PROC. INST. ACOUSTICS VOL 7 PT 3 1985

A STUDY INTO THE EFFECTS OF BUBBLES ON SUBMERGED REFLECTORS

C H Harrison and P L Cousins

CAP Scientific, 40-44 Coombe Road, New Malden, Surrey KT3 4QN

1. INTRODUCTION

The presence of entrained air bubbles in the sea has long been recognised as important in modifying the acoustic properties of sea water and in providing a new reflecting and scattering medium. Consequently a lot of the literature has concentrated on bubble resonance, the effects of a spread in size of bubbles, scattering cross section and attenuation [1].

In practice there are nearly always uncertainties in the bubble density and the bubble size, and the properties of the medium are more easily deduced directly from acoustic measurements [2] than from measurements of other physical properties, such as photographs [3]. There is also a substantial problem in the large scale shape of the medium. The foam may form a nebulous, blobby medium with irregular edges and possibly a gradual change of properties from water to foam.

There are a number of effects that this rough boundary can have on an active sonar situated in the water. The foam/water interface can act as a curved reflector or a diffuse scattering surface with a reflection coefficient depending on the impedance mismatch. A perfect, plane reflector (e.g. a plate or the sea surface) seen through a slab of foam gives an echo which is modified by the transmission coefficient at the interface, attenuation through the foam, refraction and possibly scattering at the interface on the way in and the way out.

The problem addressed by this paper is to assess the relative importance of the various contributions from (a) a foam/water boundary and (b) a reflector behind a slab of foam. Of particular interest will be rough or smooth horizontal cylinders composed of foam or in foam near the sea surface and the comparison between volume and rough surface scattering.

This contract was supported by the Ministry of Defence.

2. ACOUSTIC PROPERTIES OF FOAM

If we assume that we can treat the foam as a continuum [4,5] we can derive all its acoustic properties including such effects as multiple scattering from the air-volume-fraction β and the distribution of bubble sizes n, and we now briefly run through this treatment. The fractional change V in an element of volume of the continuum arising from the change in volume of the bubbles alone is dV/dt, and this is related to acoustic pressure p by

$$\frac{d^2V}{dt^2} = \frac{1}{\rho}\, n \cdot g\; p \qquad (1)$$

where ρ is the density of the foam ($\rho = (1-\beta)\rho_w$) and n.g is shorthand for $\int g(f_r)\, n(f_r)\, df_r$ i.e. bubble response or source strength integrated over all resonance frequencies f_r (or alternatively bubble sizes b since $f_r = (3\gamma P/\rho_w)^{1/2}/2\pi b$).

The response g at operating frequency f is

$$g = \frac{-4\pi b}{(f_r/f)^2 - 1 - i\delta} \tag{2}$$

where δ is the damping constant.

Acoustic behaviour of the mixture now depends on the wave equation with the mass-source term from equation 1

$$\left(\nabla^2 - \frac{1}{c_w^2}\frac{\partial^2}{\partial t^2}\right) p = \rho \frac{\partial^2}{\partial t^2} V \tag{3}$$

which for a plane wave $p = \exp(ikz)$ gives

$$k^2 = k_w^2 - n.g$$

$$k_w^2 = 2\pi f / c_w \tag{4}$$

From this equation we can derive (complex) values of the wave number in foam k, and then we can calculate attenuation, scattering cross section, and reflection and transmission coefficients. We take a simple bubble size distribution

$$n(f_r) = 3.88 \; f_c^2 \beta \tag{5}$$

spread over a one octave band around a central frequency f_c i.e. $0.5 f_c < f_r < 1.5 f_c$. The factor 3.88 comes from the relation between air-volume-fraction β and the integral over all bubble volumes. Combining this with equations 2 and 4 enables us to evaluate k^2 numerically.

2.1 Wavelength in Foam

Figure 1 shows the variation of the real part of wavelength (normalised by that of water) against normalised central resonance frequency $F (= f_c/f)$. (Note that the central resonance frequency increases to the right while the operating frequency increases to the left.) Curves for two values of air-volume-fraction are shown in the graphs $\beta = 0.05$ and 0.0005. Calculations have also been made for a completely different bubble size distribution (a function of the form $(f_r^3/f_c)\beta \exp(-f_r^2/2f_c^2)$, but interestingly there is very little difference between these and the plots shown here. The variation in wavelength is quite striking. Small bubbles (large F) produce a small wavelength; large bubbles (small F) produce a sound speed close to that of water; and in the middle there is a band of frequencies ($F \sim 1$) where the bubbles are just larger than resonance size giving a high sound speed.

