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FROM ADVANCED TO SIMPLE PROPAGATION MODELS

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ABSTRACT

Computer intensive models such as SAFARI and IFD are introduced. A brief discussion of their shortcomings especially when applied to realistic environments leads to the need for propagation loss benchmarks. The techniques used to establish these benchmarks are discussed and the resulting analytical formulae provide the basis of several simple models, one of which is INSIGHT.

The high speed and analytical components in INSIGHT result in a real 'feel' for the physics by providing the ability to carry out sensitivity analysis and reconcile trials data with theory. There is also scope for dealing with finite beamwidth and bandwidth.

Simple models are therefore most useful for sonar designers, operational analysts and sonar operators because frequently robustness, sensitivity and speed are more important to the user than detail.

1. INTRODUCTION

To a sonar engineer transmission loss is just another term in the sonar equation that can be simply tabulated and then forgotten about. One might even take the wealth of propagation models as proof that calculating transmission loss is straightforward. However, nothing could be further from the truth. Transmission loss is well known to depend in a complicated way on the water column, sediment and the boundaries, but it also depends significantly on signal processing and sonar parameters such as bandwidth, pulse length and beam width because these influence the predominant paths and the potential for interference between them.

Despite the sound physical basis of the mathematical models their limited regimes of validity often result in disagreements between otherwise correctly run implementations (see Fig 1). These problems are well understood (Harrison [1]) but are, nevertheless, an unnecessary burden for the uninitiated. In addition most advanced models rely on choice of some non-acoustic parameter that requires experience or multiple runs to home in on a mathematically 'correct' solution. Most assessment problems, by their very nature, already require multiple runs with one or more parameters changed, possibly in the sonar or, more likely, simply to describe a range of environments.

2. APPROACHES

There are various approaches to these problems, with pros and cons depending on application. One is to set up some rules for where each model can be used (the logic tree of an expert system). Because there are may input parameters this is non-trivial but an approach is discussed in the next paper (Ainslie [2]).

An alternative is to construct some benchmark transmission loss plots in realistic environments against which one can check a program's performance. In this context a benchmark is a plot from a computer intensive model such as SAFARI, SUPERSNAP or IFD in which we have a lot of confidence because

a) the model was used in a valid regime
b) all disagreements between models are fully understood
c) various acoustic components can be validated against simple analytical calculations.

The last approach is to construct a simple model (most likely analytical) that can not only fill in the gaps between benchmarks but can provide a reasonable prediction in its own right. The advantage of such analytical formulae is that they are robust, well understood, and they range from fast to infinitely fast, which means they are useful for sensitivity analysis. An example of such a simple model is INSIGHT which evolved from the diagnostic 'back-of-the-envelope' calculations used to establish the benchmarks. This will be described later.

It is worth pointing out that empirical models are usually fast and robust, and of course they are in some sense 'the truth'. However they do not help at all with insight or interpolation in sonar or environmental parameters because the data on which they are based can never be extensive enough.

3. COMPUTER INTENSIVE MODELS

Before describing some simple models it is worth briefly reviewing some of the more well known numerical models and their limitations. A more detailed discussion with references is given in Harrison [1] and Jensen [3]. Operational models such as FACT and RAYMODE are not discussed because they leave the user little control over the physics.

One can group the models into several types as follows, although the first four are in more common usage than the subsequent ones.

1) Ray tracing, e.g. GRASS
2) Normal mode, e.g. SUPERSNAP
3) Green's function evaluation, e.g. SAFARI
4) Parabolic equation, e.g. IFD
5) Coupled-mode
6) Finite element

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It is natural to use whichever approach is computationally most efficient for the circumstances, and so in shallow water or in a duct where there may be vast numbers of reflected rays it is more convenient to think in terms of modes. Conversely at short range or in deep water there may be only a few 'eigen rays' connecting the source and receiver but a very large number of modes, so it is advantageous to think in terms of rays. It goes without saying that all groups are relatively slow to run, but it should be remembered that although some may be extremely slow, they may provide more output. For instance the parabolic equation automatically provides solutions at all depths out to the maximum range.

3.1 Ray Tracing

A ray trajectory can be calculated in a two- or three-dimensionally varying medium by using Snell's law and allowing for boundary reflections. The intensity is then calculated at any point by including the losses at the boundaries (which may be angle and frequency dependent) and volume absorption as well as the geometric spreading term. The latter can be calculated either from the local spacing of two initially adjacent rays or by counting the number of rays arriving in a vertical bin or window at the depth and range in question. There are obvious 'housekeeping' problems with both approaches. Pros and cons are as follows:

Pros	Cons
* No symmetry required	* Corrections required at caustics
* Familiar concept	* No diffraction effects
* Beam patterns straightforward	* Choice of ray spacing

It is worth adding that one nearly always has to resort to ray tracing when scattering from one or more targets is involved (for instance reverberation calculation) because the scattering coefficients are defined in terms of arrival and scatter angle.

