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REDUCTION OF VIBRATIONAL POWER IN PERIODIC BEAMS BY USE OF A NEUTRALISER

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1. INTRODUCTION

The control of vibration by neutralisers (dynamic absorbers) is of considerable interest in practical engineering. The reduction of vibrational power transmission in a finite, periodic beam by the action of this kind of passive device has been examined theoretically. Some interesting characteristics have been found: First, a group of resonance peaks within a propagation band can be reduced in level by use of only one neutraliser, the number reduced depending on the number of bays and the location; Secondly, the necessary tuned frequency of the neutraliser is related to the first resonance frequency within the corresponding propagation band. When the neutraliser is attached at the same point as the exciting force acting upon the beam, then the optimum frequency ratio is higher than one, otherwise it is less than one. A composite neutraliser is recommended for beams with more than ten bays with the exciting force and the neutraliser not being at the same point on the structure.

2. GENERAL THEORY

Conventional analyses of the dynamic vibration absorber, see for example Refs [1-5], are limited to the case of application to simple beam structures. The characteristics of periodic beams with an applied neutraliser have not be studied extensively. A flexural wave approach is developed here in order to investigate power transmission in a finite multi-bay beam [6] and the reduction of beam vibration by application of a neutraliser [7]. Theoretical investigations of vibrational power transmission have been carried out on a finite, periodic beam with a neutraliser connected, as shown in Figure 1. For hysteretic damping, the complex stiffness of the neutraliser is denoted by $K_d = K_d'(1+i\eta_d)$. The neutraliser exerts a force upon the beam due to the harmonic motion, given simply by [7]

$$F_d(\omega) = K_{tot}\, w(x_d) \tag{1}$$

where

$$K_{tot} = (\omega^2 m_d K_d) / (K_d - \omega^2 m_d) \tag{2}$$

When a multi-bay beam is subjected to an external harmonic force F_o together with N internal forces R_j (unknowns) applied by N-1 intermediate simple supports and F_d merely by the reaction of a neutraliser, the transverse displacement at any point x on the beam ($0 \le x \le L$) can be shown [6,7] to be given by,

$$w(x) = \sum_{n=1}^{4} A_n e^{k_n x} + F_d \sum_{n=1}^{2} a_n e^{-k_n|x_d - x|} + F_o \sum_{n=1}^{2} a_n e^{-k_n|x_o - x|} + \sum_{j=1}^{N-1} R_j \sum_{n=1}^{2} a_n e^{-k_n|x_j - x|} \tag{3}$$

where $k_1 = k$, $k_2 = ik$, $k_3 = -k$, $k_4 = -ik$ and $k^4 = \rho bh\omega^2/EI$; a_n is the nth coefficient of the point response function of an infinite system. In the case of an Euler-Bernoulli beam, $a_1 = -1/4EIk^3$ and

$a_2 = ia_1$. F_d is given by equation (1). Now there are totally N+4 unknowns (Four A_ns, N-1 R_js and one $w(x_d)$), which are determined by satisfying the boundary conditions of the structure.

It can be seen from equation (3) that there is no limitation on the location of the external force F_o and the point of attachment of the neutraliser. Moreover, the theory is applicable to a non-periodic beam. Damping of the beam and of the neutraliser is also easily introduced via complex quantities EI and K_d, respectively.

3. EXPRESSIONS FOR VIBRATIONAL POWER

It is well known that the input power to a structure is

$$P_s = \frac{1}{2} |F_o|^2 \mathrm{Re}\{\beta\} \tag{4}$$

where β is the mechanical mobility of structure. In the case considered here, Re{β} can be derived from equation (3) by letting F_o=1,

$$\mathrm{Re}\{\beta\} = \mathrm{Re}\{ -i\,\omega\, w^* \} = -\mathrm{Im}\{ \omega\, w^* \} \tag{5}$$

where * indicates the complex conjugate, and

$$w = \sum_{n=1}^{4} A_n e^{k_n x_o} + \sum_{n=1}^{2} a_n + F_d \sum_{n=1}^{2} a_n e^{-k_n|x_d - x_o|} + \sum_{j=1}^{N-1} R_j \sum_{n=1}^{2} a_n\, e^{-k_n|x_j - x_o|} \tag{6}$$

