MUSICAL ACOUSTIC RESONANCE SPECTROSCOPY OF STRINGED INSTRUMENTS

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Contrary to the generally accepted view (see, for example, Benade [1], page 510), it is relatively straightforward to make high resolution measurements of the response of a stretched string when excited near resonance. From such measurements, it is easy to derive quantitative information about the effective terminating impedances at the supported ends of the string. For strings mounted on musical instruments, such as the violin, guitar or piano, the impedance at the supporting bridge reflects the characteristic vibrational response of the instrument as a whole, which ultimately determines the intensity and quality of sound produced. In this paper, we show how measurements of string resonances can be used to determine the position, width, and strength of coupling to the string of acoustically important resonances of stringed instruments.

Our method is closely analogous to the powerful resonance techniques of solid state physics, and to nuclear quadrupole resonance (NQR) in particular. The resonating string is the mechanical analogue of the NQR radio-frequency coil, which is additionally damped whenever its resonant frequency coincides with the electromagnetically excited resonances of nuclei contained with the coil. We determine the additional damping of string resonances whenever the resonant frequency of the string coincides with a coupled structural resonance of the instrument on which the string is mounted. In view of this close analogy, we refer to our technique as musical acoustic resonance spectroscopy (MARS).

In our measurements, which have been described in more detail elsewhere [2,3], we excite the string resonances electromagnetically, by passing an alternating current through the metal or metal-covered string with a magnet placed nearby to produce a sinusoidally varying Lorentz force. The direction of this force, in the plane perpendicular to the string, can be adjusted by suitable positioning of the magnet. The induced string motion is detected photo-electrically, the string vibrations modulating the light passing between a light source and photo-detector. The photo-darlington detector system described previously [2] gives a very high sensitivity but a matched infra-red LED and photo-diode (TIL 78 and 32) has a faster response.

The electronic detection system incorporates a phase sensitive detector (PSD), which is used to determine the components of string vibration in-phase with and at 90^0-phase to a reference signal derived from the current passing through the string. If the string velocity, v, is determined at the position of the localised Lorentz driving force, F, the output of the PSD gives the real and imaginary components of the complex mechanical admittance, $A = A' + j A'' = v/F$, at the point of string excitation. A typical measurement of a string resonance, is shown in Fig.1a. To avoid "ringing" from narrow string resonance, such measurements were normally scanned relatively slowly (30S for a typical trace).

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All our measurements have been made on the total string length thus avoiding problems from adjustable end-stops, which, in practice, were found to have somewhat irreproducible terminating characteristics. Measurements on a number of instruments, including several violins and cellos and a piano, have been made using string resonances from about 100Hz - 4kHz.

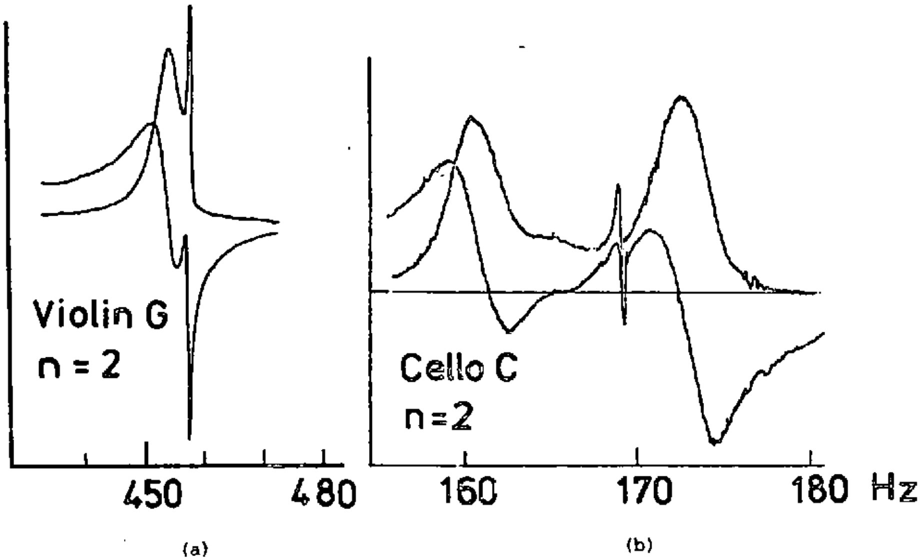

Fig.1 Typical resonance curves: (a) a weak-coupling situation showing superposition of "coupled" and "uncoupled" resonances, (b) a strong-coupling situation showing the splitting of the "coupled" resonance.

The measurements in Fig.1a were obtained for the n=2 mode ($f_n = nc/2\ell$, where c is the transverse wave velocity and ℓ the length of the string) of a violin G-string tuned slightly above its normal playing pitch. We immediately note the co-existence of two superimposed string resonances. Since either resonance can usually be excited alone, by suitable adjustment of the direction of the exciting force, the modes are to a good approximation linearly polarised in orthogonal directions. For convenience, we refer to these resonances as those of the "coupled" and "uncoupled" modes of string vibration. The origin of the two resonances lies in the directional nature of the coupling at the bridge.

