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FINITE ELEMENT ANALYSIS OF THE EFFECT OF DAMPING IN THE PISTON AND OUTER SUSPENSION OF A LOUDSPEAKER DIAPHRAGM

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INTRODUCTION

In recent years the finite element method has been used to model the mechanical behaviour of loudspeaker diaphragms [1, 2, 3]. Although this work has shown that the technique can be successful in this application, no work has been reported which discusses the measurement of appropriate values for the material properties of the model. In this paper attention is given to the method of obtaining suitable parameters for including damping in the piston and out suspension parts of the diaphragm. Measured values of material properties are used to analyse a typical diaphragm and results are compared with measurements made using a vibration interferometer.

DESCRIPTION OF THE FINITE ELEMENT METHOD

The basic equation for modelling a dynamic system by discretising the structure into a number of finite elements is

$$M\ddot{\underline{U}} + D\dot{\underline{U}} + K\underline{U} = \underline{F} \qquad (1)$$

where M, D, K are matrices representing the mass, damping and stiffness in the system respectively and $\underline{U}$ is a vector representing the displacement at each node resulting from applied forces represented by the vector $\underline{F}$. If $\underline{F}$, and therefore $\underline{U}$, are steady sinusoidal functions of time, with frequency ω , the equation reduces to

$$(-M\omega^2 + jD\omega + K)\underline{u} = \underline{f} \qquad (2)$$

in which $\underline{u}$ and $\underline{f}$ represent complex amplitudes of motion and forcing at each node.

The structure was modelled using thin shell of revolution elements [4](fig. 1). These are characterised by 3 nodes and a single radius of curvature in the meridional direction.

Mass and stiffness matrices are calculated for each element and these are added to form the global mass and stiffness matrices of (2). The element mass matrix is calculated by a consistent mass approximation and the stiffness matrix calculated from stress-strain relationships derived from Flügge's thin shell theory [4].

The construction of the matrix D, representing the equivalent viscous damping in the system, is discussed below.

THE DAMPING MATRIX

If, initially, the system is considered without damping, the dynamic equation is

$$(-M\omega^2 + K)\underline{u} = \underline{0} \qquad (3)$$

which may be solved as an eigenvalue problem. The system of equations can then be decoupled using the matrix of eigenvectors into a set of equations:

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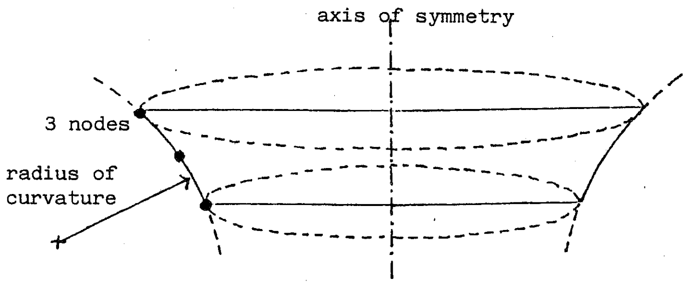
Fig 1. Axisymmetric shell element



$$-m_i\omega_i^2 + k_i = 0 , \quad i = 1,\ldots,n \tag{4}$$

each representing a vibrational mode of the system. The solution at an arbitrary forced vibrational frequency may then be obtained by modal superposition by using a measured set of modal damping ratio values,

$$\xi_i \qquad i = 1,\ldots,n .$$

However, these are not directly related to measurable material properties of the individual parts of the diaphragm, and so are not appropriate to investigating the effects of different materials.

A more useful technique of introducing damping in the equations is via a complex Young's modulus

$$E^* = E(1 + j\eta)$$

where η is the material loss factor. The loss factor can then be defined separately for the different materials in the diaphragm and a full damping matrix can be constructed from the element stiffness matrices. However, this necessitates the direct solution of (2) at each frequency of forced vibration. This greatly increases the computer time and storage required compared with the modal analysis solution.

A program for the construction of the matrices and their solution was implemented using the NAG finite element subroutine library.

MEASUREMENT OF THE LOSS FACTOR

A special driver unit was constructed in which the voice coil was attached at both ends to the chassis by a piano-wire suspension. A stiff plastic collar allowed different pistons to be screwed onto the voice coil. In this way the diaphragm could be mounted onto the driver without the need of an outer suspension and the force delivered to it was constrained to be in the axial direction.

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The motion of the diaphragm was measured using a laser vibration inferometer. The structural loss factor of a piston (without a surround present) was measured by driving it with a sinewave signal at a resonant frequency and measuring the decay curve after the driver had been open circuited. The loss factor at the chosen resonance frequency was then calculated uising the approximate relation [5],

$$\eta_i \simeq \frac{\xi_i}{2} \qquad (5)$$

assuming the motion due to coupling with other modes to be small.

