THE EFFECT OF INLET GUIDE VANES ON THE SPECTRUM OF ROTOR NOISE DUE TO INLET TURBULENCE

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INTRODUCTION

The problem of the noise generated by turbulence incident upon an aerofoil has been the subject of a large number of papers by many authors and the main characteristics of the noise are now well established. The early work of [1] found that as the factor (Λ/s) decreases; where Λ is the turbulence length scale and s is the pitch, the broad peaks at blade rate and its multiples broaden and the broadband noise increases in level. Simply using the parameter (Λ/s) ignores the relative effects of axial and tangential velocity and a rather better factor is $(DV_a/\Lambda V_r)$, where D is the diameter and V_a and V_r are the axial and rotational velocities respectively, as discussed in [2]. In general if this factor is small then there are broad peaks at blade rate and its multiples associated with the unsteady lift being correlated from blade to blade. For smaller scale turbulence at higher frequencies this factor becomes larger and the blades behave independently giving rise to broad band noise uncorrelated between blades. The work of [3] specifically includes inlet guide vanes in an approach based on describing the noise in terms of pulse modulation, however, the main effect of the inlet guide vanes which is included is the turbulent wake. The broadband noise aspect of the problem has been considered in [4] where the radiation from a single aerofoil in a turbulent stream is considered.

The work of [2] is primarily concerned with hovering helicopter rotors and in such a case significant elongation of the turbulence sucked through the rotor can occur. The work of [3] is concerned with ground testing of jet engines where again the turbulence entering the rotor may be significantly anisotropic as a result of the sucked flow. In both these examples the turbulence becomes elongated along the direction of the mean flow; this leads to a broad peak in the spectrum symmetrical about the blade rate line. The present paper is a modest extension to previous work to include the effect on the noise of the turbulence being non-isotropic and elongated in a direction not aligned with the mean flow. It will be shown that such turbulence can lead to a similar broad peak in the spectrum but in this case the centre frequency of the peak is displaced from the blade rate line. For the sake of simplicity the present analysis assumes that the rotor is acoustically compact so that the integrated force on the rotor gives rise to a simple dipole source term. The Dryden spectrum will be assumed for the turbulence and the blade response will be taken to have a two-dimensional form, the spanwise wavenumber being ignored.

THE ANALYSIS

The main elements of the problem are illustrated in figure 1. A flat blade at zero incidence and pitch angle ε rotates with a velocity V_r into a flow which is swirled at an angle ψ to the axis. The inlet flow is assumed to have variations and unsteadiness in both the X and Y directions. As a result of such unsteadiness the gust velocity normal to the blade is given by the expression

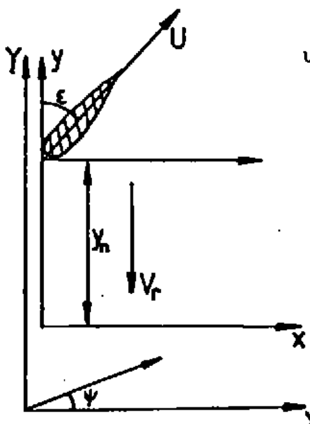

Figure 1 The geometry for the analysis.

$$u_g = u(Y)\sin\varepsilon - v(Y)\cos\varepsilon \qquad \text{-------------(1)}$$

where u(Y) and v(Y) are the x and y components respectively of the deviations of the flow from the mean value. Each of these velocities can be written as a double transform in wavenumber and frequency. If the spectrum of u(Y) is $a(k_y,k_x,\omega)$ and that of V(Y) is $b(k_y,k_x,\omega)$ then the gust velocity can be written in the form

$$u_g = \int\int \alpha(k_y,k_x,\omega) e^{i[k_y Y + k_x X - \omega t]} d\underline{k}.d\omega.$$

$$\text{where } \alpha = a\sin\varepsilon - b\cos\varepsilon \qquad \text{---------(2)}$$

If the flow were steady but circumferentially non-uniform this would correspond to the case $\omega=0$.

Transforming to the axes corotating with the blade using the relationships

$$Y = y + y_n - V_r t \qquad \text{and} \qquad X = x \qquad \text{-------(3)}$$

gives the velocity in the form

$$u_g(x,y,t) = \int \alpha(\underline{k},\omega)\exp\{i[k_y(y+y_n)+k_x x - (\omega + k_y V_r)t]\}d\underline{k}d\omega \qquad \text{-------(4)}$$

From which it can be seen that the effective frequency on the blade is $\omega_e = \omega + k_y V_r$. The full wavenumber-frequency spectrum $\alpha(\underline{k},w)$ is somewhat general for the purposes of this paper so I will assume that it is separable and that the frequency component is described by the assumption of frozen convected turbulence.

