PITCH, Π, AND OTHER MUSICAL PARADOXES

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In 1775, having won a £20,000 prize from Parliament for his work on marine chronometers, John Harrison (1693-1776), wrote *A Description concerning such Mechamism as will afford a nice, or true Mensuration of Time; together with Some Account of the Attempts for the Discovery of the Longitude by the Moon; and also An Account of the Discovery of the Scale of Musick.* Until recently the musical and acoustic significance of this book had been ignored or misunderstood.

As a result of his experiments with pendulums, monochords and a bass viol, Harrison found that the mathematics of musical harmonics were related to Π. He found that the pitches which musicians hear as harmonics and scales were caused by the multiple addition of two types of interval.

1) The Larger note as he calls it;
This is a ratio of 2 to the 2Π root or in BASIC computer terms 2↑(1/(2*Π)), which equals a ratio of 1.116633 or 190.9858 cents, approximately 1.91 frets on a conventional guitar. (L)

and

2) The lesser note, which is half the difference between five Larger notes (5L) and an octave. i.e.(2/(2↑(1/(2*Π)))↑5)↑(1/2), giving a ratio of 1.073344 or 122.5354 cents, an interval of approx. 1.23 frets. (s)

The equivalent of the fifth (i.e. seventh fret on guitar) is composed of three Large (3L) plus one small note (s) i.e. (3L+s) = (190.986*3) + (122.535) = 695.493 cents or ratio of 1.494412.

The equivalent of the fourth (fifth fret) is 2L+s = 504.507 cents.

I appreciate that this totally contradicts, what we were all taught in 'O' level physics about the harmonic series being caused by whole number frequency or string length ratios, but follow this idea carefully.

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Assume that the octave ratio is two. That is by halving the length of a string exactly the pitch or harmonic which is sounded is double the frequency (an octave). Let this octave be a complete circle. Imagine a clockface. The twelve hour positions each being thirty degrees apart represent the twelve notes of the twelve note equal temperament scale. Each interval being one semitone or 100 cents. Save this image we will come back to it later.

Now considered the layout of a conventional piano keyboard. In each octave there are seven 'white' notes and five 'black' notes. The 'white' notes represent the naturals, and the 'black' notes serve as sharps or flats. The interval between the white notes which are separated by a black note is a Large interval (L). That is C to D; D to E; F to G; G to A; and A to B.

The interval between the adjacent white notes is a small interval (s). E to F; and B to C.

An octave consists of 5 Large plus 2 small intervals.
The major scale is in the pattern L + L + s + L + L + L + s = 5L + 2s.
The minor scale (white notes starting from A) is L + s + L + L + s + L + L = 5L + 2s.

Now we return to the clockface.

The radian is the angle where the distance around the circumference is equal to the radius. Imagine a slice from a circular cake where all three sides are of equal length. This slice with an angle of 57.2958 degrees represents one Large interval. If we cut the five Large intervals, we are left with a slice of 73.5210 degrees, which represents two small intervals. By cutting this remaining slice into two equal pieces of 36.7605 degrees we now have five Large slices and two small slices, which any crumbs left on the plate or knife, represent the complete octave.

Back to the piano. If we build the pattern L+L+s+L+L+L+s which represents the major scale starting from F we need to flatten the B to B flat. If we do the same starting from G we will need to sharpen the F to F#. We can continue doing this ad infinitum, and generate any number of notes we would like in an octave, the flats being generated through the fourths (F), and the sharps through the fifths (G). In consequence we can describe any frequency ratio or interval in musical terms. It may require many orbits or octaves of sharps or flats but eventually we will find it by cumulative addition to whatever level of accuracy we should require.

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The result of this system is beautifully simple, for not only does it work musically and mathematically, it also suggests something very significant about our understanding of musical acoustics. The harmonic series based on whole number ratios seems to be true of the octave, as a mirror image on each side of the node is produced, but for all other integer ratios are only poor approximations. The traditionalists usually now ask about beat frequencies, to which I reply.
"How accurately have you measured the pitch of your *integer* harmonics?"

It seems that what is happening is that the movement of a vibrating string or sine wave, which had previously been assumed to be in two dimensions, is really in three or more dimensions, in a pattern which is similar to a coiled spring, and that the 'old' thinking had only been considering the cross-section of this pattern.

Independent musical and mathematical research seems to substantiate my findings, for the mathematical relationship between the Fibonacci series and Π, has now been established and scales based on the Golden ratio have been proposed by Jacques Dudon, which also explain how such a system has been used in the music of other cultures. These other *foreign* scales are also explained by further patterns of Large and small intervals.

At this point I leave Mervin K. Hobden of the Harrison Research Group and Tim Oakes to give you their historical, physical and musical findings.

The Physical Reasoning Behind Harrison's Musical Scale

John Harrison's division of the musical scale was not arbitrary and was the result of careful study of the physics of the vibrating string.
Harrison is known to have used two identically constructed monochords for this purpose. As a result of this study, Harrison appears to have abandoned the classical model of the transverse mode as the primary source of musical sound, substituting a progressive wave structure formed from the superposition of impulses travelling in a longtitudinal mode only. The transverse mode is therefore seen as a standing wave pattern and Harrison's model is similar to that proposed by Oliver Heaviside at the end of the 19th century for the formation of electrical waves on transmission lines.

A comparison is made with the ideas of Dr. Robert Smith, Master of Trinity College, Cambridge. Smith published his "Harmonics, or the Philosophy of Musical Sounds" in 1749. Harrison was later to claim that Smith had plagarised his ideas without fully understanding them. This claim is examined and shown to be justified to the extent that Smith misunderstood Harrison's ideas on pulses travelling along the vibrating string, substituting a model that depends very heavily on the classical transverse mode coupled to Newton's ideas on the successive compression and rarefication of the surrounding air.