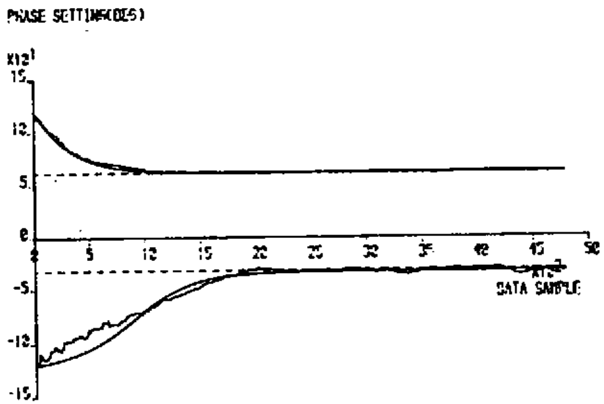

Fig 3b Convergence of the phase settings corresponding to the situation of Fig 3a.

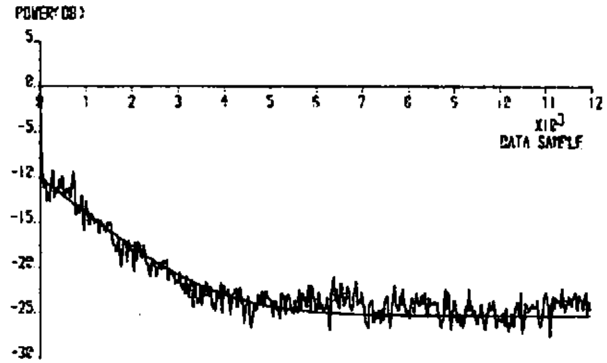

Fig 5 A typical output power convergence curve for the two-receiver perturbation (relaxation) algorithm on the conventional real-and-quadrature-weights system.

SIN(JAMMER 2 ANGLE OF ARRIVAL)

SIN(JAMMER 1 ANGLE OF ARRIVAL)	-1.0	-0.8	-0.6	-0.4	-0.2	-0.0	0.2	0.4	0.6	0.8	1.0
-1.0	63	208	163	158	137	121	111	104	99	95	91
-0.8	208	88	194	172	146	129	116	109	103	99	95
-0.6	163	194	94	166	161	136	122	114	105	104	99
-0.4	158	172	166	102	177	147	126	120	115	110	105
-0.2	137	146	161	177	115	151	135	131	125	116	112
-0.0	121	129	136	147	151	139	167	154	142	132	123
0.2	111	116	122	126	136	167	111	160	165	151	139
0.4	104	109	114	120	131	154	160	99	166	175	160
0.6	99	103	106	115	125	142	165	166	92	196	165
0.8	95	99	104	110	116	132	151	175	196	87	218
1.0	91	95	99	105	112	123	139	160	165	218	82

SIN(JAMMER 1
ANGLE OF ARRIVAL)

Table 1 Number of data samples required for convergence against the two jammers' directions for a three-element two-receiver Davies Beamformer (SDO2 algorithm). Jammer 1 and 2 both have powers -3dB.

SIN(JAMMER 2 ANGLE OF ARRIVAL)

SIN(JAMMER 1 ANGLE OF ARRIVAL)	-1.0	-0.8	-0.6	-0.4	-0.2	-0.0	0.2	0.4	0.6	0.8	1.0
-1.0	84	264	204	173	155	142	131	122	114	107	101
-0.8	501	66	206	193	167	152	139	129	120	111	104
-0.6	225	602	94	130	163	161	146	134	123	113	104
-0.4	202	210	355	103	121	170	151	135	121	111	105
-0.2	205	237	211	222	119	127	133	125	120	119	120
-0.0	146	146	145	141	141	140	150	161	176	170	164
0.2	111	114	120	134	151	123	111	267	206	211	240
0.4	106	114	126	140	155	175	122	99	403	213	203
0.6	106	115	125	136	146	163	165	136	92	776	230
0.8	104	112	120	129	140	152	166	194	216	67	471
1.0	100	107	114	122	131	142	155	173	204	265	63

SIN(JAMMER 1
ANGLE OF ARRIVAL)

Table 2 Number of data samples required for convergence against the two jammers' directions for a three-element two-receiver Davies Beamformer (SDO2 algorithm). Jammer 1 and 2 have powers 0 dB and -15 dB respectively.