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# CREEPING WAVE THEORY APPLIED TO IMPULSE PROPAGATION IN THE ATMOSPHERE.

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## INTRODUCTION

In a refractive atmosphere, due to wind and/or temperature gradients, sound follows curved ray paths and a shadow boundary can form through which sound is assumed to penetrate by means of creeping waves. Acoustic impulses form a convenient way of studying both the region prior to and beyond the boundary, as the resultant waveform is sensitive to time shifts between sounds following different paths. Furthermore, the presence of a ground wave component can be distinguished by its shape, whereas with single frequency measurements the ground wave is just a different magnitude sinusoid shifted in time relative to the source. In the following work, a 2ms duration impulse created by the discharge of a blank in a rifle has been used to probe around the shadow boundary. The impulse waveform measured 2m from the muzzle was used as the reference. After digitally sampling and storing the waveform, it was analysed into its frequency components using a DFFT algorithm and then the components multiplied by the appropriate continuous wave theory coefficients before generating the predicted pulse waveform using an inverse DFFT. Comparison of experimental and predicted impulse waveforms, as well as the attenuation of the peak values above that due to geometric spreading, i.e. the excess attenuation, provide a stringent test of propagation theories. All experimental waveforms are the ensemble average of at least ten individual shots, which effectively eliminates minor fluctuations in the waveform due to turbulence. All waveforms have been scaled to remove differences in peak levels for ease of comparison. Their relative size can be judged from the corresponding excess attenuation graph.

## LOCATING THE BOUNDARY BY RAY THEORY

Although ray theory is, at best, only applicable prior to the boundary, its simplicity makes it a useful starting point. Compared to a neutral atmosphere, refraction alters the path lengths and hence the delay between the direct and reflected rays reaching the receiver. The angle of incidence of the reflected ray is also modified, altering the phase change on reflection. This results in excess attenuation occurring prior to the boundary, as indicated in Fig.1, where the predicted curve B is compared to that for a neutral atmosphere, A, for a source receiver above grassland with a measured effective flow resistivity of 300,000 rayl. Here, a linear sound speed gradient of $1.6s^{-1}$ was required to fit the attenuation data, although this is in disagreement with the prevailing meteorological conditions which indicated a non-linear gradient and a wind speed at 1.2m around $3ms^{-1}$. It should be noted that the theoretical excess attenuation curve is discontinuous at the boundary, which appears to be near 46m in this case.

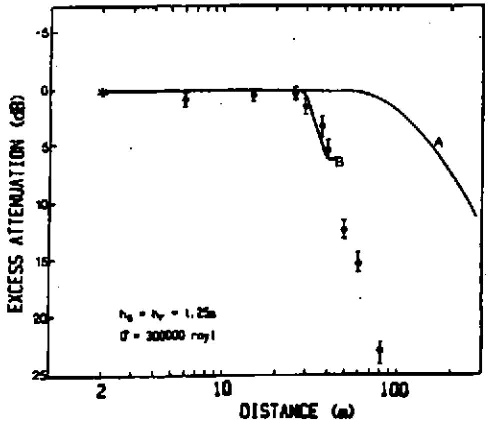


Fig. 1: Comparison of neutral atmosphere (A) and ray bending (B) predictions with experimental data taken over grassland.

Although it is feasible to deduce the boundary position from the effective linear gradient required to fit the excess attenuation data, an alternative technique is suggested by inspection of waveforms at various distances around the boundary. In Fig. 2, experimental waveforms are compared with shapes calculated assuming both a 1.6$s^{-1}$ linear gradient and a non-linear one similar to that present during the measurement period. It is apparent that the linear gradient predictions are in good agreement, in contrast to the use of the more realistic non-linear gradient which fails to generate a sufficiently large low

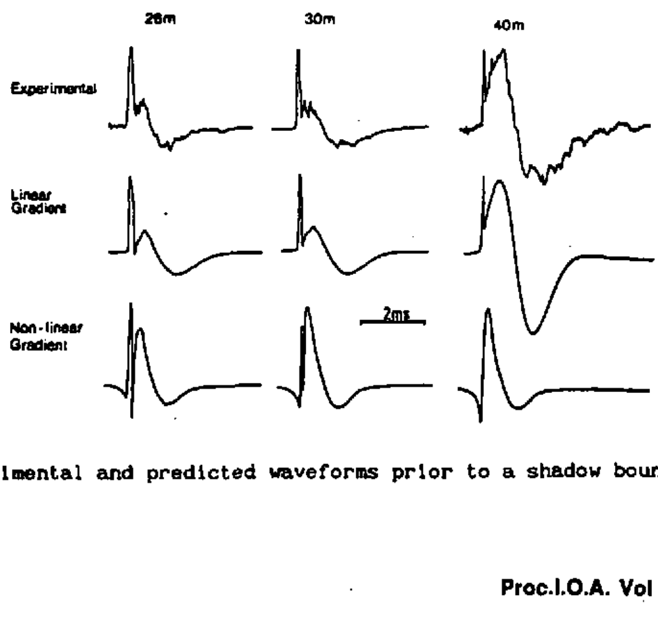


Fig. 2: Experimental and predicted waveforms prior to a shadow boundary at 46m.