The time averaged transmitted power associated with internal shear force and bending moment at any point ($0 \le x \le L$) on the beam, respectively, are

$$P_u(x) = \frac{1}{2}\,\mathrm{Re}\{ S\,(-i\,\omega)\, W^* \} = \frac{1}{2}\,\mathrm{Re}\{ S^*\, i\,\omega\, W \} \tag{7}$$

$$P_m(x) = -\frac{1}{2}\,\mathrm{Re}\{ M\,(-i\,\omega)\, \theta^* \} = -\frac{1}{2}\,\mathrm{Re}\{ M^*\, i\,\omega\, \theta \} \tag{8}$$

where

$$w = \left\{ A_1E_1 + A_2E_2 + A_3E_3 + A_4E_4 + a_1[E_5 + iE_6] + F_d a_1[E_7 + iE_8] + \sum_{j=1}^{N-1} R_j a_1 [E_{1j} + iE_{2j}] \right\}$$

$$M = EIk^2 \left\{ A_1E_1 - A_2E_2 + A_3E_3 - A_4E_4 + a_1[E_5 - iE_6] + F_d a_1[E_7 - iE_8] + \sum_{j=1}^{N-1} R_j a_1 [E_{1j} - iE_{2j}] \right\}$$

$$\theta = k \left\{ A_1E_1 + iA_2E_2 - A_3E_3 - iA_4E_4 + a_1(jf)[-E_5 + E_6] + F_d a_1(jd)[-E_7 + E_8] + \sum_{j=1}^{N-1} R_j a_1 (jr)[-E_{1j} + E_{2j}] \right\}$$

$$S = EIk^3 \left\{ A_1E_1 - iA_2E_2 - A_3E_3 + iA_4E_4 - a_1(jf)[E_5 + E_6] + F_d a_1(jd)[E_7 + E_8] - \sum_{j=1}^{N-1} R_j a_1 (jr)[E_{1j} + E_{2j}] \right\}$$

and

$$E_1 = e^{kx},\; E_2 = e^{ikx},\; E_3 = e^{-kx},$$
$$E_4 = e^{-ikx},\; E_5 = e^{-k|x_o - x|},\; E_6 = e^{-ik|x_o - x|},$$
$$E_7 = e^{-k|x_d - x|},\; E_8 = e^{-ik|x_d - x|},\; E_{1j} = e^{-k|x_j - x|},\; E_{2j} = e^{-ik|x_j - x|}.$$

where jd, jf and jr are the sign operators, jd=-1 if $x_d>x$, jd=1 if $x_o<x$, jf=-1 if $x_o>x$, jf=1 if $x_o<x$, jr=-1 if $x_j>x$, and jr=1 if $x_j<x$. Using equations (7) and (8), the total vibrational power transmitted along the beam is the sum of the powers associated with the shear force and bending moment conponents, that is

$$P_a(x) = P_u(x) + P_m(x) \tag{9}$$

4. CALCULATION AND DISCUSSION

Emphasis is focused on reduction of the input power and the power transmitted along the beam due to attachment of a neutraliser. The non-dimensional frequency kl ($\omega \sqrt{\rho bhl/EI}$) has been used in figures from 3 to 7. Only hysteretic damping is considered here, and the mass ratio is expressed as

$$\mu = m_d / M \tag{10}$$

where M=ρhblN denotes the total mass of a N-equal-bay beam. The tuning frequency ratio within the first propagation band which appears in the following figures is written as

$$\Omega_m = \omega_d / \omega_m \tag{11}$$

where m=1 denotes the first mode of the structure. In the figures, the solid curves represent the vibrational power input to or transmitted along the beam to which no neutraliser is attached. The dashed curves are the predicted behaviour with a neutraliser attached.