2.2 Scattering Cross Section

The scattering cross-section σ_s is given by the usual formula

$$\sigma_s = n . |g|^2 / 4\pi \tag{6}$$

in which we integrate over all bubble sizes. Figure 2 shows variation of σ_s against F. Cross-section is simply proportional to β, but again there is a peak for $F \sim 1$ dropping off rapidly for large or small F. The Rayleigh scattering law $f^4 b^6$ corresponds to large F (in this terminology $F^{-6} f^{-2}$).

2.3 Attenuation in Foam

The attenuation coefficient (dB per unit length) can be derived directly from the imaginary part of the wavenumber k_i since

$$\alpha = 10 \log\left(e^{-2k_i}\right) = 8.68\, k_i = .364 \, (k_i/k_w)\, f \qquad (7)$$

with α in dB/cm and f in kHz.

Plots of attenuation versus F are shown in Fig 3. The most striking feature is the severe attenuations (9 dB/cm/kHz and 0.3 dB/cm/kHz) at resonance for each of the values of β.

3. REFLECTION AND TRANSMISSION FROM A PLANE BOUNDARY

3.1 Reflection Coefficient

We assume that neglect of the back surface of the foam is justified by use of pulsed transmission.

Treating the foam/water interface as a simple impedance mismatch across a smooth plane boundary we can calculate the Rayleigh reflection coefficient.

$$R = \frac{1 - z}{1 + z} \qquad \text{where} \qquad z = \frac{\rho_w k}{\rho k_w} \qquad (8)$$

and k is complex. Figure 4 shows R against F. For small F (large bubbles) the asymptote is slightly larger than zero because although the wavenumber tends to that in water the densities still differ and $z = (1-\beta)^{-1}$. For β =0.05 we have R=0.03. For large F the asymptotic wavelength seen in Fig 1 clearly leads to an asymptote in R. In fact, the values are R=0.93 and 0.5 for β =0.05 and 0.0005 respectively. Clearly variations in reflection coefficient may approach the limits of zero or one, and they may occur in a relatively small range of frequency (or bubbles size).

3.2 Transmission Coefficient

The transmission coefficient T is also of interest since a perfect reflector behind a slab of foam is seen through the foam/water interface twice. Clearly T varies in an inverse relation with R through

$$T^2 = \frac{\rho\, k_w}{\rho_w\, k} \left(1 - R^2\right) \qquad (9)$$

It is tempting to assume that the perfect reflection is simply reduced by the two way transmission coefficient at the foam/water boundary, but this can lead to anomalous results if care is not taken over the effects of the complex wavenumber. Essentially, in the region where R is changing over from 0 to 1 its phase is non-zero (even for normal incidence), and there is strong attenuation in the medium. This means that the effects of the simple transmission coefficient are always swamped by either attenuation in the medium or multiple reflections in the foam slab (which are capable of reducing the transmission even at normal incidence).

In summary, when the reflection coefficient is low (small F) the transmission coefficient tends to one; as the reflection coefficient changes over near resonance there is strong attenuation and poor penetration; when the reflection coefficient becomes large (large F) the attenuation in the medium drops but the transmission coefficient then reduces penetration.

3.3 Gradual Changes in Velocity

It is conceivable that the transition from water to foam could be slow so that reflections are weak or negligible. Brekhovskikh [6] has given some formulae for various cases including this one, and provided

$$\frac{1}{c}\frac{dc}{dz} \ll \frac{\pi}{\lambda}\cos^3\theta$$

we have

$$R = \frac{1}{c}\frac{dc}{dz}\frac{\lambda}{8\pi\cos^3\theta} \qquad (10)$$

As an example assume the sound speed to change from 1500m/s to 750m/s in 3 wavelengths. We then have a reflection coefficient at normal incidence of -22dB. Treating the same velocity difference as a step change (neglecting density differences) we have a reflection coefficient of -4.7dB.