frequency ground wave component. Choice of other non-linear gradients and including intensities in the calculations only reduces the agreement. The spike at the onset of the pulse waveform is the remnant of the direct and the almost totally inverted specularly reflected component. For the linear gradient case, the path difference for these components becomes effectively zero at the boundary and consequently they cancel exactly leaving only the rounder ground wave term. Thus, inspection of the waveforms to determine at what distance the spike disappears would seem to be a useful method of locating the boundary [1].

Fig.3 presents a graph of the variation in the risetime of the leading edge of the impulse waveform as a function of distance. A marked change occurs about 46m, supporting the earlier estimate. By repeating these measurements with a variety of source-receiver heights, a consistent profile of the shadow boundary has been obtained.

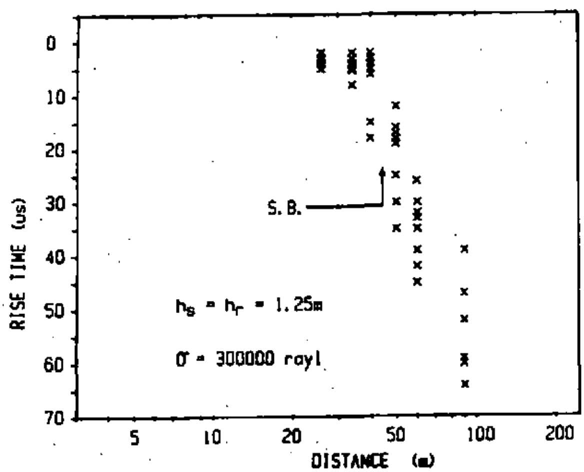


Fig.3: Pulse risetimes measured from waveforms captured at various distances from the source.

When the ground impedance approximates to a flow resistivity of 300,000 rayl or above, as in Fig.1, there is no apparent discontinuity in the excess attenuation measurements at the boundary. However, such a change was observed in measurements taken over the same field but with very dry ground, corresponding to, say, 150,000 rayl. The distance at which such discontinuities occur agrees with the prediction of the boundary location by the risetime technique, supporting the validity of the latter method.

As ray theory is not applicable beyond the boundary it is necessary to resort to creeping wave theory to continue the analysis.

## CREEPING WAVE PREDICTIONS

Based on the diffraction of sound waves [2], this theory envisages sound entering the shadow zone by following the limiting ray from the source down to the ground, where it creeps along continuously shedding energy up along paths parallel to the limiting ray. Earlier attempts [3] to apply the theory were limited by approximations involving a creeping wave layer of thickness $l$, which should be small compared to the wavelength: a major limitation when considering the relatively broad band nature of pulses. More recently, several residue series solutions have been presented which avoid this limitation [4,5,6].

If the pressure at a receiver at a distance r from a point source above a locally reacting surface of impedance $Z_2$ is written as

$$p = \frac{1}{r} \int_0^\infty V_\omega a_\omega \exp[i(kr - \omega t)]d\omega \quad . \tag{1}$$

then, for a linear sound speed gradient, the residue series solution for $V_\omega$ is [2,4,5]

$$V_\omega = (4\pi\xi)^{1/2} e^{i\pi/4} \sum_n \frac{e^{i\tau_n \xi} Ai[b_n-(h_s/l)e^{2i\pi/3}]\, Ai[b_n-(h_r/l)e^{2i\pi/3}]}{(\tau_n-q^2)[Ai(\tau_n)]^2} \tag{2}$$

where $\tau_n = b_n e^{-2i\pi/3}$ and $\xi$, $l$ and q depend on the wavenumber on the ground, $k_0$, and the radius, R, of the limiting ray in a linear gradient. The impedance enters through the variable q. Ai represent Airy functions while $b_n$ are the roots of the equation

$$Ai'(b_n) + q[\exp(i\pi/3)]\, Ai(b_n) = 0 \; , \tag{3}$$

which must be solved at each frequency for $b_n$ and the value substituted into Eq.(2).

