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RADIATION EFFICIENCES OF MUSICAL INSTRUMENTS

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In this paper we describe two quite different methods for determining the radiation efficiency of the various vibrational modes responsible for determining the tonal quality of a musical instrument. The two methods are those of Acoustic Resonant Scattering and the comparison of Q-values for an instrument in free-space and in a rigid-walled enclosure. Preliminary measurements on Helmholtz resonators and a violin are presented.

When a wind or stringed instrument is played in the normal way, a large number of vibrational modes of an instrument will in general be excited at the various harmonics of the fundamental note sounded. The intensity and quality of the resulting sound will depend on the position of these harmonics relative to the natural vibrational modes of the instrument and on their effectiveness in radiating sound. We shall define the radiation efficiency, ε, of a particular vibrational mode as the energy lost by radiation as a fraction of the total energy losses, which will include internal mechanical damping and viscous air losses.

Acoustic Resonant Scattering

The first method for determining acoustic radiation efficiencies that we shall describe is Acoustic Resonant Scattering. This technique is closely related to the optical and Mossbauer resonant scattering techniques that have proved to be very powerful in atomic and nuclear spectroscopy, but we are not familiar with any previous application to acoustical problems of the kind we describe here. The method is essentially that of measuring the sound radiated by a musical instrument when it is excited by incident sound - a familiar phenomenon to anyone who has played an instrument close to a piano with the dampers lifted.

The theory for the absorption and re-radiation of sound by an acoustically excited mechanical resonator is given by Rayleigh (1). Assuming that the resonator acts as a monopole source of sound and is small in comparison with the acoustic wavelength, λ, the energy absorbed and re-radiated at resonance is equal to the energy flux of the incident, uniform sound wave crossing an area λ^2/π. At a distance d from the resonator the pressure of the scattered wave, P_S, as a fraction of the incident sound pressure, P_O, is therefore given by $\lambda/2\pi d$.

This simple, yet remarkable, result shows that, at a given distance, the amplitude of scattered sound is independent of the size of the scattering object and depends only on the acoustic wavelength. It is also independent of the nature of the resonantly excited mechanical system - it could be the oscillating air column of a wind instrument or the cavity, string or structural resonance of a stringed instrument. It does assume, however, that the damping of the resonating system is by acoustic radiation alone. If there are additional viscous or internal friction losses the mechanical resonances will not be so strongly excited. The pressure of the scattered wave will therefore be reduced by the radiation efficiency, ε, defined above, so that

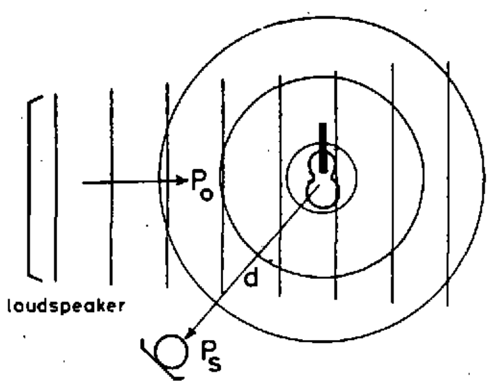


Fig.1 Experiment to measure re-radiated sound.

$$\frac{P_S}{P_O} = \epsilon \frac{\lambda}{2\pi d} \quad . \qquad (1)$$

By measuring this quantity and comparing it with the theoretical expression, an absolute and rather direct determination of the radiation efficiency can in principle be obtained.

A schematic representation of the experimental arrangement is shown in Fig.1. The measurements were made in an anechoic chamber of modest quality. A QUAD electrostatic speaker was used as the source of sound, and the frequency was swept over the range of interest using a voltage-controlled oscillator (VCO) to drive the loudspeaker amplifier. The resonator or musical instrument to be studied was placed at a central distance of about 2 m from the loudspeaker with the microphone at a distance of typically between 0.25 m and 1 m from the source of re-radiated sound.

The microphone receives sound direct from the loudspeaker in addition to sound re-radiated by the scattering object. To distinguish between the direct and re-radiated sound we have to measure the components of the sound pressure in-phase and in phase-quadrature with the output of the VCO both before and after the scattering object is placed in position. A typical set of measurements for our "standard musical instrument" - a milk bottle - is shown in Fig.2. The distance and, therefore, relative phases have been adjusted to give difference signals with the familiar absorption and dispersion curves of a simple resonator.

We measure the sound pressure at the position of the scattering object before the resonator is placed in position so that we can normalise the re-radiated sound pressure to the incident sound pressure. In Fig.3 we show some normalised measurements for the scattering amplitude as a function of scattering distance, d, plotted in such a way to allow comparison with eq.(1). The predicted inverse dependence on distance is confirmed and a value for the radiation efficiency of order 20% is obtained for the milk bottle.