An alternative way of looking at this is to calculate the small step change in velocity that would give the same reflection coefficient $R = -\Delta c/2c$.

Substituting we find

$$\Delta c = \frac{dc}{dz}\frac{\lambda}{4\pi}; \qquad (11)$$

Δc is the change in velocity along the gradient in a fraction of a wavelength.

4. THE EFFECT OF ROUGH BOUNDARIES

In reality the reflecting or transmitting boundary between water and foam may not only be diffuse but completely irregular in shape and time varying. The irregularities can be viewed in many ways according to the relative size of the acoustic wavelength to the roughness height and length scales and the shape of the irregularities (which may be very poorly known). Exact scattering theories can be very difficult especially when the boundary has steep slopes. Here, we give a very much simplified approach where it is assumed that the height of the roughness is greater than a wavelength so that one can add the powers from the incoherent scatterers.

4.1 Rippled Boundary

Treating the boundary as a rippled surface the angular spread θ_s after scattering is of order twice the slopes of the surface θ_s. If the normal range to the surface is z and the pulse length is $2p$ then the area of the first return disc is $\pi p z$. The power originally falling on this area is now spread over an area $\pi\theta_s^2 z^2 = 4\pi\theta_s^2 z^2$. If the latter area is the larger, the power will be reduced by the factor $p/4\theta_s^2 z^2$. Thus the intensity I (relative to the intensity at unit distance I_o) is

$$I = I_o \frac{p}{4z^3\theta_s^2} R^2 \qquad (12)$$

If, on the other hand, the new area is smaller than the old ($\theta_s^2 < p/4z$) the intensity remains at the value for specular reflection (a glinting surface).

$$I = I_o \frac{1}{4z^2} R^2 \qquad (13)$$

For later delay times similar arguments can be used. The area of the annulus is $2\pi p r$ which means the intensity will be of the same order as the first return as long as there are steep enough facets, except that the formula depends on oblique range r rather than normal range.

$$I = I_o \frac{p}{2 r^3 \theta_s^2} R^2 \tag{14}$$

4.2 Hemispherical Bosses

If we imagine the scattering surface to be blobs of foam in water a better model might be hemispherical bosses of radius a and spacing L superimposed on a plane. For each hemisphere (much larger than a wavelength or a bubble) we have

$$I = I_o \frac{a^2}{4 z^4} R^2 \tag{15}$$

The number of hemispheres in the first return disc is $\pi z p / L^2$ so that the total power is

$$I = I_o \frac{\pi a^2 p}{4 L^2 z^3} R^2 \tag{16}$$

There is an interesting case where the hemispheres touch (L=2a), and we have

$$I = I_o \frac{\pi p}{16 z^3} R^2 \tag{17}$$

which is independent of the hemisphere radius. This is what one would obtain from a completely incoherent scattering surface if it were realisable.

At the same delay time there may be a specular contribution from the part of the plane surface in between the hemispheres. Because this comes from the first Fresnel zone where, by definition the scattering is coherent, the intensity is proportional to area squared. Therefore the remaining specular return is

$$I = I_o \frac{1}{4 z^2} \left(1 - \left(\frac{\pi a^2}{L^2}\right)\right)^2 \tag{18}$$

When the spheres touch (L=2a) the specular reflection vanishes although strictly the packing of the hemispheres in this model requires a spacing $L = \sqrt{\pi}\, a$ for zero intensity. The specular reflection rapidly returns as L is increased; it is with 3dB of maximum when the spacing is only 1.18 times the zero intensity spacing.

For later returns the intensity is given by

$$I = I_o \frac{\pi a^2 p}{2 L^2 r^3} R^2 \tag{19}$$

Dependence on p and r is as before, but interpretting a/L as slope, the slope dependence for bosses is exactly opposite to that for ripples.

4.3 Rough Cylinder

One could model a long rough cylinder of foam in a similar manner by imagining the many clouds of bubbles as hemispherical bosses of radius a superimposed on a much larger cylinder of radius b. Following the earlier method we still have intensity for one sphere given by equation 15, but the first return disc is now an ellipse of area $\pi \sqrt{pz} \sqrt{pb}$ so the number of spheres (assumed touching) is $\pi p \sqrt{bz} / \pi a^2$.