3.2 Normal Mode

The Helholtz equation can be separated in range and depth assuming cylindrical symmetry and vertical stratification. The solution of the resulting one-dimensional equation in depth can be expressed as the sum of the discrete normal modes and one or more branch cut integrals. The acoustic pressure is given by

$$p = \frac{i}{4} \sum_{n} \phi_n (z_s) \phi (z_r) H_o{}^{(1)} (K_n r) \tag{1}$$

where ϕ_n are the normal modes evaluated at the source and receiver depth z_s and z_r and K_n are the eigenvalues. The modes represent up- and down-going rays travelling at an angle defined by the horizontal wavenumber $K_n = k(z) \cos \theta (z)$, and they can be calculated by standard shooting methods such as Runge-Kutta or reformulated as a more stable algebraic eigenvalue problem. Weak variations of velocity profile and water depth are incorporated into SUPERSNAP via the adiabatic approximation. The modes are evaluated at the source and receiver ranges, and the wavenumber in the Hankel function is interpolated between the values at source and receiver. Brekhovskikh and Lysanov [4] show that the Hankel function or complex exponential turns into a WKB type integral which requires the horizontal wavenumber at various ranges for it evaluation.

Pros

* Diffraction effects
* Low frequency
* Adiabatic approximation
* Surface roughness
* Shear wave effects
* Relatively robust

Cons

* Stratified medium
* Short range effects missing

3.3 Greens' Function Evaluation

Again in cylindrical coordinates the Helmholtz equation can be separated (by taking a two-dimensional or Bessel transform) into a one dimensional problem in depth. The complete solution of the inhomogeneous Helmholtz equation for a stratified medium can be written as the Bessel transform of the vertical Green's function.

$$\phi(r,z_r) = \int_0^\infty G(K, z_r, z_S)\, J_0(Kr)\, KdK \qquad (2)$$

This is the basis of SAFARI. The Green's function G is calculated for the given source and receiver depths and all possible horizontal wavenumbers by a matrix method, and then the above Bessel transform is approximated by a Fourier transform.

The distinction between the Green's function and the normal modes can be visualised by the analogy with a violin string stretched across the water column. The normal modes are the shapes of the violin string at its many resonances (the violin frequency corresponds to the horizontal wavenumber). Between resonances the amplitude is zero, and so the sequence is discrete. The Green's functions are the shapes of the violin string when driven by a harmonic source at some particular position along the length (i.e. the source depth). These exist for any frequency of the harmonic source, although there will be a large amplitude at each resonance. The effect of introducing losses into the medium or boundaries is to broaden the resonance peaks and this manifests itself in each mode as an exponential decay with range.

Since the method has to take the discrete Fourier transform of this resonance curve to get back to range space it is clear that sampling will be a problem unless the spikes are broadened. In practise broadening may have to be introduced artificially with a complimentary superimposed exponential increase with range. A rigorous description of this process is given in Schmidt [5] and the choice of broadening clearly requires some user expertise. Because of the usual Fourier transform relation between wavenumber and range there is an inverse relation between maximum range and maximum wavenumber (for a fixed transform size N). Therefore large velocity contrasts result in attaining only a small maximum range, and vice versa.

Pros

* Complete solution
* Shear effects
* Surface roughness

Cons

* Stratified medium
* Limited range or angle capability
* Sampling problem

3.4 Parabolic Equation

The parabolic equation takes a different starting point from SAFARI or SUPERSNAP. For waves travelling predominantly within a small range of angles the elliptic wave equation can be approximated to a parabolic equation by taking out the main oscillating part of the solution in a function S(r). Thus the velocity potential ϕ can be written in terms of a slowly varying function ψ of range and depth

$$\phi = \psi(r, z) S(r) \tag{3}$$

and the parabolic equation is

$$\frac{\partial^2 \psi}{\partial z^2} + 2ik_0 \frac{\partial \psi}{\partial r} + k_0^2 (n^2 - 1) \psi = 0 \tag{4}$$

where the range and depth dependent wavenumber k is written in terms of a refractive index n and reference wavenumber k_0, $k = k_0 n$. If ψ is completely defined over a vertical line at some given range then $\partial^2\psi/\partial z^2$ is known and consequently $\partial\psi/\partial r$ is known, so ψ can be calculated at the next range step $r + dr$, and so on. This marching solution clearly allows for diffraction in an arbitrary medium provided angle changes are small. Two implementations of the parabolic equation are IFD whose solution uses finite differences, and PAREQ which relies on a simple algebraic relationship between the vertical Fourier transform of ψ at one range point and the next.

One undesirable effect of running the parabolic equation in a medium where there are significant steep angle returns is that these returns are mapped into a smaller angle. Typically one see features shifted outwards in range.

Pros

* Range-dependent
* Diffraction

Cons

* Choice of reference sound speed
* Range offsets
* Angle limitation
* Compression wave only
* Choice of range/depth step size

4. SIMPLE MODELS

Simple models have been around for some time; Weston [6], in a paper of 20 years ago quotes references to the three-halves mode stripping law back to 1943. In the meantime computer intensive methods appear to have shifted emphasis away from analysis. The problem now is that computers provide detail but not necessarily accuracy, and this is the logic that led to the benchmarks already mentioned.