As an independent check on the validity of this method for determining radiation efficiencies, we undertook a series of measurements on a collection of Helmholtz resonators. For radiation damping alone, the Q-value, Q_R, of a Helmholtz resonator of volume V is given by $Q_R = \frac{1}{2\pi^2}\frac{\lambda^3}{V}$ (Rayleigh (1)). The measured Q-value, Q_M, is reduced by additional viscous and thermal damping. We can therefore make an independent determination of the radiation efficiency of a Helmholtz resonator simply by comparing the measured Q-value with the radiation limited value, $\epsilon = Q_M/Q_R$. Weinreich (2) has argued that for an optimally designed Helmholtz resonator viscous losses will equal those from radiation, so that when thermal losses are included the acoustic efficiency of a Helmholtz resonator

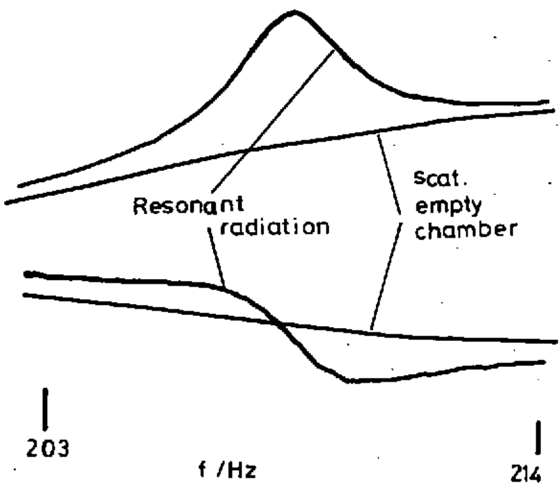


Fig. 2

should be less than 50%.

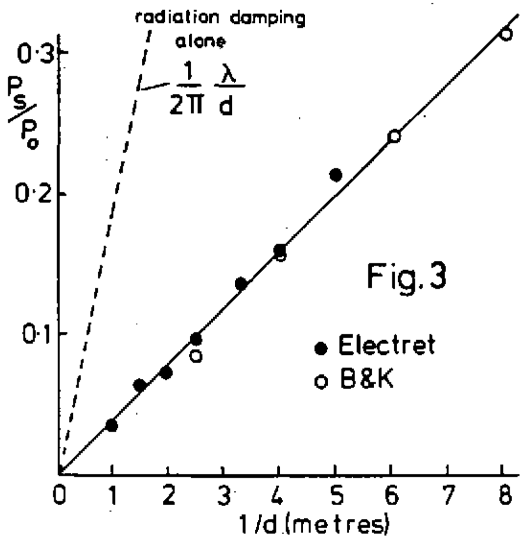
Figure 4 shows an example of measurements made with the aid of a small microcomputer, which records data via an 8-bit A/D converter and performs the necessary algebra to display the amplitude of the scattered sound as a function of frequency. From such measurements it is straightforward to derive Q-values, which can be used to derive values for the radiation efficiency, as described above. In the following table we compare values of radiation efficiencies obtained from measurements of the amplitude of resonantly re-radiated sound and from Q-values. With one exception, these values are consistent with Weinreich's (2) prediction that the acoustic efficiency of a Helmholtz resonator will always be less than 50%. The general

Frequency	192	256	320	440	512	576	640	704	768
ε scatt.	.39	.30	.27	.39	.39	.35	.48	.37	.48
ε Q-val.	.40	.25	.19	.33	.38	.41	.39	.39	.55

level of agreement between these two quite independent determinations of radiation efficiencies encourages us to believe that acoustic resonant scattering could indeed provide reliable values for the acoustic efficiency of resonant modes in systems for which no independent theoretical estimate of the natural line widths (radiation limited) can be obtained - as, for example, the structural modes of vibration of the violin.

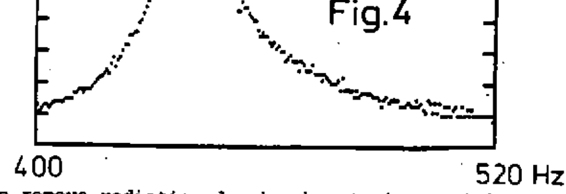
In the acoustically important range of the violin it is never a very good approximation to assume that the size of the violin is much less than the acoustic wavelengths involved in exciting structural resonances. Consequently, even in the absence of excited resonances, the violin will scatter a significant amount of the incident sound giving a background signal that will vary slowly with frequency. Moreover, the sound radiated by the violin will also include non-negligible contributions from dipole and higher-order components, which will complicate the interpretation of any measurements. Nevertheless, it seemed worthwhile to investigate the resonantly scattered radiation from a violin, if for no other reason than to test the limitations of this technique for locating resonances and for determining their radiation efficiencies.

In an attempt to overcome the background problem of non-resonantly scattered radiation arising from the finite size of the violin, we measured the difference in scattered radiation with the violin first in its natural state and then modified in some way to remove the resonant modes of interest. The difference



between such measurements gives information on the resonant modes that have been removed, with contributions from non-resonantly scattered radiation and radiation from unchanged resonances automatically subtracted from the result. Figure 5 shows the difference in scattered sound from a Vuillaume violin before and after its strings were damped, its f-holes covered and its table heavily loaded to remove the air and main mechanical resonances from the frequency range of interest. The measured difference should therefore provide information on the principal acoustic resonances of the violin with any non-resonant background scattering automatically subtracted. A number of resonances of the strings, air and structure can be identified from these measurements. In future we anticipate measuring the difference in scattering between violins in their natural playing state and a solid violin with no internal resonances to avoid artificial modification of the instrument being studied.

