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RANGE-DEPENDENT ENVIRONMENTS IN PROPAGATION MODELLING

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ABSTRACT

Oceanographic and bathymetric range-dependence occurs on many scales in two or three dimensions. Modelling sound propagation in these environments is hazardous, and different approaches are valid in different regimes. For instance, the parabolic equation (e.g. IFD) is most reliable for shallow angle rays whereas the adiabatic normal mode approach (e.g. SUPERSNAP) is most reliable for steep ray angles. This combined with the abruptness of the environmental changes defines limits of validity in a subjective way. An attempt is made to tighten up these limits, and reference is made to comparisons between IFD and SUPERSNAP.

1. INTRODUCTION

An important requirement of propagation modelling is to be able to deal with range-dependent environments. The motivation originates partly as a simple craving for realism, but it is also partly due to the promise of new phenomena or regimes that may be put to tactical use. This, in turn, is inspired by a growing awareness of the detail in ocean circulation systems as seen through satellites added to the obvious undulations of the seabed. Above all, the available computing power makes it feasible to contemplate more complicated problems.

There are a number of methods for handling range-dependence numerically, and several of them are also suitable for analytical calculations. The purpose of this paper is to investigate the limitations of some of these approaches from a theoretical point of view but adding some examples in an attempt to narrow down the limits.

2. APPROACHES

A number of approaches spring to mind :

1. Ray tracing
2. Ray invariants
3. Flux
4. WKB phase integral (for calculating mode number)
5. WKB approximation applied to horizontal propagation
6. Adiabatic approximation
7. Coupled-mode
8. Parabolic equation
9. Finite Element method

These are by no means independent, and in fact there are many cross-relations, as has already been noted by Weston [1]. Items 2, 3, 4, 5, 6 are closely related and the adiabatic approximation is, by definition, an absence of mode coupling. Some of these approaches are in practise used solely as computer-intensive numerical methods. These include ray tracing, coupled-mode solution, parabolic equation and finite elements. However, there is no fundamental distinction between the methods; each could be the basis of a computer-intensive model or a back-of-the-envelope calculation, in principle. Nevertheless in this paper we are interested in rules of thumb and simple calculations since these lead ultimately to reduced computation time. In particular there are various ways of looking at the adiabatic approximation and ray invariants that lead to some insight into their validity. Some comparisons will be made later between adiabatic normal mode (SUPERSNAP) and the parabolic equation (IFD).

In what follows the working example is a duct bounded by reflecting sea surface and ocean bottom, but as will be seen the arguments are just as applicable to refracting rays in a range-varying duct. One can often treat the shape of the velocity contours in the same way as the shape of the bottom boundary.

2.1 Ray invariants

Purely by considering the geometry of successive bottom reflected rays it is possible (Harrison [2]) to show that in three dimensions with isovelocity water but arbitrary bathymetry the ray trajectory is governed by two ray invariants. The first is

$$H \sin\theta = H_o \sin \theta_o \qquad (1)$$

and states that as the water depth H decreases the ray's grazing angle θ increases (regardless of ray azimuth) keeping H sin θ constant. The second is

$$\cos \theta \sin \phi = \cos \theta_o \sin \phi_o \qquad (2)$$

which states that the azimuth or heading of the ray ϕ (measured relative to the downslope direction) is dependent on the grazing angle so that rays tend to bend in the horizontal plane towards deep water. Some interesting trajectories and horizontal shadow zones can be found in various basin and trough shapes (Harrison [3, 4]).

In the case where the sea bed is a tilted plane forming a wedge it can be easily shown by considering images (i.e. a kaleidoscope) that the invariants are exact if applied only at reflection points. For other geometries, though, they are an approximation, and the derivation in [2] suggests that for validity the first invariant requires

$$2\gamma \cot \theta \cos \phi << 1. \qquad (3)$$

In the up- or down-slope direction the ray angle must be much bigger than the bottom slope γ. In other directions the limit is set by the component of slope, which disappears in the across-slope direction. Thus to first order we need

$$2\gamma << \theta \qquad (4)$$

which might also be interpreted as the condition that *the ray must continue to hit the boundaries or 'cycle'*. An additional interpretation is found by dividing both sides by 2H. Now the proportional change in depth with range H^{-1} dH/dr needs to be smaller than the reciprocal of the cycle distance $(2 H \cot \theta)^{-1}$, or conversely *the cycle distance needs to be shorter than the characteristic 'scale of variation' of the sea bed* (Brekhovskikh [5] eq (7.2.27')). Whatever the interpretation, it is clear that the adiabatic approximation works best for steep rays - quite the opposite of the parabolic equation, whose paraxial approximation makes it best for small deviations from horizontal.