4.1 Shallow Water and Ducts

In shallow water or in a shallow duct it is relatively straightforward to construct a simple model (starting from a ray or a mode standpoint). For instance, in a surface duct with surface

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reflection coefficient R and velocity gradient c' one can disregard the effects of refraction on intensity (but not on cycle distance). As a result there are a number of rays with different numbers of surface bounces, all spreading as 20 log r. The resulting geometric series in R has upper and lower limits in bounce number (N, M respectively) that depend on the minimum and maximum cycle distance. It is then easy to show that the intensity is

$$I = I_o \frac{4}{r^2} R^M (1-R^{N-M+1})/(1-R) \qquad (5)$$

where the maximum number of bounces is

$$N = r/r_c \qquad (6)$$

$$r_c = 2 (2 c z_{r,s} / c')^{1/2} \qquad (7)$$

and $z_{r,s}$ is the depth of the deeper of source and receiver. The minimum number of bounces is

$$M = r/r_m \qquad (8)$$

$$r_m = 2 (2 c H / c')^{1/2} \qquad (9)$$

where H is the depth of the duct.

This formula accounts quite naturally for the transition between spherical and cylindrical spreading as can be seen by taking R close to unity. Formula (5) reduces to

$$I = I_o \frac{4}{r^2} (N-M+1) \qquad (10)$$

which is

$$I = I_o \frac{4}{r} \left(\frac{1}{r_c} - \frac{1}{r_m} + \frac{1}{r} \right) \qquad (11)$$

Using a similar approach but converting the sum over arrivals into an angle integral Weston's mode stripping formula can be derived, on the assumption that reflection loss is proportional to grazing angle (RL = $\alpha\theta$ = - 10 log R).

$$I = I_o (20 \pi/\log_e 10)^{1/2} H^{-1/2} \alpha^{-1/2} r^{-3/2} \qquad (12)$$

This law relies on the freedom of rays to travel almost horizontally (strictly isovelocity environment). In reality, of course, there will be a lower angle limit θ_m corresponding to the maximum cycle distance in a refracting environment. The effect of this is to introduce a significant decay with range through a complimentary error function.

$$I = I_o \, (20\,\pi/\log_e 10)^{1/2} \; H^{-1/2} \; \alpha^{-1/2} \; r^{-3/2} \; \mathrm{erfc}\left(\theta_m(\alpha \; r \; H^{-1} \; \log_e 10/20)^{1/2}\right) \qquad (13)$$

In a similar manner it is just as easy to incorporate a boundary loss law of the form RL = $\alpha\theta + \beta$, and even range-dependence by including the adiabatic approximation.

4.2 Deep Water

In deep water behaviour is more complicated because there are a number of different types of arrival including convergence zones, and the source and receiver are usually relatively close to the surface so that there are some large scale interference effects. The diagnostic analytical formulae that were used to check the benchmarks provide the basis of a deep water simple model (which incidentally also works in shallow water) called INSIGHT, Refs. [7-8].

The components in INSIGHT include Lloyd's mirror, surface duct, bottom reflection, bottom refraction, and convergence zone, as shown in Fig 2. For each component a coherent and an incoherent formula is available, and the components themselves may be added coherently or incoherently.

The meaning of coherent is clear from Fig 6. As is well known, the standard Lloyd's mirror formula represents beats resulting from two rays connecting source and receiver (direct and surface reflected). In this respect it is 'coherent'. The version in INSIGHT includes the distortion due to refraction in a linear surface duct. One can also construct an 'incoherent' Lloyd's mirror formula by setting the $\sin^2$ term to $\frac{1}{2}$ for large argument but the $(\text{argument})^2$ for small argument.

Similarly it is easy to see that for a fixed order of bottom reflection or refraction there are always 4 rays connecting source and receiver (with 0, 1, 1, or 2 surface bounces), and by considering source images one can write a coherent or incoherent formula for these components. Note that these formulae are strictly true for isovelocity water, but because the corresponding rays are usually relatively steep when these components are significant the horizontal offsets caused by refraction in the water are small, and in any case the offsets can be calculated as a correction term extremely efficiently.

The surface duct contribution is more difficult to calculate quickly if depth and frequency dependence are to be retained, and inevitably one must resort to a normal mode solution. Nevertheless we have developed some approaches in Harrison [8] which enable us to calculate the surface duct contribution with a formula rather than a numerical sum of modes. The incoherent version of this is still valid as receiver depth passes through source depth where one might expect problems with caustics.

5. INSIGHT EXAMPLES

An example of the incoherent components is shown in Fig 3, and on a dB scale one can mentally add them by taking whichever component is greatest. This should be compared with the benchmark (SAFARI) (Fig 4) for the same case, which was a downward refracting Atlantic-type profile at 600Hz with source and receiver at a few hundred metres depth. Considering the smooth nature of the incoherent curves one could not possibly hope for a better fit (except possibly the convergence zone at about 55km which was not modelled in this case). The Lloyd's mirror return is truncated by downward refraction leaving at first bottom reflections then, as grazing angles reduce at larger range, bottom refracted paths. One important conclusion from this and many other examples is that even with incoherent addition as is valid for, say, a broad band receiver, transmission loss certainly does not behave like spherical or cylindrical spreading, as is often assumed.