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The lift per unit span at a point(x.y) on the blade is given by an expression of the form

$$\Delta p(x,y,t)=\pi\rho Uc\int K(x,y;k_y.k_x)\alpha(k_y.k_x)\delta(\omega-\underline{k}.\underline{V}) \\ *\exp\{i[k_y(y+y_n)+k_x x-(\omega+k_y V_r)t]\}d\underline{k}d\omega \qquad \text{--------(5)}$$

where K is the gust response function of the blade. This could be that appropriate to Sear's analysis or the steady state form. Expression (5) gives the chordwise lift distribution in response to a defined gust, however, because of the stochastic nature of the inflow it is more appropriate to consider the lift cross-correlation function, that is

$$R_{12}=\langle\Delta p(x_1,y_1,t_1)\Delta p(x_2,y_2,t_2)\rangle$$

for the nth blade. The total lift cross correlation is then given by the integration over the blade area and summation over all the blades. Using the ergodic hypothesis the ensemble average is replaced by a temporal integration. If the blades are taken to be identical then the blade summation gives an expression of the form

$$\sin(k_y\pi r)/\sin(k_y\pi r/N) \qquad \text{-------(6)}$$

This function has a peak value of N when k_y= N/r and subsidiary zeros at k_y= q/r, where q is an integer. These values correspond to the allowed wavenumbers if the condition of circumferential periodicity is imposed, consequently if such a condition is imposed the only values of k_y allowed are at multiples of N/r. The integration over the blade yields a lift response function in terms of the wavenumber along the blade, that is $(\omega+k_y V_R)/U$. For example this lift response function for an isolated blade would be the Sears' function. Changing coordinates in k-space to $k_\parallel$ and $k_\perp$ that is the wavenumbers parallel to and normal to the blade. one finally obtains the following expression for the lift spectral density per unit span by Fourier transforming the lift auto-correlation.

$$\Phi_{11}(\omega_r)=(\pi\rho Uc)^2\left|L\left(\frac{\omega_r}{U}\right)\right|^2\int|A(k_\perp,\omega_r/U)|^2\ \Phi(\omega_r/U,k_\perp)dk_\perp \qquad \text{----(7)}$$

where L is the lift response function which may take the form of the Sears' function or a quasi-steady form. A is the cascade response function given by (6) and Φ is the spectral density of the turbulence projected normal to the blade. Equation (7) is a purely two dimensional result and does not include the spanwise wavenumber. The above derivation is very similar to that of reference [1] but differs in the explicit inclusion of the cascade response function A. It is this function which gives rise to the particular features to be discussed. In the absence of A equation (7) reduces to the results discussed in [2] and [4] where blade interaction terms were not appropriate. Because A is purely a function of k_y, its presence within the integral gives the k_y axis a preferred status. Thus, even if Φ is isotropic, rotating the wavenumber axes does not produce the simplification in the analysis which occurs in, for example, [2].

TURBULENCE INDUCED FORCES ON A ROTOR

Figure 2 is a sketch representing some of the terms which contribute to equation (7). The value of $k_{\parallel}$ gives the effective frequency (ω_r) and the integration in (7) is along lines parallel to the $k_{\perp}$ axis at the appropriate value of $k_{\parallel}$. The intersection of the first peak in the function A with the line ω=0 gives the value of k equivalent to ω_r =2πnN i.e. blade rate. The line ω=0 represents the wavenumber appropriate to the circumferential distortions of the steady flow. Because A is such a sharply peaked function the dominant contributions to the integral in (7) are from the intesections of the peaks in A with the lines at constant k . Thus, the contribution at the fundamental blade rate comes from the intersection of the line 1-1 with the peaks in A. If only the steady flow is considered then the only effect is from the value along the line ω=0. The contribution to the unsteady force due to turbulence at a particular frequency ω_r is given by an integration along a line such as 2-2 which is drawn

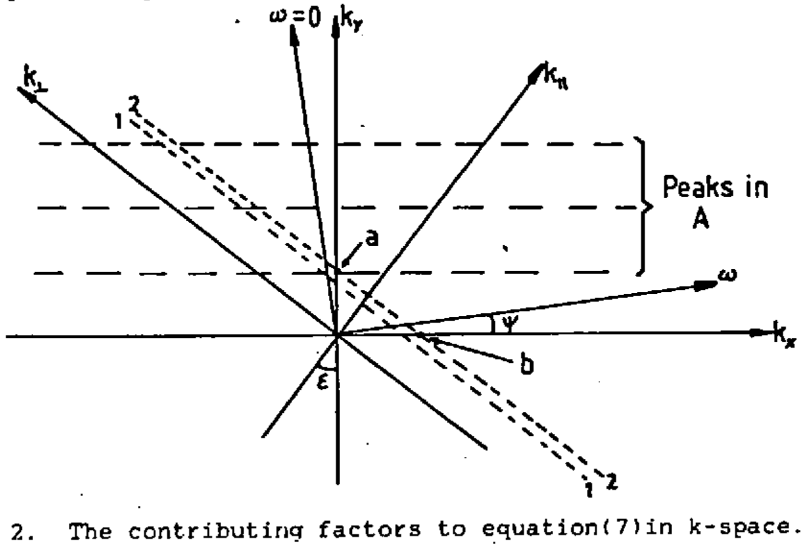

Figure 2. The contributing factors to equation(7)in k-space.