Therefore the intensity is

$$I = I_0 \frac{p}{4z^3}\sqrt{\frac{b}{z}} \quad R^2 \tag{20}$$

Note that the factor $\sqrt{pb}$ limits at the radius of the cylinder b, and so for small b we should replace $p\sqrt{p}$ by $b\sqrt{p}$ in equation 20.

Interestingly, the intensity only depends on the diameter of the cylinder and not the spheres of which it is composed. The range dependence is only marginally worse than a "flat rough" surface.

Alternatively, one can regard the return in terms of a scattering strength per unit length for a cylinder [1]

$$I = I_0 \quad \frac{S\,L}{z^4} \tag{21}$$

Equating this to equation 20 and putting $L = \sqrt{pz}$ we have

$$S = \frac{\sqrt{pb}}{4} \quad R^2 \tag{22}$$

With R^2 close to 0dB this formula gives close agreement with experimental values quoted in Urick [1].

4.4 Transmission Through a Plane Rough Boundary

There are a number of interesting cases where a plane reflector or an isolated scatterer are viewed through a slab of foam with a rough boundary. Most of these become difficult to treat exactly even as a phase screen because of the double transit of the rough boundary.

Here, we deal only with a perfect plane reflector behind a thin slab of foam. The two-way path length in the foam must be short enough so that refraction or diffraction at either boundary never shifts rays too far along the slab. The phase change on crossing the two boundaries is just twice the value for a single boundary and depends on the difference in wavenumbers between water and foam. From the ray point of view two phase screens spread the energy into twice the angle. Following an angle spreading argument similar to that in Section 4.1 we find

$$\theta_c = 2\,\theta_s \,\left|\left(c_w/c_f\right) - 1\right|$$

and

$$I = I_0 \frac{p}{4 z^3 \theta_s^2 \left(\left(c_w/c_f\right) - 1\right)^2} \quad T_A \tag{23}$$

where T_A represents the attenuation in transitting the interface and the medium. The transition from rough to coherent (the equivalent of specular) transmission is at a roughness height a, where

$$a = \left[\,2\,\left|\left(k_f - k_w\right)\right| \cos\theta\right]^{-1} \tag{24}$$

and this is usually larger than the equivalent reflection transition height (same formula without k_f). For example, a change in wavelength of 10% gives a transmission transition height 10 times the reflection transition height. However, large or, particularly, small sound speeds in foam may result in a very small transition height (see intermediate and large F in Fig 1) so that it is quite possible that specular reflections from a plate can be reduced to complete incoherence by a thin rough layer of foam. The returned pulse shape would be similar to that of the rough reflecting surfaces already considered.

5. COMPARISON OF SURFACE AND VOLUME SCATTERING

5.1 Volume Scattering Formulae

Consider a plane bounded semi-infinite slab of foam insonified from the water side. At the instant when the centre of the pulse has just reached the boundary the illuminated area is a disc of area $\pi p z$. There are three possible scattering volumes of interest. One is the low attenuation, thick slab case where it is a sector of a sphere of volume $\pi p p_f z/4$. The effective slab thickness is more or less the pulse length in foam $p_f = p \lambda_f / \lambda$. Another is the case where the layer thickness H provides the limit on depth of the volume. Lastly there is the high attenuation case where the volume is a flat disc of area $\pi p z$ and thickness the decay length $1/4k_i$.

Thus we can write for the low attenuation, thick slab case

$$I = I_0 \frac{\pi p p_f \sigma_s}{4 z^3} T_A \qquad (25)$$

whereas for a thin slab

$$I = I_0 \frac{\pi p H \sigma_s}{z^3} T_A \qquad (26)$$

For high attenuation we have

$$I = I_0 \frac{\pi p \sigma_s}{4 k_i z^3} T_A \qquad (27)$$

5.2 Comparison of Surface and Volume Formulae

Clearly one cannot simply add all the contributions, and there are some cases where it is physically impossible for two mechanisms to be important at the same time. For example, energy may be scattered from the front face and the volume simultaneously, but obtaining the maximum reflection from the front prohibits volume scattering, and similarly, obtaining the maximum scattering from the volume prohibits reflection.