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An alternative derivation of eq (1) in two dimensions is that the change in ray angle $\Delta\theta$ produced by an additional bottom reflection (i.e. over a distance Δr which is equal to the cycle distance r_c) is just

$$\frac{\Delta\theta}{\Delta r} = \frac{-2\gamma}{r_c} = \frac{-2\,dH/dr}{r_c} \qquad (5)$$

with dH/dr evaluated at the reflection point. Taylor expanding the bottom profile provides a formula for the depth change between bounces ΔH

$$\Delta H = \frac{dH}{dr}\Delta r + \frac{d^2H}{dr^2}\Delta r^2/2 + \ldots \qquad (6)$$

Substituting the first order term in eq (5) with $r_c = 2H\cot\theta$ gives

$$\Delta\theta = -\Delta H \tan\theta / H \qquad (7)$$

Assuming that there are many reflections so that $\Delta\theta \to d\theta$, $\Delta H \to dH$ this equation can be integrated, and the result is eq (1). On the other hand, if the second order term is also included there will be an extra term $+ r_c\, d^2H/dr^2$ on the right of eq (7). Therefore a condition for validity of eq (1) is that this additional term must be negligible, and so there is an upper limit on the bottom curvature given by

$$|d^2H/dr^2| << |2\gamma/r_c| \qquad (8)$$

The magnitude of the curvature must be less than the slope divided by the cycle distance. This makes good intuitive sense and shows why eq (1) is exact for the wedge.

This condition can also be expressed in terms of the Fourier components of the reflecting surface. If eq (6) is extended to include all Taylor terms and then the individual terms are Fourier transformed in range (denoted by a bar) the depth change due to the slope term needs to be greater than the sum of the depth increments due to the higher derivatives. Thus

$$\left| \bar{H}(K) \sum_{n=2}^{\infty} \frac{(iK\Delta r)^n}{n!} \right| << \left| \bar{H}(K) \ (iK\Delta r) \right| \qquad (9)$$

The condition clearly relaxes for absent high spatial frequencies (i.e. $\bar{H}(K) \to 0$) but otherwise the condition is true provided

$$K\Delta r \sim < 0.5 \qquad (10)$$

(since the series is equal to $\exp(iK\Delta r)-1$). This means that *the undulation wavelength must be at least 12 ray cycle distances* or alternatively*the undulation correlation length must be twice the cycle distance.*

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2.2 Mode coupling
The sound field can also be represented formally as a system of local normal modes that are dependent on local water depth and sound speed profile, but the modal excitation at one range must be dependent, in general, on the excitation of all other modes at neighbouring ranges because the mode equations are coupled. In effect energy is scattered out of one mode into another. The coupling coefficients (Rutherford and Hawker [6]) express the degree of modal scattering, and Milder [7] has provided a criterion for the validity of the adiabatic approximation, namely that one of the coefficients which is defined as

$$B_{mn}(r) = \int_0^\infty \rho \varphi_m (\partial \varphi_n / \partial r) dz \qquad (11)$$

has an upper limit determined by a jump to an adjacent mode (i.e. n = m + 1). The criterion is

$$|B_{m,m+1} X_m| << \pi \qquad (12)$$

where X_m is the ray cycle distance for the mth mode. This may be compared with the earlier criterion for ray invariant validity by evaluating B_{mn} and X_m for isovelocity water where, with perfectly reflecting boundaries, the normalised mode shapes are

$$\varphi_m = (2/H)^{\frac{1}{2}} \sin(\pi z m/H) \qquad (13)$$

Inserting this in the earlier formula (with density ρ = 1) and integrating for O to H we obtain