for the particular case of the intersection of the fundamental peak in A with the k_y axis. This illustrates the fact that if the turbulence spectrum is peaked along the k_y axis then the peak in the turbulence induced spectrum will be at a higher frequency than the steady flow induced blade rate peak. Such a turbulence induced peak will only occur if the contributions from each peak in A do not overlap as a result of the breadth in the turbulence spectrum. For example, the contribution from the point(b) must be significantly less than that from the point(a) if a distinct turbulence induced peak is to be seen. To express this interms of the wavenumbers, a peak will only occur if the following inequality is satisfied:

$$k_x \tan\varepsilon \ll k_y$$

where k_x is the width in wavenumber space of the turbulence spectrum and k_y is the first peak in the function A. Written in a different form this becomes

$$\Lambda \gg (r/N).(V_a/V_t) \qquad \text{-------(8)}$$

where Λ is the turbulence length scale. This is very similar to the form already quoted from [2] except that in equation (8) the transverse velocity V_t includes the swirl velocity in addition to the rotational component. There is an implicit assumption that the turbulence has a sufficiently small scale in the y-direction. For the sake of simplicity it will be assumed that the turbulence spectrum is separable and of the Dryden form resulting from an exponential correlation function in each of the x and y directions. Now, because of the very sharp peaks in A, it can be regarded as a sequence of delta functions at $y = y_n$ so that the integral in equation (7) can be replaced by a summation to give

$$\Phi_{11}(\omega_r) = (\pi\rho c N)^2 U |L(\omega_r/U)|^2 \Sigma\Phi_x((\omega_r/U\sin\varepsilon) - k_{y_n})\Phi_y(k_y) \quad (9)$$

where N is the number of blades. For the Dryden form of the spectrum Φ_x will peak when $k_y = \omega_r/U\cos\varepsilon$ which gives the peak turbulence level at

$$f_r = Nn + \frac{NV}{2\pi}\frac{\psi}{r}$$

This is shifted above blade rate by the amount $NV_\psi/2\pi r$. The width of the peak is determined by the width of the turbulence spectrum which in this approximation is given by the expression

$$\Delta f = 2V_a/\Lambda_x$$

It is also clear from equation (9) that the amplitude of the peak is a function of the turbulence spectrum in the y-direction and if this decays sufficiently rapidly then the peak is not seen.

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The sort of unsteady force spectrum that may result from turbulence of the appropriate form is illustrated in figure 3.

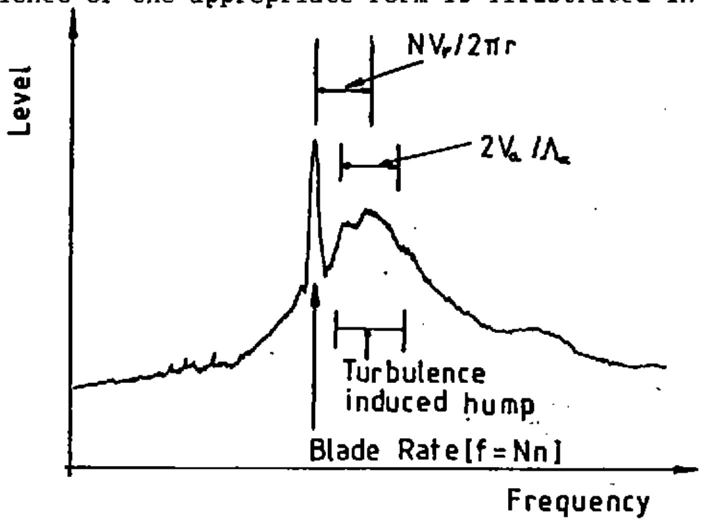

Figure 3. A typical spectrum in the presence of swirl.

DISCUSSION

The preceeding analysis is clearly highly simplified and is intended merely to illustrate a particular feature that may occur in the spectra of turbulence induced rotor noise. For a spectrum of the form of figure 3 to result it is clear that the turbulence must have a specific form. It must be elongated along the axis of the rotor and be sufficiently narrow in the circumferential direction and still be convected with the mean flow in the swirled direction. If the turblence were simply elongated along the swirled mean flow direction then the induced hump would be symmetrical about the blade rate peak. Such a specific form for the turbulence is unlikely to result purely from convection of inlet turbulence through the guide vanes and is more likely to result from secondary flow effects introduced by the guide vanes themselves. The details of such effects fall outside the scope of this short paper. Other cases besides that considered here are possible; if the angle of elongation of the turbulence were greater than the swirl angle to the axis then the turbulence induced hump would appear lower in frequency than the blade rate line. In effect the alignment of the non-isotropic turbulence with an axis not coincident with the mean flow direction introduces a doppler shift in the turbulence induced spectrum. In a more detailed analysis fuller account would have to be taken of the form of the cascade function A and of any three-dimensional effects, however, such effects would unnecessarily complicate an analysis in which the fundamental quantity, the turbulence spectrum, is extremely uncertain.

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