In deriving Eq.(2), a Hankle function of the first kind and of order zero, $H_0^{(1)}$ has been replaced by an asymptotic limit. To avoid this approximation [6], the residue series can be written as

$$V_\omega = \frac{\pi r\, e^{i\pi/6}}{l e^{ikr}} \sum_n \frac{H_0^{(1)}(k_n r)\, Ai[b_n-(h_s/l)e^{2i\pi/3}]\, Ai[b_n-(h_r/l)e^{2i\pi/3}]}{[Ai'(b_n)]^2 - b_n[Ai(b_n)]^2} \tag{4}$$

where $k_n^2 = k_0^2 + b_n \exp(-i2\pi/3)/l^2$. Deep within the shadow boundary only the leading term, n = 1, is required in either Eq.(2) or (4), however up to ten terms are necessary close to, and especially prior to, the boundary. It is claimed that Eq.(2) is not applicable prior to the boundary [2], whereas Eq.(4) avoids this limitation [6]. In fact, under the experimental conditions of this work, both equations give results within 0.1dB and so no further attempt will be made to differentiate between them.

Fig.4 shows creeping wave predictions assuming an effective sound speed gradient deduced from the impulse risetimes. The example shown is one where there is a marked discontinuity in the excess attenuation measurements around the predicted boundary position of 13m. Excellent agreement with the experimental excess attenuation measurements is obtained close to the boundary and, indeed, all the way back to about 4m from the source, after which it became difficult to get the theory to converge. However, deep within the shadow zone the pulse attenuation is grossly overpredicted. Impulse waveforms are correctly calculated at all distances, Fig.5, suggesting that the frequency and phase dependence of the residue equations is correct. The theory also accounts correctly for the effect of changing ground impedance. Altering the effective sound speed gradient shifts the predicted attenuations but with little change to the slope of the line.

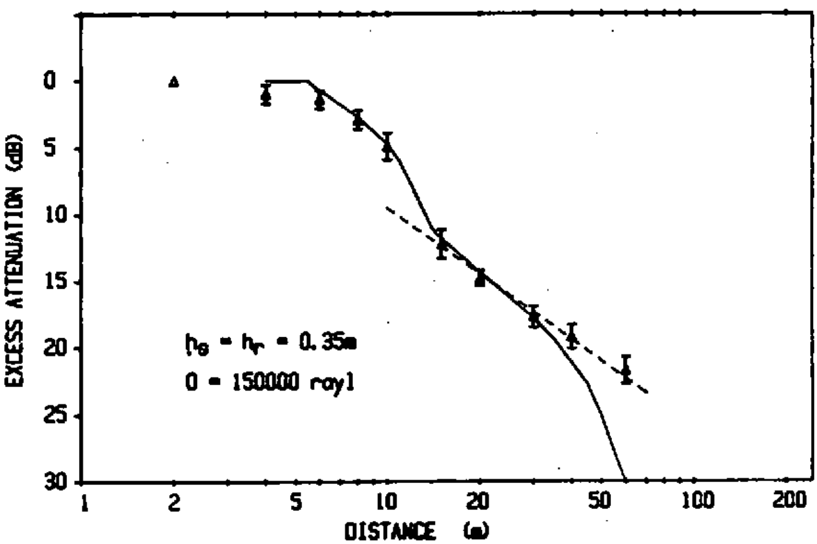


Fig.4: Creeping wave predictions compared with experiment, assuming a shadow boundary around 13m.

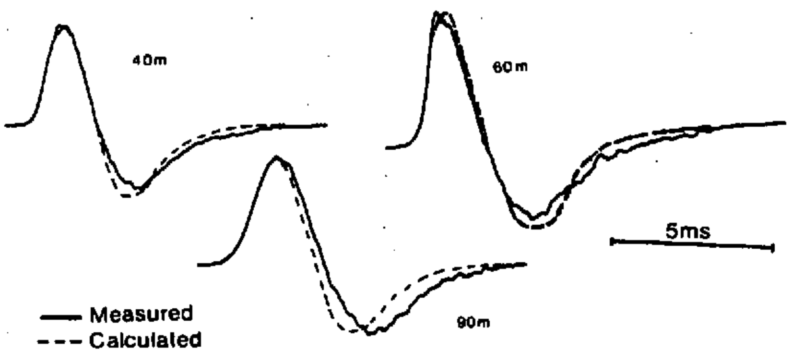


Fig.5: Comparison of experimental pulse shapes and creeping wave predictions.

