BRITISH ACOUSTICAL SOCIETY: SPRING MEETING at Chelsea College
London S.W.3. on Wednesday 25th April to Friday 27th April 73.

VIBRATIONS: SESSION 3: MUSICAL ACOUSTICS

Paper No.
73 MA2

Theory of Woodwind Instruments

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Summary

The resonance frequency of any note on a woodwind instrument can be calculated. The basic calculations are simple, but they require many corrections. The results of the calculations when compared with blowing tests reveal systematic differences, which have not yet been fully explained.

The sound level of descant recorders was found to be related to the density of the wall material of which they are made.

Introduction

A woodwind instrument is judged on: 1. the purity of its tuning, 2. its tone quality, 3. the ease with which it can be blown by the player, and the stability and flexibility of its tone.

The purity of the tuning can be measured fairly well by measuring the frequency of the fundamental; it also can be calculated according to methods described below.

The tone quality is related to the initial transient and to the composition of partials (1). No objective criteria are known, though listeners usually agree on the quality of a particular instrument.

Similarly, players agree on the ease of blowing woodwind instruments.

We shall restrict ourselves mainly to the calculation of the frequency.

Excitation of woodwind instruments

A woodwind instrument consists of a long narrow cylindrical or conical tube, and is blown at one end. Blowing compensates for losses due to radiation and "friction" along the walls. The losses at the walls are larger than radiation losses, in narrow tubes by a factor of 100 (2, 3).

The flute is excited by blowing a jet of air from between the lips across the embouchure hole. The motion of this jet is coupled to the pressures in the tube (4, 5).

In reed instruments (clarinet, saxophone, oboe, bassoon), the slit height between mouth and tube depends on the reed position, while the latter is determined by the pressures in the tube (3, 6).

In both types of excitation, the excitation action shifts the resonance of the passive system. In flutes this can either be a sharpening or a flattening (5, 7). In reed instruments, the effects of reed damping and the displacement of the air by the moving reed in the tube wall will produce a lowering of the pitch. In the clarinet, for instance, the reed compliance induces an effective increase in tube length of about 1 cm.

Resonances in tubes

Flutes, being open at both ends, overblow in all integral harmonics, and clarinets, closed at one end, in odd harmonics. Conical tubes, truncated and closed by a reed, blow as flutes having the length of the non-truncated cone (Fig. 1B). This holds only up to a certain upper frequency limit. This limit can be higher when a mouthpiece cavity (either a real one or a fictitious one due to reed compliance effects) of the volume of the truncated tube part is present (Fig. 1C)(3).

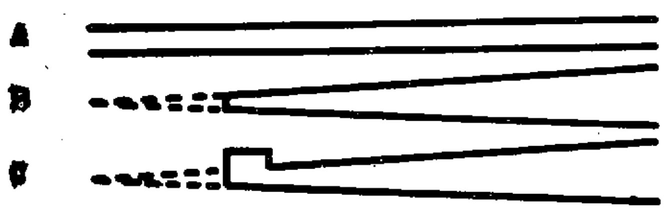

Fig.1. Cylindrical and conical woodwind instruments, sounding in the same frequency.

Holes

To find the resonance frequency of a tube with a side hole, we calculate the tube-piece L_x replacing the branched tube with the side-hole:

$$\frac{S_1}{j\rho c \tan kL_x} = \frac{S_H}{j\rho c \tan kL_H} + \frac{S_1}{j\rho c \tan kL_R}, \qquad (1)$$

where k = wave number, ρ = density, c = velocity of sound in free space. When the hole is near to the right-hand open end, the tangents can be replaced by their arguments:

$$S_1/L_x = S_H/L_H + S_1/L_R. \qquad (2)$$

This equation is independent of frequency, and it can be used for all registers. It is also applicable to clarinets and conical instruments.

When more holes are opened, and when they are close to one another, eq.(2) can be used for every hole one after the other.

Deviations from the simple expression may occur in certain fork fingerings.

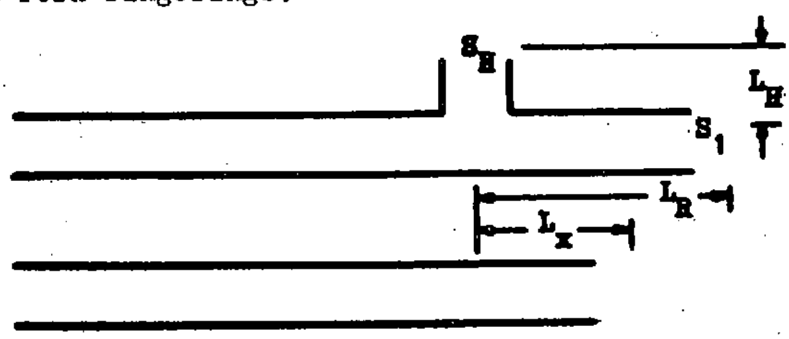

Fig.2. Flutes, blown from the left, sounding in the same frequency, one with, and one without a hole.

Corrections

A number of corrections must be applied.

- End corrections for holes, at the inside and outside of the tube, and for keys suspended above the holes.
- Deviations from idealized cylindrical or conical bore cause gradual frequency shifts for parts of registers.
- Changes in the velocity of sound due to interactions with the walls. For a clarinet the magnitude of this shift is plotted in Fig. 3 as D_w.
- Closed side-hole corrections, plotted as D_V in Fig.3, cause substantial shifts, which are different for different registers.
- Overblowing holes, made to aid excitation in a higher mode, are only in the proper place for a single note. As a side-effect, other notes are sharpened. See, for example, D_{reg} for the clarinet in Fig. 3.

