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REAL TIME ANALYSIS.

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Analysis of Fan Noise in Ducts.

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Introduction

The generation and propagation of sound in ducts is conveniently described in terms of duct modes or eigenfunctions. These are the natural sound pressure variations and each mode can be considered to be independent and have its own individual behaviour as a function of frequency in a simple duct with no flow or plug flow. At low frequencies only modes with low spatial frequencies (long wavelengths) can propagate; all other modes become attenuated quickly in the duct. As the frequency increases, more modes propagate and so the effectiveness of the source increases. Each mode when it reaches the end of the duct has its own individual behaviour which depends on frequency. Part of the mode is radiated in a particular pattern and part is reflected from the termination.

If the amplitudes of the modes in the duct can be determined, it is possible to obtain useful information about the behaviour of the source and also the duct radiation.

The discussion in this paper will be mainly concerned with the annular ducts associated with axial flow fans, but similar analysis may be made on rectangular and other shapes of duct. The eigenfunctions for an annular duct are of the form (Ref. 1) such that the pressure at any position is given by

$$p_{\theta rt} = \sum_{m} \sum_{n} \sum_{\omega} p_{mn\omega} \left\{ J_m(k_{mn}r) + Q_{mn}Y_m(k_{mn}r) \right\} \epsilon^{i(m\theta+\omega t)} \quad \ldots\ldots (1)$$

Thus in the circumferential direction they may be characterised by travelling waves with m cycles and in the radial direction by Bessel functions of the first and second kind. k_{mn} and Q_{mn} are the eigenvalues given by solving the duct boundary conditions and r is the radius normalised on the outer duct radius.

The problem is to measure sound pressures in the duct and from them determine the values of the complex quantities $p_{mn\omega}$. There are many ways of performing this analysis, the method chosen in a particular case depending on what else is known about the source. The frequency spectrum may be discrete or continuous. If it is discrete, a single microphone may be traversed to many places within the duct. If it is continuous, either many microphones must be used or else the statistical behaviour may be obtained from cross-correlation of two microphone signals (Refs 2 and 3). When the measurements are made at a large distance from the source, only a certain range of modes will be present at each frequency and the amount of data to be collected is correspondingly reduced.

The values of $p_{mn\omega}$ are valid for only one axial plane. In general there will be waves travelling in both axial directions. These may be separated by processing the values of $p_{mn\omega}$ obtained in at least two different axial planes in a duct of uniform cross section.

Analysis of pure tones

If the source consists of only pure tones it is possible to measure the amplitude and phase of the frequency components at each duct position sequentially using an rms meter and phase meter and then transform the data as described later. More often, the tones which are required are buried in broad band noise. In this case a process of time domain averaging may be used to separate the repetitive signal from the total signal provided that a reference signal is available. This technique is particularly useful for fan noise analysis where the propagation of harmonics of shaft order is to be studied (Ref. 4). A series of samples of the signal is taken following each pulse obtained from a transducer once per shaft revolution. These samples are then averaged for corresponding delays as shown in Fig. 1. The result is the part of the waveform which is caused by the repetitive pressure field of the rotor.

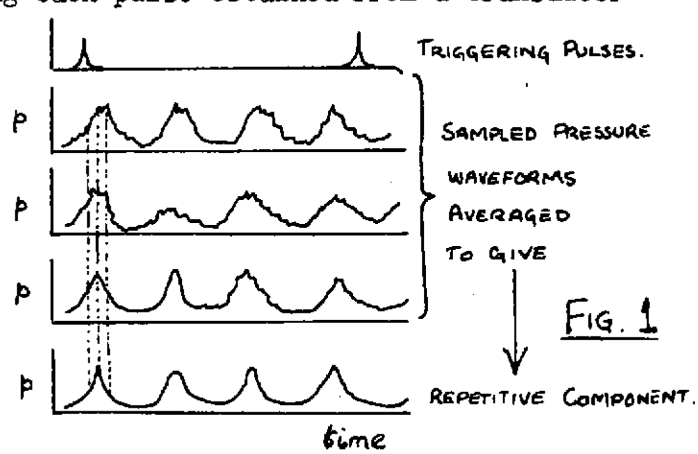


FIG. 1

If measurements are made close to fan blades the pressure profile of each blade is determined. This information from any point in the duct may be transformed into the frequency domain by simply Fourier transforming the waveform to yield the magnitude and phase of each harmonic of shaft order. This process is conveniently performed using a fast Fourier transform routine on a computer.

Circumferential transform

The temporal data, where obtained from phase and rms meters or by an averaging technique, results in an array of sound pressure amplitudes and phases which may conveniently be represented by phasors (complex numbers having magnitudes and phases equal to the magnitude and phase of the sound pressure harmonic). Eqn 1 may be rewritten in the form

$$p_{\theta r\omega} = \sum_{m=-\infty}^{\infty} p_{mr\omega}\, \epsilon^{im} \qquad \ldots\ldots \quad (2)$$

Thus $p_{\theta r\omega}$, which has been measured, is the Fourier transform of $p_{mr\omega}$ which is the next step in obtaining $p_{mn\omega}$. Thus by transforming $p_{\theta r\omega}$ at each radius we can obtain the amplitudes and phases of the circumferential modes at each radius.

This process may also be performed using a fast Fourier transform computer routine, but this time the input is complex instead of real for the time domain transform. With a real input, the outputs for negative frequencies are the complex conjugates of the values for the corresponding positive frequencies. The outputs for the spatial transform has independent positive and negative frequency components. These correspond to modal components travelling in opposite circumferential directions.