Comparing the last three formulae for volume scatter with equation 12 for ripples, 16 and 17 for bosses, and 23 for transmission through a slab we see that there is a good deal of similarity. Range dependence in all cases is z^{-3}, and pulse length dependence is p^1 in all cases except equation 25. The reduction factor from specular reflection (equation 13) is of order p/z for surface scattering, but $(p/z)\sigma_s \times$(effective slab thickness) for volume scattering.

The magnitude of the factor may be quite small and one can easily imagine the ratio being 1000 i.e. -30dB, so that the effect of a rough surface cannot be neglected.

The reason for this similarity is that behind each element of area there is a column of volume which will scatter in just the same way as the elementary area does. In fact, because of the possibility of volume absorption it is safe to say that the volume scattered intensity (near the first return) can never exceed the maximum possible surface scattered intensity i.e. that given by a perfect incoherent scattering surface. Smoother scattering surfaces (equation 12) will of course give a stronger signal still.

6. CONCLUSIONS

The acoustic returns from clouds of foam and from reflectors enveloped by foam are difficult to compute exactly. We have presented some simple formulae which are intended to give some insight into the importance of the various reflecting and scattering mechanisms.

From numerical plots of the variation of wavelength with central resonance frequency and operating frequency (whose ratio is F) we have derived the reflection coefficient at a water/foam boundary and the attenuation and scattering cross section in the foam. Near bubble resonance there is strong attenuation in the medium and the reflection coefficient changes from a low value to a high value (being high at large central resonance frequencies or low operating frequencies).

The effect of surface roughness can be important in reducing the strength of echoes from the foam/water boundary and particularly in turning a perfect plane reflector within the foam into an apparently rough one. It is shown that for most practical purposes range and delay time dependence is identical for surface and volume scatter although pulse length dependence may differ and frequency dependence will certainly differ. At low operating frequencies (compared with central resonance frequency) interface reflection will dominate, and the boundary will tend to appear smooth. Conversely, a perfect, plane reflector will be seen through foam best at high frequencies but may appear rough. Volume scatter will be strongest in a band near the central resonance frequency.

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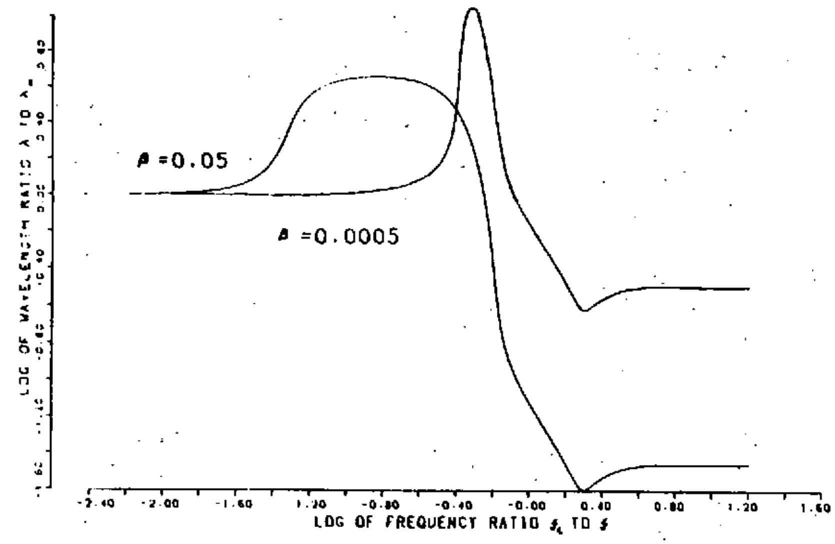