The above theory assumes a linear sound speed gradient. To improve agreement between experiment and predictions, a non-linear gradient can be incorporated by extending a technique suggested by Daigle et. al. [7] to the case of a source located above the ground. Based on the experimentally measured non-linear gradient, the corresponding caustic can be deduced by using the method suggested by Van Moorhem [8]. This effective boundary is non-circular. At any given receiver distance inside the shadow zone, the circle which joins the source, the ground and the caustic at that distance, see Fig.6, is used to determine the radius corresponding to an effective sound speed gradient. This technique ignores the intercept of the caustic with the ground and results in an effective gradient which decreases with distance from the source. Using this distance adjusted gradient in either residue equation is a significant improvement, although it now under-predicts the attenuation at larger

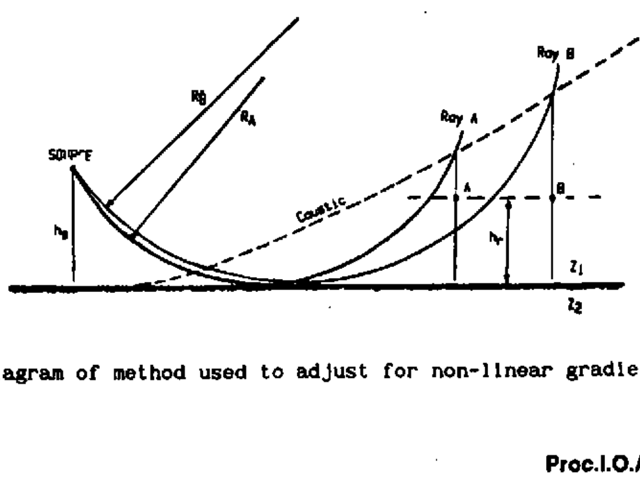


Fig.6: Diagram of method used to adjust for non-linear gradient.

distances, Fig.7, with only minor changes to the predicted waveforms deep inside the shadow zone. Improved agreement is retained if the geometry or ground impedance is altered.

When either residue solution is used to calculate impulse waveforms around the boundary using the optimum linear gradient value, it is apparent that a spike remains for several metres beyond the boundary. This contrasts with ray theory which predicts that it vanishes at the boundary. The implication is that the use of risetimes to locate the boundary is invalid. However, at most the error is only a few metres and assuming that the boundary is closer does not improve agreement with the distant excess attenuation data. When the distance adjusted gradient is used, remnants of the fast risetime remain even at 200m, because the use of the variable gradient is effectively moving the boundary out to near the distant measuring point.

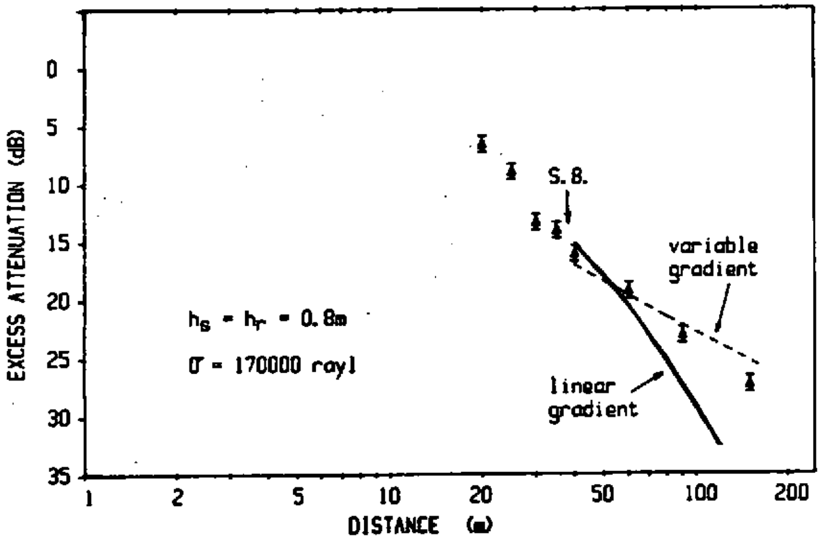


Fig.7: Results calculated using the distance adjusted gradient model.

## CONCLUSION

At this stage it appears that the fast risetime of the impulse may remain until 5m or more beyond the shadow zone, although this could be an artifact of using a linear sound speed gradient. Never the less, a boundary located by the risetime technique is consistent with discontinuities in the excess attenuation data over low impedance ground and does produce the optimum linear sound speed gradient for use in the theory. Although reasonable agreement with excess attenuation measurements can be achieved by invoking a linear gradient varying with distance from the source, this is physically difficult to justify. There is a need for a rigorous treatment of creeping wave theory with non-linear gradients to resolve the unsatisfactory aspects of the current approach.

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