The insight and understanding provided by the model is demonstrated in the following coherent comparisons with SAFARI (Figs 5-9). Examples have been purposely chosen to compare like with like. Figure 5 (SAFARI) has isovelocity water, 4000m deep with a change in density at the bottom (to 1.92) providing an angle - independent reflection loss of 10 dB. At first sight it is by no means obvious why there is a null at about 4 km, and why there is quite deeply modulated interference out to 60 km. The simple model (Fig 6) firstly agrees extremely well, and secondly shows straight away that there are three significant components: Lloyd's mirror (at short range) and the first and second bottom reflections. Lloyd's mirror is coherent despite the appearance; the few lobes are at very short range. The reason for the large lobe shape of the bottom returns is obvious from the form of the formula and is merely a result of the incorporated surface interference (the four ray paths constituting each component). At higher frequencies there would be more lobes within the same range. The rapid interference is seen in the SAFARI plot (Fig 5) when there are two components with comparable amplitude. Modulation is most deep when amplitudes are the same. Indeed these beats between components could be calculated without too much effort although modelling them is of less interest than the gross features.

Figure 7 shows a comparable SAFARI example with the reflecting bottom replaced by a refracting bottom, i.e. a positive gradient of $1s^{-1}$ with continuous velocity at the boundary and density of one. Again SAFARI gives no clues about the gaps or the interference. The simple model (Fig 6), on the other hand, shows that predominant contributions are the first and second bottom refraction and Lloyd's mirror. Again the fit is remarkably good, and there is interference when the two components have comparable amplitudes. The reason for the gap out to about 10 km is that for shorter ranges the bottom refracted ray is forced to dive deeply into the sediment and the returning ray is shadowed by the boundary between sediment and underlying rock. The simple model includes an edge diffraction term under these conditions which results in the slight oscillations at around 15 km. Had the sediment been deeper there would ultimately have been a caustic at somewhat less than 10 km. This would have been modelled as an Airy function.

If these two environments are combined so that the bottom partially reflects and refracts the result given by SAFARI is shown in Fig 8. The more complicated beat pattern can still be explained by superimposing the earlier bottom reflection and refraction components as shown in Fig 6. Out to 10 km Lloyd's mirror and the first bottom reflection dominate, but when the bottom refracted arrival appears it has a comparable amplitude to the reflection and the small phase difference results in the slow beats seen from about 10 to 20 km. From here on the beat pattern is, not surprisingly, more chaotic because there are now four components with comparable amplitudes.

As a demonstration that the effects of refraction in the water column are small Fig 9 is for an Atlantic profile with otherwise the same parameters as Figs 6 and 8. Comparing with Fig 6 the fit is equally good except that the noticeable convergence zone return at about 55 km was not modelled in this example. The shifts in the detail of the interference patterns are unimportant.

6. SENSITIVITY ANALYSIS

The obvious approach to investigating sensitivity to the many environmental and acoustic parameters with conventional models is a daunting task because without the benefit of hindsight one is forced to try out and tabulate vast numbers of combinations. To see the magnitude of the task the very basic parameters to be varied might include frequency, source depth, receiver depth, water depth, sediment density, absorption, initial sediment sound speed, velocity gradient, bedrock velocity, sea surface sound speed and a velocity gradient every 100m down to 1000m. If these 20 parameters are allowed three values each (high, middle, low), which is extremely crude then $3^{20} = 3.5\ 10^9$ runs are required!

Simple models, and in particular, the model INSIGHT provide a neat way round this problem since the components generally have analytic form. This means that for a given set of conditions, as we have seen, one can easily find the dominant component or mechanism at ranges of interest. A glance at the formula then immediately shows what dependence to expect on any variable. For instance, returning to Fig 6, it is obvious from the simple model (but not at all obvious from SAFARI) that the section between about 3 and 10 km does not depend at all on sediment absorption whereas from 15 km onwards there is a strong dependence on absorption and frequency but only little on density. Out to 3 km there is no dependence on the bottom at all. Thus the number of cases that would actually require multiple runs to deduce sensitivity is already drastically reduced. As a bonus the simple evaluation of formulae means that multiple runs are feasible virtually in real time.

An example of the spread in transmission loss (incoherent formulae) produced by varying several parameters simultaneously is shown in Fig 10. Lloyd's mirror has source depth varying between 305 and 610m. Bottom reflections have density varying between 1.92 and 3.84. Bottom refracted paths have absorption varying between .045 and .09 dB per wavelength.

7. APPLICATIONS

Aside from straightforward calculation of loss and sensitivity there are a number of other applications of INSIGHT. One is the reconciliation of trials or exercise transmission loss data with the nominal acoustic parameters and environment. Inevitably there will be discrepancies, but the new approach to sensitivity means that it is relatively straightforward to find out which parameters can explain them.

A frequent problem with trials design is that one does not know beforehand which are the most important parameters to measure. Again the approach to sensitivity means that one can say immediately what is or is not important. More significantly one can immediately see that there may be parameters which, if not measured (e.g. sediment absorption) may completely jeopardise any comparisons of theory with experiment.