Whenever a structural resonance of the instrument is excited, the bridge will rock on its two feet in its own plane causing the supported string to move in a particular direction. At low frequencies, on instruments of the violin family, the most important coupling is expected to be on the "air" and "main body" resonances involving a nearly rigid rocking motion of the bridge about the foot of the bridge nearest the soundpost. Transverse string vibrations polarised in the associated rocking direction will be strongly perturbed, whereas string vibrations polarised in the orthogonal direction will be unperturbed by such coupling, though they may be weakly coupled to other structural resonances involving different coupling directions.

To describe such measurements, we have developed a simple theory for string resonances based on ideas first proposed by Kock [4] for piano strings and extended to the physics of the violin by Schelleng [5]. The vibrating string is considered as a mechanical transmission line of characteristic impedance $Z_o = cm/\ell$, where m is the mass of the vibrating string. One end of the string is assumed to be a perfect node and is represented as an open-circuit termination. In the simplest version of the model, the end supported on the bridge is terminated by a series resonant circuit comprising the effective mass, M, compliance, C, and resistance, R, of the single structural resonance to which the string is assumed coupled. The associated resonant frequency $f_B = 1/2\pi\sqrt{MC}$ and quality-factor $Q = \omega M/R$.

When the string is excited at a frequency $f \approx f_B$, the series resonant circuit can be represented, to a very good approximation, as an equivalent length ε of transmission line terminated by a resistance r, where

$$\frac{\varepsilon}{\ell} = \frac{1}{n\pi} \cdot \frac{Z_o}{R} \frac{2Q\delta}{[1+(2Q\delta)^2]} \quad (1)$$

$$r = R[1+(2Q\delta)^2] \quad (2)$$

$$\text{and} \quad \delta = (f_B - f)/f_B. \quad (3)$$

For string resonances at a lower frequency than the coupled structural resonance, the above equations show that the effective length of the string is increased by the coupling resulting in a lowering of the resonant frequency for the "coupled" mode relative to the "uncoupled" mode. This is illustrated in Fig.1a, where the coupled main-body resonance is at 465 Hz. The increase in effective length of string arises because, for string resonance at frequencies below the coupled structural resonance, the bridge is forced to move in-phase with the force inducing the motion, so that the effective nodal position on the string is on the far side of the bridge from the string. The relative positions of the "coupled" and "uncoupled" string vibrations is reversed when $f > f_B$, since the bridge is then forced to move in anti-phase with the string. The damping of the string resonances determined from eq.(2) results in a Q-value for string resonance, $Q_s = (n\pi/2).(r/Z_o) = f/\Gamma$, where Γ is the width of the string resonance at half-height. We note that this damping is halved at the equivalent half-height positions of the structural resonance. Therefore, since $Z_o/R = (Q/n\pi)(m/M)$, by observing the magnitude and frequency dependence of the width (or amplitude) of string resonances as the string is tuned across a structural resonance, it is possible to derive the position of the structural resonance, its Q-factor and its effective mass. Furthermore, by adjusting the direction of the exciting force, so that only the "coupled" mode of string vibration is excited, the coupling direction can also be determined.

Although the above analysis is appropriate for many of the weakly coupled resonances of strings on musical instruments, where Q-factors frequently exceed 100, the coupling on instruments of the violin family is often too strong to ignore the changes in effective length of a string that can take place over the width of a string resonance (eq.(1)). For sufficiently strong coupling, such changes result in the single resonance splitting into a pair of resonances, closely analogous to the double resonance of an over-strongly coupled I.F. radio transformer.

This is most easily discussed in terms of the normal modes of the coupled system, where the relative motions of the string and structural vibrations are either in-phase or in anti-phase with each other. When the unperturbed string and structural resonances would otherwise have coincided, the frequency of the normal modes of the coupled system are given by

$$f_{\pm} = f_B[1 + \frac{1}{2Q}(j \pm \sqrt{K^2-1})]^{\frac{1}{2}} \quad (4)$$

where the coupling factor $K = (2Q/n\pi).(m/M)^{\frac{1}{2}}$ and $j = \sqrt{-1}$.

The character of the normal modes clearly depends on the magnitude of K relative to unity. For K << 1, we recover the predictions of our previous analysis. However, for K > 1 we have a double resonance with a splitting approaching $K/2Q = (1/n\pi)(m/M)^{\frac{1}{2}}$, both modes being equally damped with half the resonant-width of the coupled structural resonance.

An example of such a double-resonance is shown in Fig.1b for the n = 2 mode of a C-string on a cello known to suffer from a bad wolf-note at these frequencies, which arises when the frequency of a string resonance is shifted appreciably by the coupling. The direction of excitation has been chosen to suppress the narrow "uncoupled" resonance, which is only weakly excited between the two much broader resonances in the coupling direction. From such curves it is again straightforward to derive values for all the relevant parameters of the coupled structural resonance. Such measurements should provide helpful diagnostic information when attempting to adjust an instrument to minimise problems from wolf-notes.

We hope that this brief account will have illustrated the potential value of the MARS technique for deriving reliable quantitative information about the acoustically important resonances of any stringed instrument. In an accompanying paper [6] we describe how these techniques can be used to study the physics of the piano.

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