$$B_{mn} = \frac{-m}{H} \frac{2n}{n+m} \frac{1}{n-m} \frac{dH}{dr}$$

$$B_{m,m+1} \approx \frac{-m}{H} \frac{dH}{dr} \qquad (14)$$

Remembering that the cycle distance $X_m = 2H \cot \theta$ the criterion is

$$\tan \gamma << \frac{\pi}{2m} \tan \theta \qquad (15)$$

Comparing this with eq (4) and ignoring the $\pi/2$ factor there is a discrepancy of m^{-1}. For large mode numbers (high frequencies, deep water) this condition is more stringent than the ray condition. In fact, bearing in mind that for the mth mode

$$m\lambda = 2H \sin \theta \qquad (16)$$

θ/m is just the angular separation of the modes (or $\lambda/2H$). So it is the mode angular separation rather than the ray angle that should be bigger than the bottom slope.

Whether this criterion is correct or over-stringent depends on your view point. At high frequencies a single ray is represented by many neighbouring modes with very similar eigenvalues and similar numbers of zero crossings. Indeed this spread of modes with its slowly changing vertical wavelength is essential to provide the ray with a 'width' or vertical localisation. The Milder/adiabatic criterion is the condition that there is no significant jumping from one mode to adjacent ones. This makes sense if we are interested in detailed interference patterns, but makes no sense if we are only interested in incoherent ray intensities, because the original mode and the mode into which it scatters both constitute the same ray. There are therefore two adiabatic criteria. One is the Milder stringent one; the other the weaker one, more akin to the ray invariant criterion.

Remembering the earlier curvature criterion it is interesting to note that in the case of a wedge shaped ocean the Cartesian mode solution is approximate and requires mode coupling. However, in the polar coordinate system with axis along the wedge axis the solution is separable with the vertical modes defined in terms of angle about that axis (Buckingham [7]) and no coupling or approximation is required. This is consistent with the ray invariant becoming exact under these circumstances.

2.3 WKB Phase Integral

The WKB approximation can be used as a means of calculating the normal modes provided the sound speed changes slowly with depth. In a refracting duct the mode shape approximates to Airy functions at the outer edges, and marrying these with the WKB middle portion results in the 'Phase Integral' (Morse and Feshback [8], 5).

$$\left(n + \tfrac{1}{2}\right)\pi = \int\left(k^2(z, r) - K^2(r)\right)^{\frac{1}{2}}dz \qquad (17)$$

This formula is just a refracting equivalent of eq (16) since k is the range and depth-dependent wavenumber ($k = 2\pi f/c$) and K is the horizontal wavenumber. The integral limits are the upper and lower point where $K(r) = k(z, r)$. As the duct varies with range the nth mode always fills out the space between these two points stretching and shrinking like a concertina. The horizontal wavenumber K can be thought of as $K(r) = k(z, r) \cos\theta(z, r)$. Here the angle θ is equivalent to a ray angle locally determined by Snell's law, but from one range to the next the horizontal wavenumber is determined by the mode number n and the variations of the sound speed k. Explicitly in terms of θ and c eq (17) is

$$\left(n + \tfrac{1}{2}\right) = 2f\int\frac{\sin\theta}{c}dz \qquad (18)$$

According to the adiabatic approximation each mode exists and propagates independently of the others. So for a particular mode we have n and f as constants which leaves $\int(\sin\theta/c)\,dz$ as constant. This is precisely Weston's invariant [9] which reduces to eq (1) for isovelocity water.

2.4 Horizontal WKB expansion

Brekhovskikh [5] derives a formula for the adiabatic normal mode solution by use of the WKB approximation in the horizontal.

$$p(z, r) = (2\pi i)^{1/2}\sum_n \varphi_n(z_s, 0)\,\varphi_n(z_r, r)\,(K_n(r)r)^{-\frac{1}{2}}\exp\left(i\int_0^r K_n\,dr\right) \qquad (19)$$

Here the modes are evaluated at the source and receiver (depth and range) only, and the intervening medium is only involved in the mode phases which are manifest in the WKB-type phase term. In an incoherent mode sum even this is absent and the solution is

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$$\left|p(z, r)\right|^2 = (2\pi / r) \sum_n \varphi_n^2 (z_s, o)\ \varphi_n^2 (z_r, r)\ K_n(r)^{-1} \qquad (20)$$

The lack of dependence on the intervening medium is reminiscent of the ray invariants. In fact it is easy to see that the mode shape at the source range obeys eq (17) (or (16)). There is a similar relation at the receiver for the same mode number, and the consequence is the ray invariant (eq (18) or (1)).