The INSIGHT model also allows a reliability or margin of error to be attached to predictions of quantities such as Probability of Detection (PoD). This should bring out the dangers of using performance metrics such as detection range.

Finally INSIGHT clearly has uses in educating sonar operators, OR analysts and research physicists alike.

8. CONCLUSIONS

There are many computer intensive propagation loss models that are capable of detailed and accurate mathematical solutions given sufficient environmental detail. Reconciliation of these models is a surmountable problem, and indeed some benchmarks for realistic environments have been constructed.

A much better understanding of the many mechanisms and components is provided by simple models. These have the obvious advantage of explicit dependence on input parameters and rapid computation. Usually new mechanisms can easily be added. Provided there is reasonable agreement with the benchmarks one can rely on simple models for sensitivity analysis.

The model INSIGHT evolved as a predictive tool from the diagnostic calculations used for checking benchmarks. INSIGHT is designed for sensitivity analysis, and its speed and robustness suggests a number of applications, other than simple loss calculation and range prediction, where most models would be impractical or impossible. Such applications include broad band sonar (finite pulse length) and reconciliation of trials data with models.

9. REFERENCES

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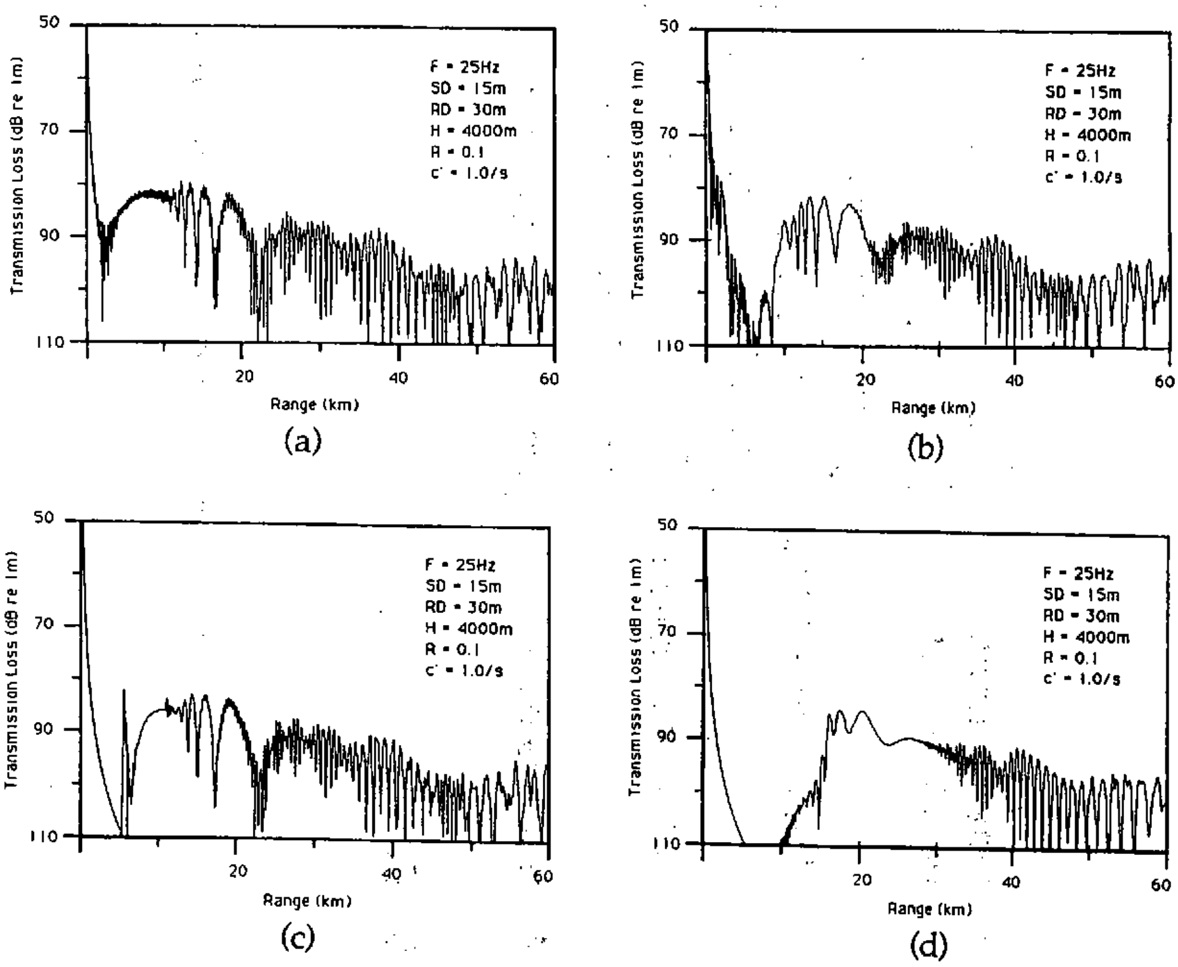

Fig 1 : Typical discrepancies between wave treatments : (a) SAFARI, (b) SUPERSNAP, (c) IFD, (d) PAREQ.