Fig 1 Log of wavelength versus frequency ratio F

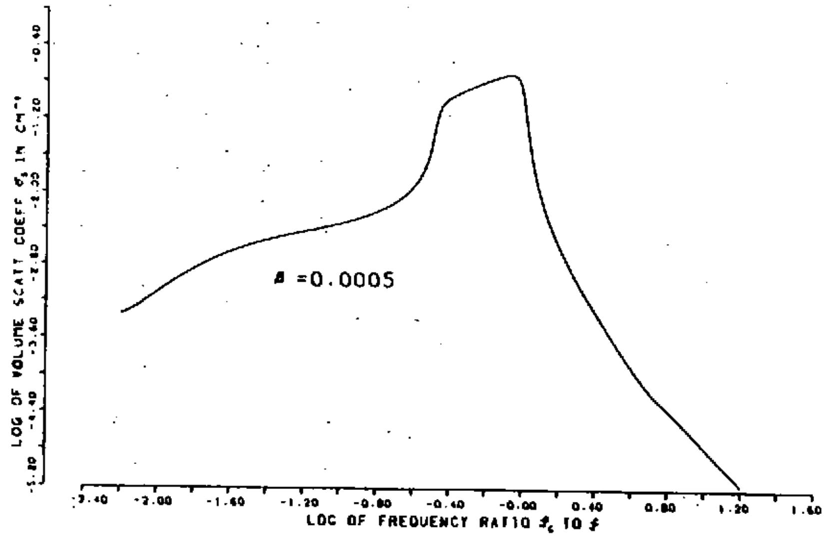

Fig 2 Log of volume scattering coefficient versus frequency ratio F

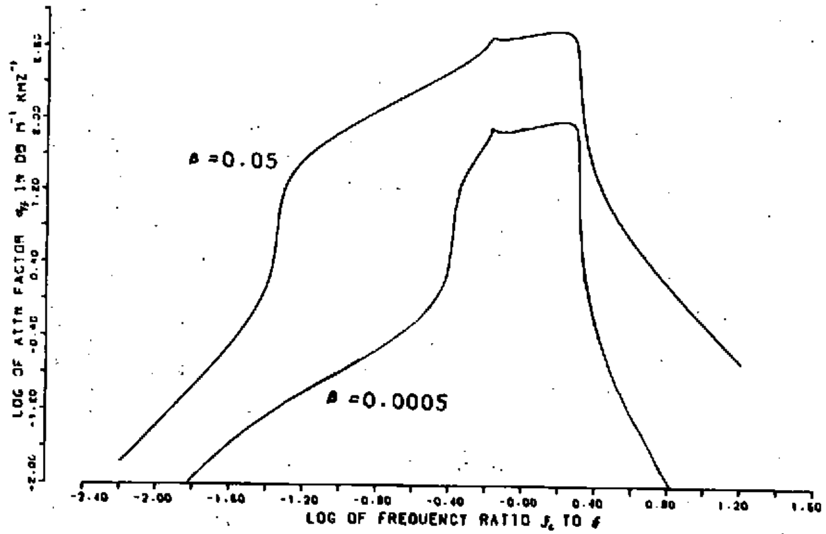

Fig 3 Log of attenuation factor versus frequency ratio F

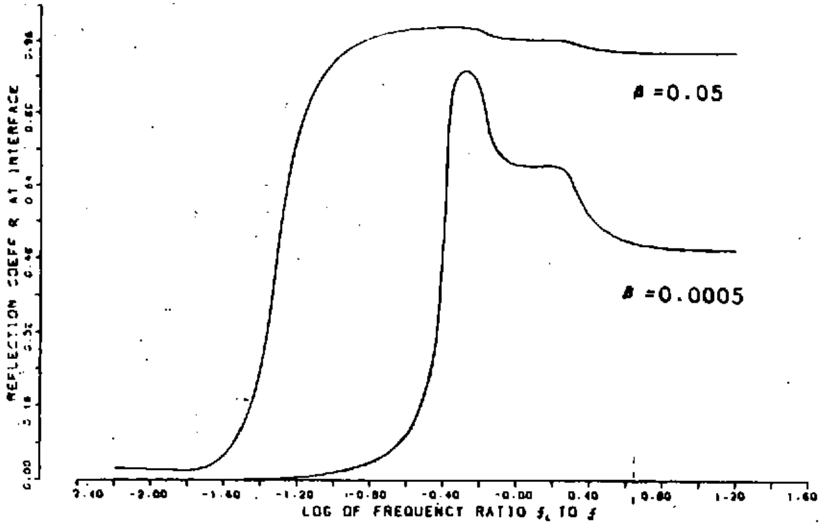

Fig 4 Reflection coefficient versus frequency ratio F