3. APPLICATIONS

As an illustration of the applicability of the adiabatic approximation some runs of SUPERSNAP (adiabatic normal mode) are compared with IFD (parabolic equation) in troughs of various depths containing isovelocity water overlying sediment of velocity 1665m/s (Fig 1).

The first is the control case with flat bottom. Figures 2 and 3 show extremely close agreement between SUPERSNAP and SAFARI as one would hope. The slightly irregular beats confirm the presence of three modes. However, IFD (Fig 4) has a clear range discrepancy increasing to about one third of a beat at 500 wavelengths. The critical angle is 27° which is enough to induce the well known 'phase' or angle errors.

In contrast the SUPERSNAP and IFD plots for the shallow trough (Figs 5 and 6) agree extremely well at all ranges. Although the critical angle is unchanged the ray angles at the deepest point where the water is 50% deeper are correspondingly reduced (according to the ray invariant) to 17°. The fact that the two plots agree so well suggests that both are correct; the angles are within the IFD limit for most of the range, and the adiabatic approximation is still valid.

Finally the SUPERSNAP and IFD plots for the deep trough (Figs 7 and 8) also agree extremely well. Here the ray angle at the deepest point (100% depth increase) is 13°. Clearly IFD is retreating from its limit, but more interestingly the stringent adiabatic approximation is still holding as the water depth changes by a factor of two. The bottom slope is of order 1° with 13° rays representing the steepest of three modes.

4. CONCLUSIONS

A number of approaches to dealing with range-dependent environments have been briefly examined emphasising the cross-relationships between ray invariants, mode coupling, the adiabatic approximation and the WKB approximation, and in particular looking at their respective criteria for validity. It appears that there are two adiabatic approximations - the first being stringent, ensuring that detailed interference patterns are correct, and the second being weaker, ensuring only that mean intensities are correct. Some examples are discussed.

5. ACKNOWLEDGEMENT

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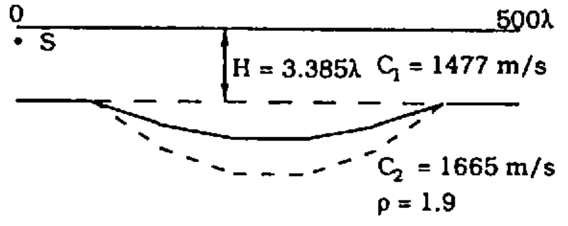

Figure 1 - Bathymetry for the following examples
1. Flat bottom. 2. Shallow trough.
3. Deep trough. Water is isovelocity