1. Lloyd's mirror (inc.refraction)
2. Bottom reflections
3. Bottom refraction
4. Surface duct
5. Etc

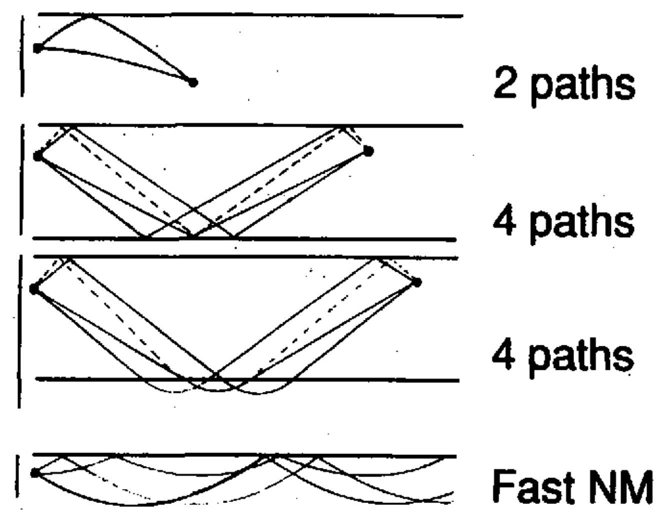

Fig 2 : Separate components derived from coherent or incoherent formulae.

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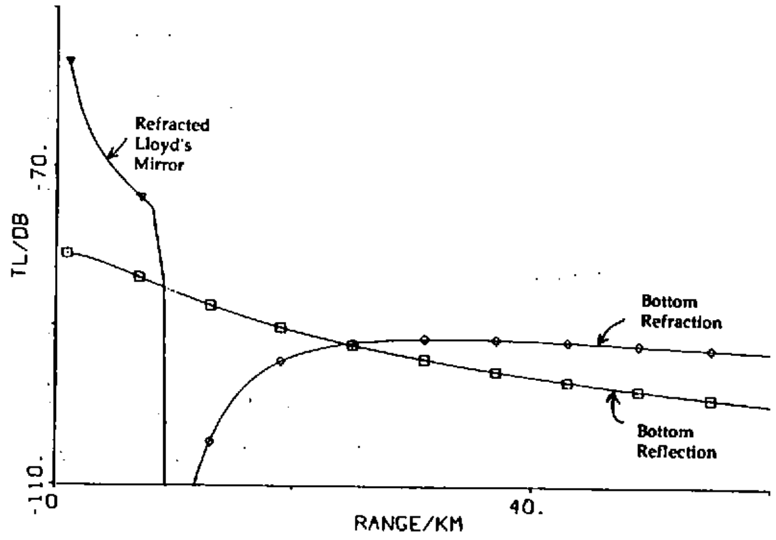

Fig 3 : Incoherent components from the simple model INSIGHT.

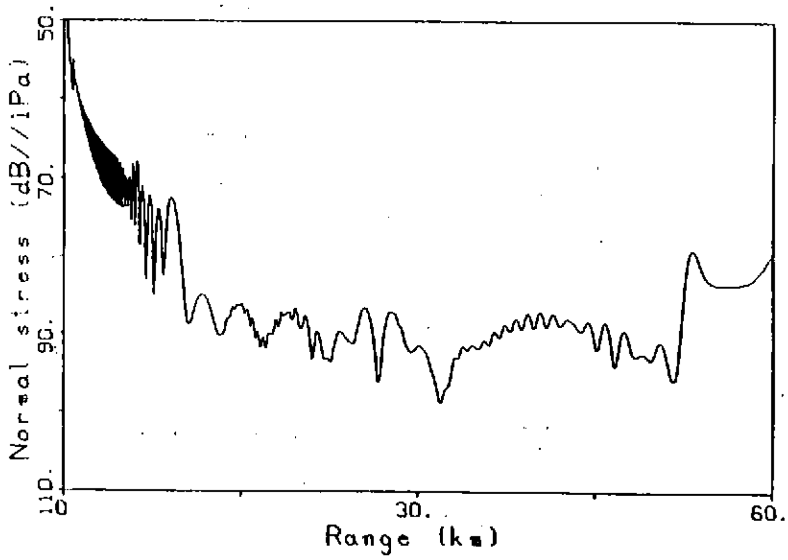

Fig 4 : Transmission loss benchmark (SAFARI) for comparison with Fig 3.