A problem which occurs with the circumferential transform as it does in the temporal transform is aliasing. The fact that complex numbers are transformed adds some understanding to the true nature of aliasing. To obtain a frequency transform of a temporal signal, at least two samples must be taken during every cycle of the highest frequency present. If this is not so, aliasing occurs and the signal is reflected about half the sampling frequency as shown in Fig. 2 producing an incorrect spectrum. In the transform required for circumferential mode analysis, two measuring positions are required on each wavelength of the highest order mode present. If not, aliasing occurs, but this time the true nature of aliasing is apparent. A positive mode beyond the equivalent Nyquist frequency produces a negative mode. In fact ghost spectra are generated displaced from the original spectrum by a complete sampling frequency as shown in Fig. 3.

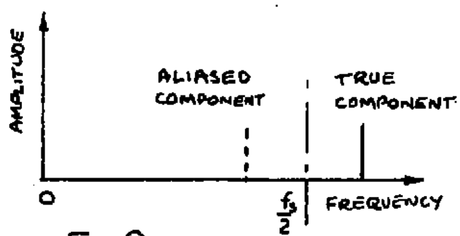


Fig. 2.

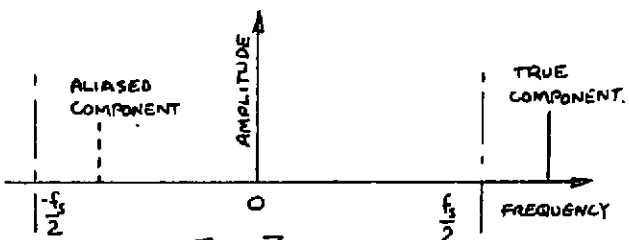


Fig. 3.

Radial transform

The remaining transform is in the radial direction. For each value of m and ω there is a series of radial functions represented by the expression in parenthesis in Eqn 1. These functions are orthogonal as are the circumferential functions and can be separated by multiplying the actual distribution by the shape of the mode required and integrating with respect to r^2 over the duct radius.

A more accurate method of analysis which is more suited to digital techniques is to write the circumferential mode phasors at each radius in terms of $p_{mn\omega}$ and solve the resulting set of simultaneous equations. The number of equations required, and hence the number of measuring radii, depends on the number of radial modes which can be present for particular values of m and ω. This method yields a greater accuracy than the previous method.

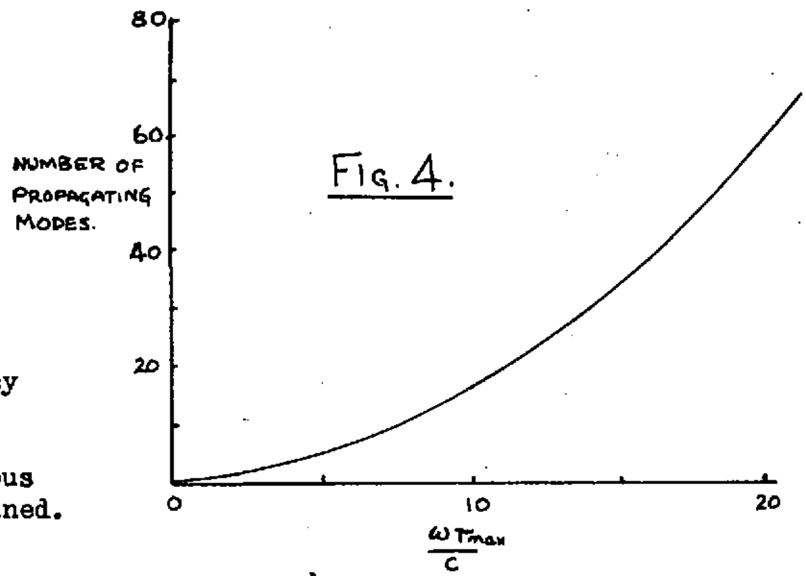
Continuous spectra

When the spectrum of the source is continuous, consisting of random noise or randomly modulated tones, the sound pressure phasor can no longer be meaningfully defined for all temporal frequencies. If the instantaneous behaviour of the modes is to be observed a large array of microphones must be sampled simultaneously and processed to determine the instantaneous modal distribution which can finally be analysed in the time domain. This technique requires an extremely fast rate of data acquisition and would at present be very difficult and expensive to implement digitally.

If only the statistics of the modal distribution are required it is possible to use two microphones and correlation techniques as described in Refs 2 and 3. The correlation functions are then Fourier transformed to determine the cross spectra which are the equivalent of the phasor representation of the pure tones. The steady tones produced by blade row interactions may also be analysed using correlation as described in Ref. 5.

Limited modal range

If the modal measurements are made in a parallel section of duct at a large distance from the source or any obstruction, then only propagating modes will be present. The propagating modes at any frequency may be easily defined by analyses such as that given in Ref. 1. In general more modes are present at high frequencies than at low frequencies as shown for a cylindrical duct in Fig. 4. It is evident that at low frequencies, less measured data is necessary to define the modes. In fact, if correct microphone positions are chosen as for instance in Ref. 2, only two microphone positions per propagating mode are necessary. The sound pressure equation for each position in terms of modal pressures may then be written down and solved for the modal pressures. If insufficient points are taken at a particular frequency or if there are residual decaying modes present erroneous results will be obtained.



Conclusions

The measurement of the modal distribution of in-duct fan noise enables both the study of noise sources and the prediction of far field radiation from ducts. Several methods of analysis of sound pressures into modes may be used, the chosen method depending on the nature of the source and duct geometry.

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