If this technique can be shown to give reproducible results, it could be useful for comparing instruments in different laboratories, since nothing has to be attached to the violin which might otherwise change its properties. Moreover the method relies only on a comparison of sound amplitudes to give absolute values for "the radiation efficiency". Even if such measurements turn out to be difficult to interpret, especially at the higher frequencies, the method is potentially valuable since it is free from any calibration problems.



Q-values in an enclosure

The second method for determining radiation efficiencies arose out of an experiment in which we planned to remove radiation damping by placing a violin in an evacuated enclosure. To our initial surprise simply placing the instrument in the enclosure (a thick-walled stainless steel cylinder closed at both ends) without evacuation resulted

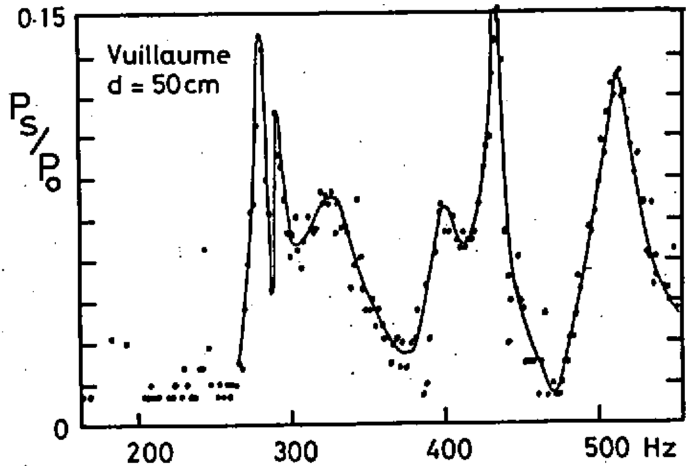


Fig. 5

in a dramatic increase in the Q-value of the main-body resonance at around 460 Hz. The increase in Q-value arises because, when placed in the rigid-walled enclosure, there are no radiative acoustic modes to which the vibrational modes of the instrument can couple. The standing waves within the enclosure cause a largely reactive loading of the vibrational modes of an instrument leading to a general depression of the frequencies of the lowest vibrational modes of an instrument but with very little frequency dependence unless the vibrational modes of the instrument accidentally coincide with the frequency of a standing wave of the enclosure.

Therefore to determine the radiation efficiency of a particular mode it is only necessary to measure the Q-value of the mode in free space, Q_{free}, and in an enclosure, Q_{encl}, with dimensions chosen to remove accidental coincidences of vibrational modes. From these measurements it is easy to show that the radiation efficiency is given by $(Q_{encl.} - Q_{free})/Q_{encl.}$.

Figure 6 illustrates the large effect that placing a violin in an enclosure has on the string resonances, when these are strongly coupled to the main-body resonance. From such measurements, we can deduce that, for this particular instrument, half the energy associated with the main-body resonance is radiated as sound. This has important consequences in relation to the wolf-note problem, since efficient acoustic radiation will tend to decrease the Q-value thus reducing the risk of exciting a wolf-note (Schelleng (3)). We hope to present some more recent examples to illustrate this technique for both wind and stringed instruments from our current measurements.

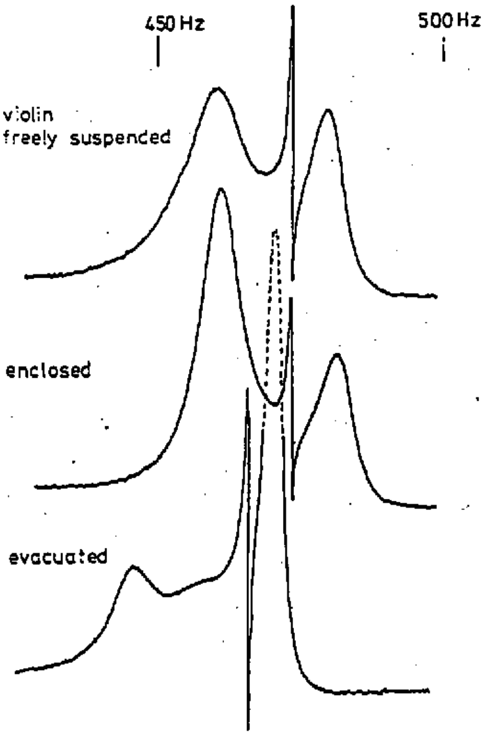


Fig. 6 Influence of air loading on string resonances and main body resonance.

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2. Weinreich, G., private communication.
3. Schelleng, J. The violin as a circuit, Journal Acoust. Soc. Amer. 35 (1963), 326-336.