When a neutraliser is attached at the centre of a three-equal-bay beam and is tuned to the first resonance frequency in the first propagation band, the input power and power transmitted along the beam are as shown in Figs 3 and 4 respectively, where two resonant peaks are effectively depressed by use of only one neutraliser. It is interesting to see that this type of result appears again in Figs 6 and 7, for a five-equal-bay beam with a neutraliser, where five resonance peaks are reduced. In the general case of multi-equal-bay beam, a group of resonance peaks in the corresponding propagation band can be reduced by application of one neutraliser when it is tuned to the first resonance frequency in the band.

As is known for a finite periodic beam, the number of resonances in each propagation band is equal to the number of bays of the beam. However in Figs 3 and 4, the second resonance did not appear in the plots as the external force was applied at the node of the second mode, see Fig 2.

Another characteristic of a periodic beam with a neutraliser is that the optimum tuning frequency ratio Ω, having the minimum peak value in the band, is greater than 1 when the neutraliser is attached at the same point as the exciting force, otherwise it is less than one, see Fig 5.

However, if the bay number N is large enough, such as N≥10, and if the neutraliser is not attached at the same point as the excitation force applied at the beam, it is, usually, extremely difficult to attenuate every resonant peak within a propagation band. This is because when the bay number is increased, the number of modes in a band is also increased. In this condition, it is very difficult to find a point which is not exactly at or near the nodal points of the modes within the propagation band. In this case, it is recommended that the tuning ratio should be in match togther with the use of a composite neutraliser, that is a main neutraliser with a smaller neutraliser coupled in series or parallel.

5. CONCLUSIONS

Reduction of input vibrational power and transmitted power in a periodic beam by use of a neutraliser has been examined theoretically. Some novel results have been found: one neutralizer can reduce the level of a group of resonance peaks within a propagation band, the number of which, of course, depends on the number of bays; the tuned frequency Ω is related to the first resonance frequency in the propagation band. The optimum tuning frequency may be greater or smaller than the first natural frequency depending on the location of the neutraliser relative to the excitation point of the structure. When the number of bays is greater than ten, use of a composite neutraliser is proposed.

6. ACKNOWLEDGMENT

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7. REFERENCES

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NOMENCLATURE

a_n	nth coefficients of infinite-system point response functions
b, h, l	breadth, thickness and the bay length of the beam
E	Young's modulus
F_o	magnitude of externally applied transverse point force
I	second moment of area of cross section of beam
k_n	nth wavenumber
K_d	complex stiffness of the neutraliser = $K_d'(1+i\eta_d)$
K_{tot}	equivalent stiffness

m_d	mass of the neutralizer
N	number of bays
w	transverse displacement in y direction
$w(x_d)$	transverse displacement at point x_d at which the neutraliser is attached
x	distance along the beam
x_o, x_d	force excitation point and the neutraliser attachment point
η, η_d	loss factor of beam and neutraliser
ρ	material density
ω	circular frequency, rads/sec
ω_d	tuning circular frequency of neutraliser, rads/sec

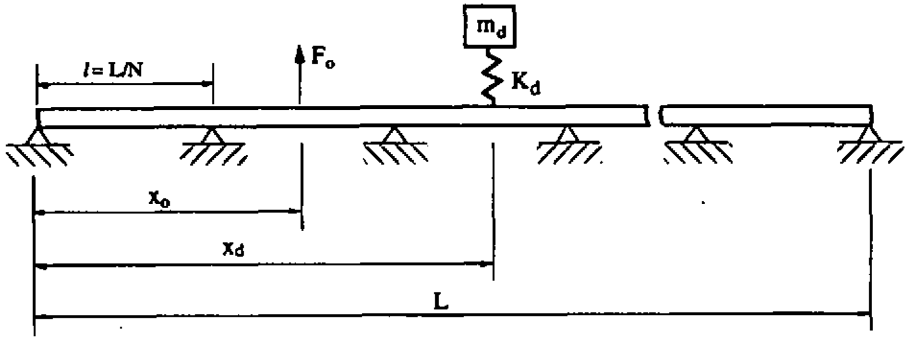

Figure 1. Neutraliser with hysteretic damping attached to arbitrary point on a finite, periodic beam excited at arbitrary point by a sinusoidally varying force