The total correction, D_{corr}, for the clarinet is the heavy line in Fig. 3.

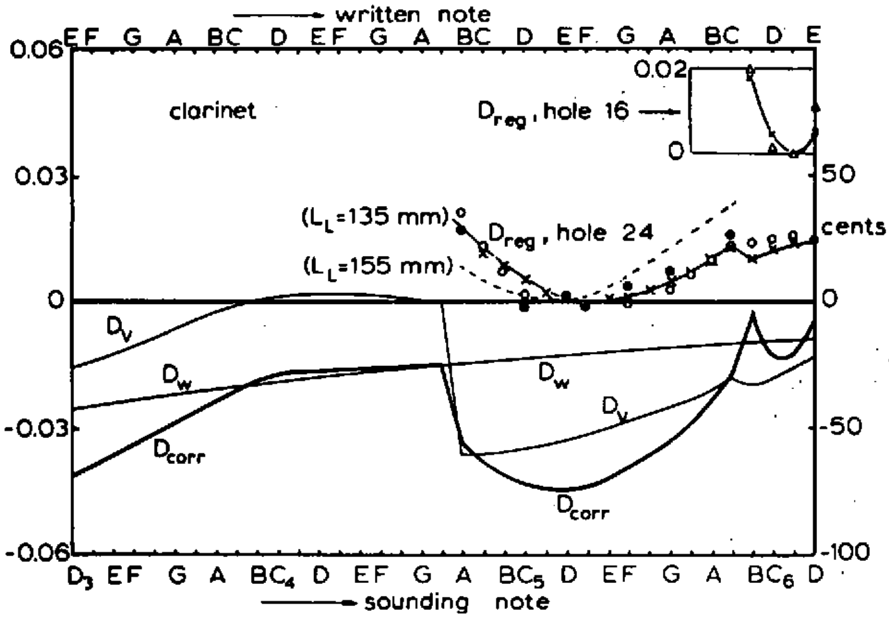

Fig. 3. Corrections for clarinet, expressed in relative frequency shift, and in cent. Drawn lines and crosses represent calculated, and circles measured **values**.

Results

For a number of woodwind instruments, the blown frequencies were compared with those calculated from the dimensions (3). Systematic uncertainties were encountered in the exitation. Even seemingly alike instruments blown by the same player sometimes gave differences that were too large to be explained.

Typical results are shown in Fig. 4 for two clarinets. Crosses denote calculated, and circles blown deviations from the nominal pitch. A known shortcoming of clarinets, a sharpness of part of the low register, is clearly visible. From Fig. 3 it appears that this is due to the combined action of D_V and D_{reg}.

In flutes we found discrepancies between theory and experiment equivalent to differences in length of about 1 cm.

In soprano saxophones, the upper part of the second register often is too sharp. It was found that this is due to excessive truncation of the cone and to deviations of the true conicity.

In many, even in professional instruments, deviations from smooth tuning were detected which could easily have been avoided.

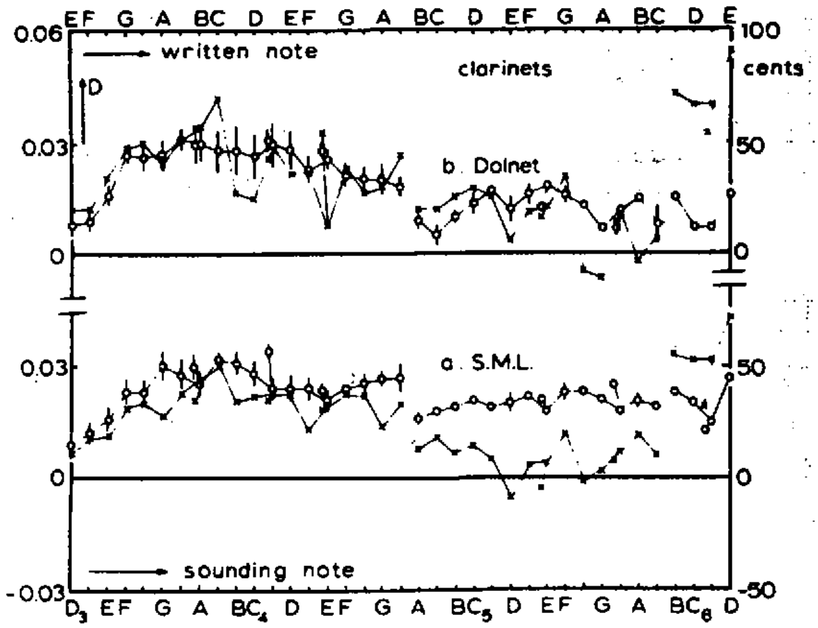

Fig. 4. Tuning deviations for two clarinets.
o = as measured, x = as calculated.

Wall density

The loudness of wooden descant recorders increases with their paraffin uptake after impregnation. A 5 gramme increase in the weight of a 80 gramme instrument entails an increase of about 3 dB in the sound level of the bottom note (7). To explain this we hypothesize a coupling between jet vibrations and tube body vibrations (bending in the vertical plane).

References

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