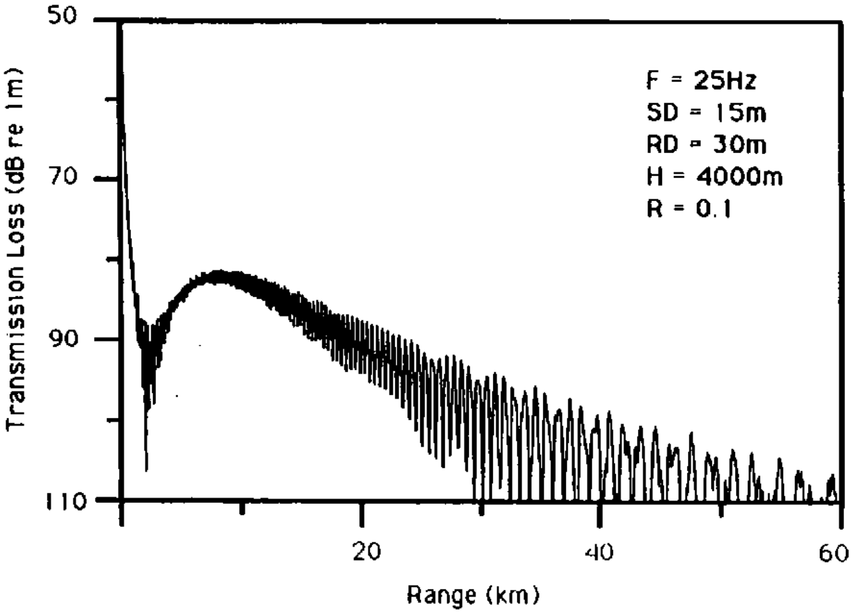

Fig 5 : SAFARI plot for isovelocity water with a bottom reflection loss of 10 dB.

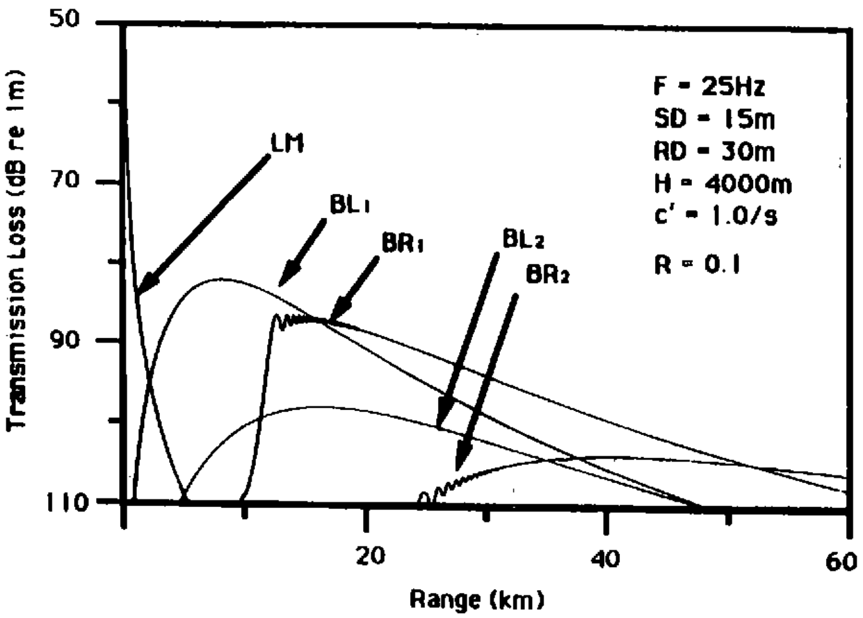

Fig 6 : INSIGHT : Lloyd's mirror (LM), first and second bottom reflection (BL), and first and second bottom refraction (BR).

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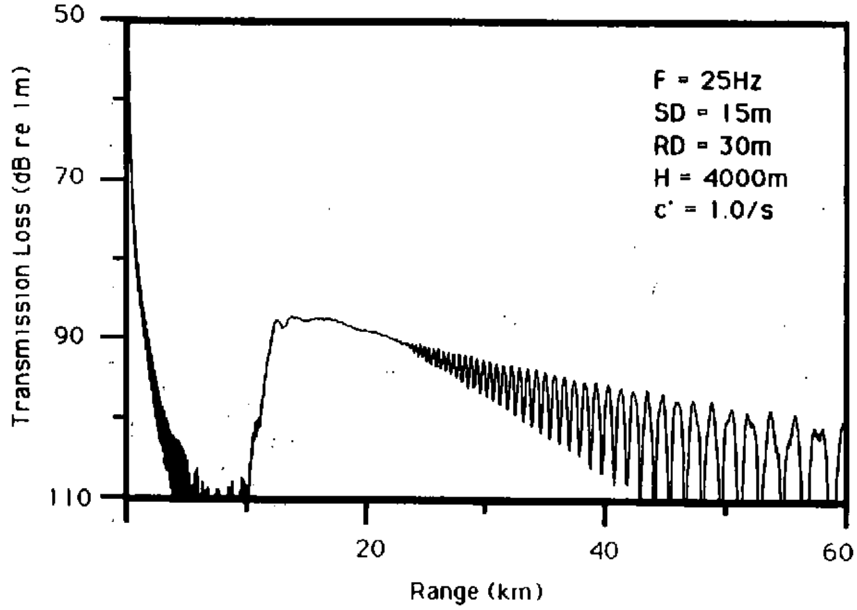

Fig 7 : SAFARI plot for isovelocity water with a perfectly transmitting refracting bottom.