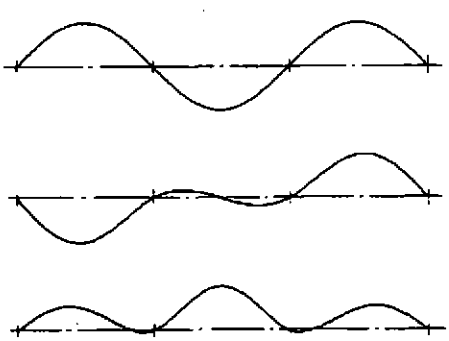

Figure 2. The first three modes of displacement of a three-equal-bay beam with simple supports

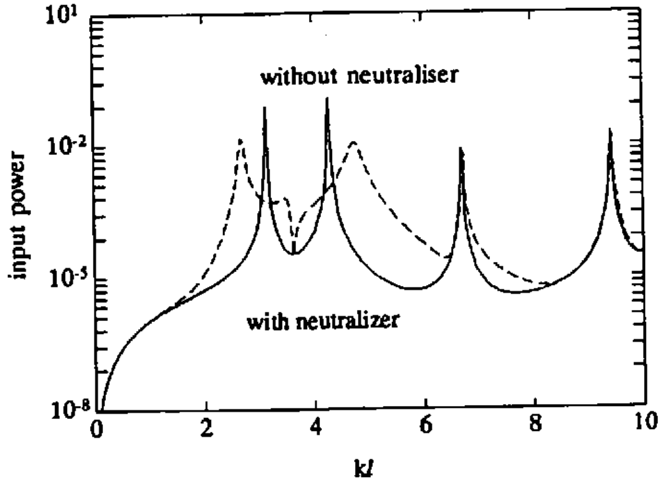

Figure 3. Vibrational power input to a three-equal-bay beam with simple supports, mass ratio $\mu = 0.2$, $\eta = 0.005$, $\eta_d = 0.25$, $x_o = x_d = 1.5l$, $\Omega = (\omega_d/\omega_1) = 1.15$

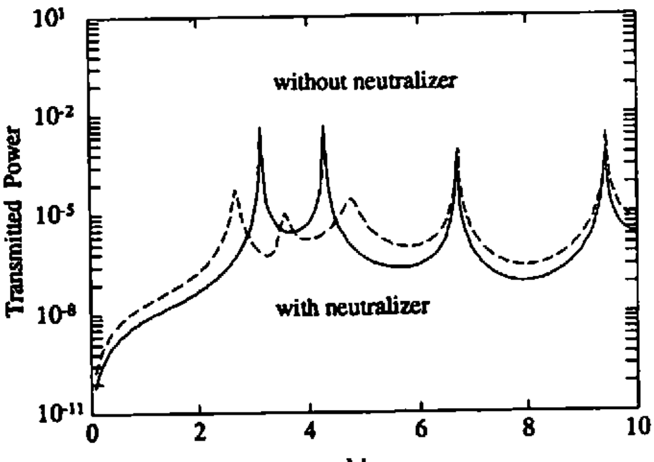

Figure 4. Power transmission in a three-equal-bay beam with simple supports, mass ratio $\mu = 0.2$ $\eta = 0.005$, $\eta_d = 0.25$, $x_o = x_d = 1.5l$, z=2.5 l, $\Omega = (\omega_d/\omega_1) = 1.15$