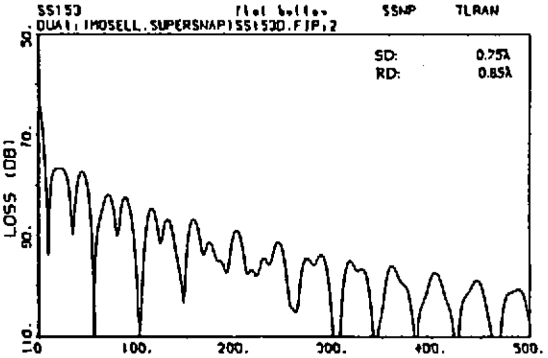

Figure 2 - SUPERSNAP for flat bottom

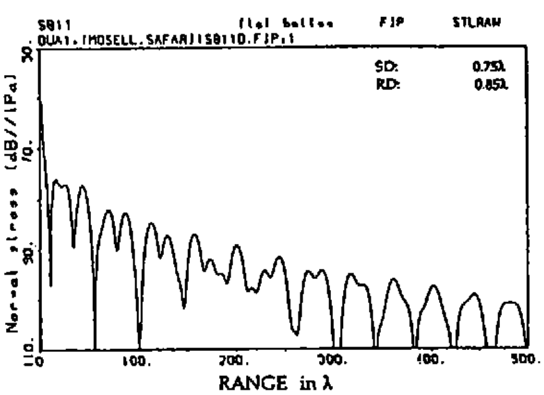

Figure 3 - SAFARI for flat bottom

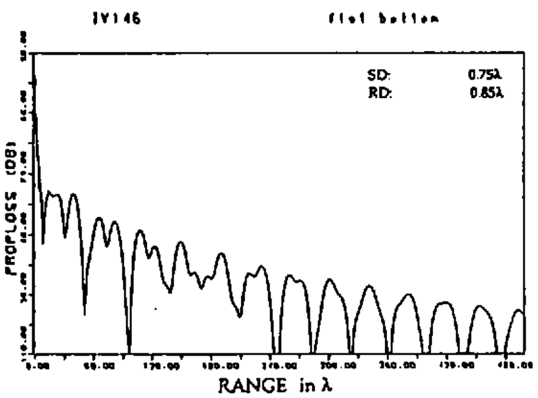

Figure 4 - IFD for flat bottom

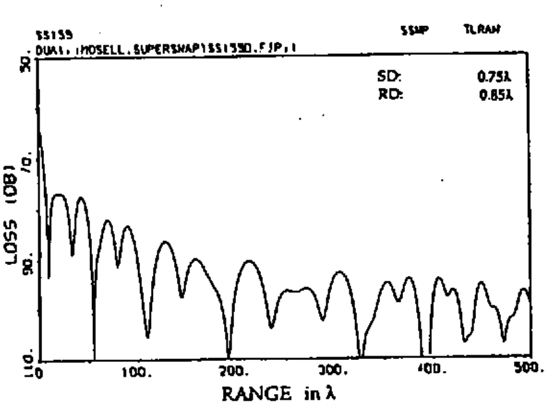

Figure 5 - SUPERSNAP for shallow trough

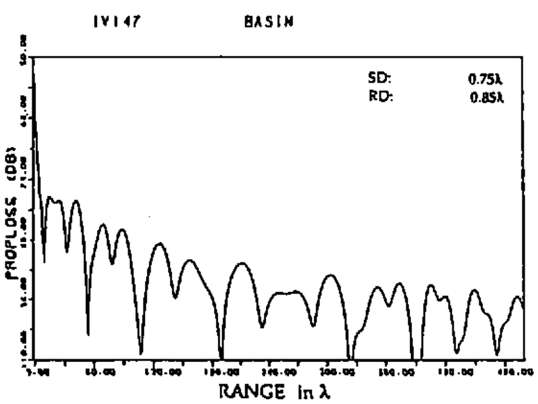

Figure 6 - IFD for shallow trough

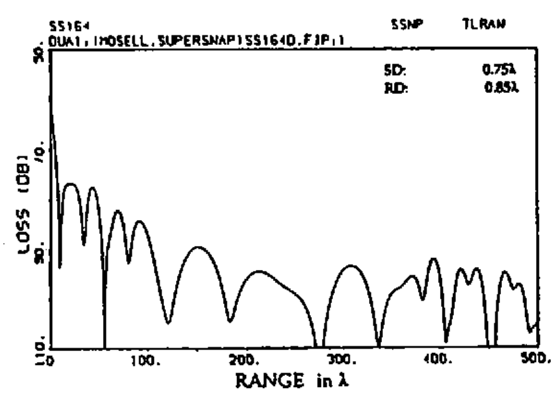

Figure 7 - SUPERSNAP for deep trough

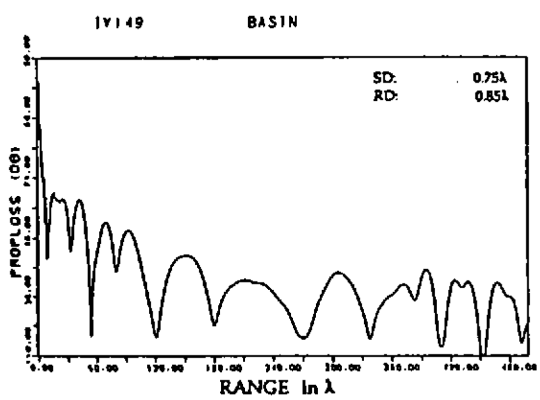

Figure 8 - IFD for deep trough