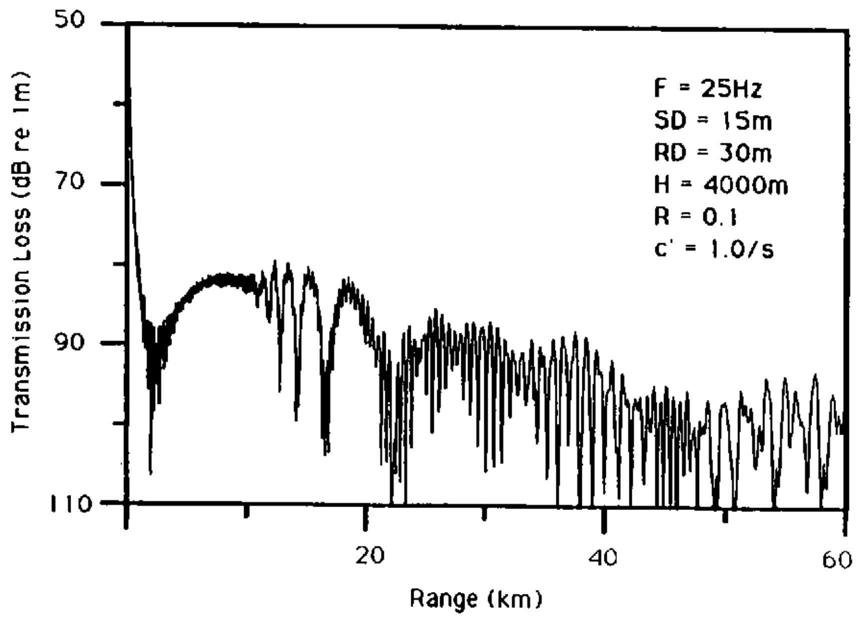

Fig 8 : SAFARI plot for isovelocity water with a reflecting and refracting bottom.

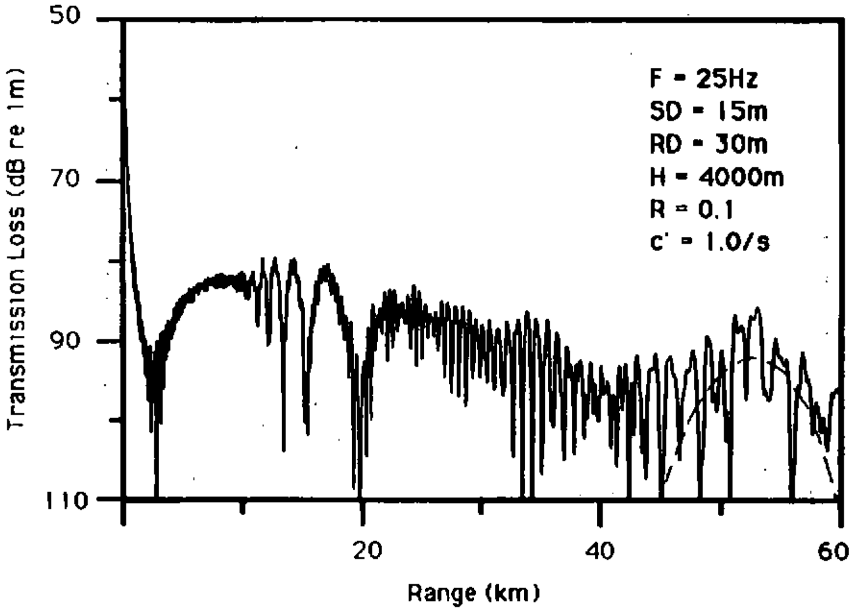

Fig 9 : SAFARI plot for the bottom environment as in Figs 7 and 8 but with substitution of an Atlantic profile in the water.

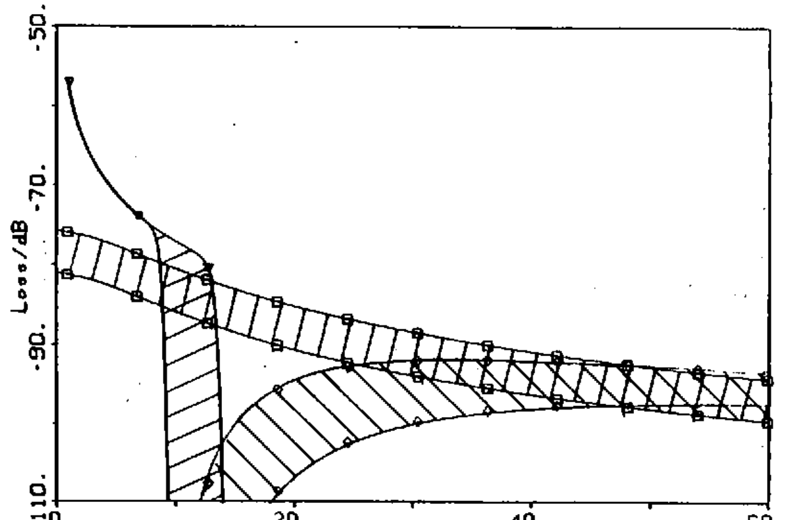

Fig 10 : Spread in transmission loss caused by varying source depth (in LM), bottom density (in BL), and bottom absorption (in BR), cf. Fig 3.