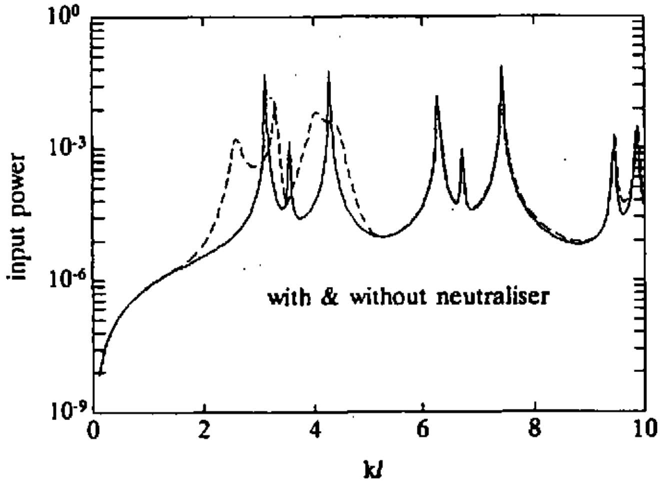

Figure 5. Vibrational power input to a three-equal-bay beam with simple supports, mass ratio $\mu = 0.2$ $\eta = 0.005$, $\eta_d = 0.40$, $x_o = 1.3l$, $x_d = 0.5l$, $\Omega = (\omega_d/\omega_1) = 0.99$

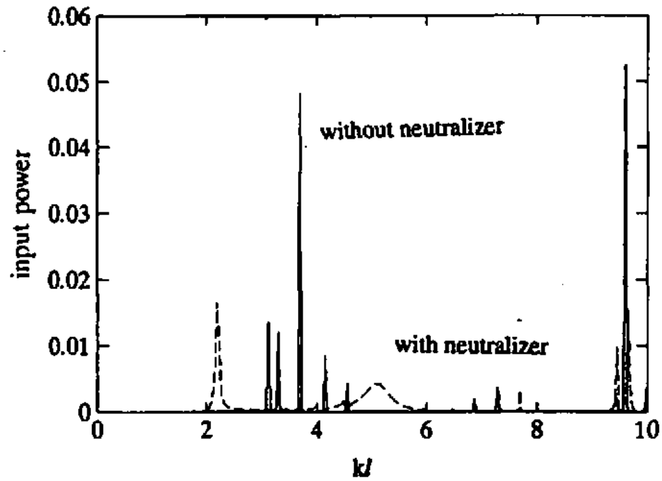

Figure 6. Vibrational power input to a five-equal-bay beam with simple supports, mass ratio $\mu = 0.40$ $\eta = 0.001$, $\eta_d = 0.35$, $x_o = x_d = 0.5l$, $\Omega = (\omega_d/\omega_1) = 1.04$

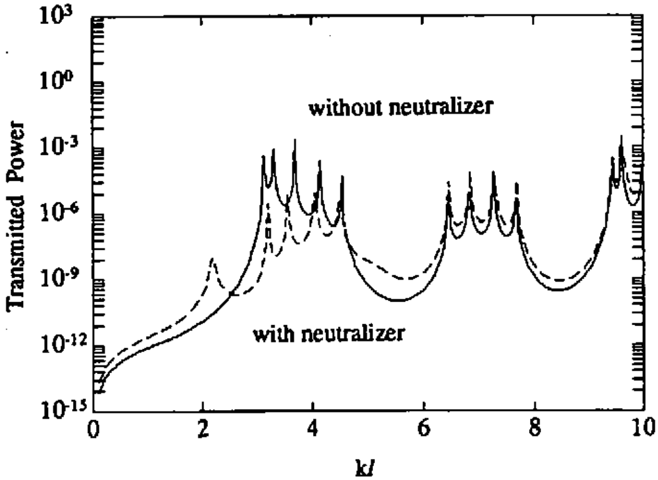

Figure 7. Power transmission in a five-equal-bay beam with simple supports, mass ratio $\mu = 0.2$
$\eta = 0.005$, $\eta_d = 0.40$, $x_o = 1.3l$, $x_d = 0.5l$, $z = 4.5l$, $\Omega = (\omega_d/\omega_1) = 1.04$

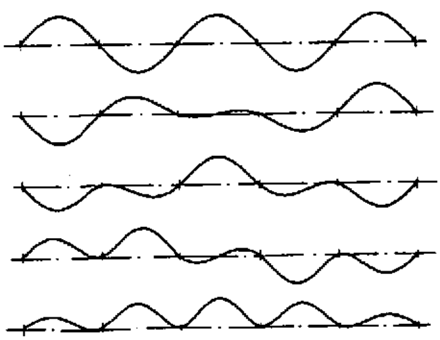

Figure 8. The first five modes of displacement within the first propagation band
of a five-equal-bay beam with simple supports