As an example, the results of this measurement for a 6", high impact polystyrene piston are presented in figure 2. These vary from mode to mode indicating structural dependence. However, there is no simple overall frequency dependence and the modal values cannot be predicted for an unmeasured diaphragm. The mean is therefore taken as a frequency independent measure of the damping.

The F.E. model (1) includes no structural-acoustic coupling. The proportion of the observed damping arising from radiation losses was therefore assessed by performing the experiment at low pressure (0·2 atmos). A comparison of the means of the two sets of data,

$$\bar{\eta}_{1\ \text{atmos}} = \cdot 043 \pm \cdot 001 \quad , \qquad \bar{\eta}_{\cdot 2\ \text{atmos}} = \cdot 034 \pm \cdot 007$$

shows a small though significant difference. The measured equivalent loss factors, presented below, therefore include a contribution for this effect, but it is not thought that any gain in the accuracy of the results would justify the great increase in computing needed for a coupled structural-acoustic model.

The loss factor obtained in this way gives a consistent value for different sizes and shapes of piston while also being a distinct measure of the performance of different materials. A selection of values measured for materials often used in loudspeakers is given in table 1.

In order to use the model in assessing diaphragms which have not yet been constructed, a measurement on a simple clamped beam of the material was devised. A calculation of Young's modulus was made from observations of the resonant frequencies. The size of the beams was arranged so that these occurred at audio frequencies. Measurements of the loss factor were made at the same time. These results are also shown in table 1.

Although the structural loss factor measured on these beams is lower than that for a diaphragm it may be used as an adequate indication in evaluating materials. The loss factor given for the surround material was found at the "surround resonance" of a complete diaphragm. At this frequency the motion of the outer suspension dominates while the piston remains relatively rigid in behaviour. The Young's modulus of this rubber sheet like material was measured under static extension on a material testing machine as a beam could not be constructed from it.

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Table 1. Young's modulus and measured loss factors of diaphragm materials

Material	$\eta_{diaphragm}$	η_{beam}	Young's Modulus $/10^9$ Nm^{-2}
High Impact Polystrene	•043 ± •001	•028 ± •001	2•09 ± •03
uPVC	•046 ± •002	•039 ± •001	2•69 ± •03
Polypropylene	•116 ± •008	•097 ± •005	1•94 ± •06
Plasticised PVC (used for surround)	•29 ± •02	-------	•0081

RESULTS FOR A PVC DIAPHRAGM

A typical diaphragm was analysed using the F.E. program. The piston was divided into 28 elements and the outer suspension into a further 9. The material parameters enetered for the two parts are given in table 2. At the region where the two materials overlap the density was adjusted to allow for the total mass of these elements. This region was attributed the stiffness of the piston and the loss factor of the surround as these are the dominant property in each case.

Table 2. Material properties used in the model

	Young's Modulus (Nm^{-2})	Poisson's Ratio	Density (kg m^{-3})	Thickness (mm)	Loss factor
Piston (uPVC)	$2{\cdot}69 \times 10^9$	•33	1310	•327	•047
Suspension (plasticised PVC)	$8{\cdot}1 \times 10^6$	•33	1180	•533	•29

The results of the calculation for the piston without a surround attached are shown in figure 4, and those for the complete diaphragm are presented in figure 5.

The plots show amplitude of acceleration normal to the surface, along a meridian of the diaphragm. This may be more easily related to features in the sound pressure level response than displacement. Solutions were obtained for vibration at 50 frequencies in the range 39-7695 Hz. A sinusoidal force was applied at the neck of the cone in the axial direction; the same at each frequency.

In figure 4 the resonances of the piston can be clearly identified.

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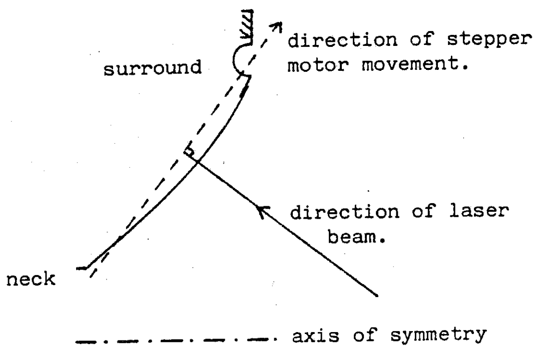
They are characterised by a maximum of amplitude occurring at a particular position on the meridian. This confirms the well known conclusions of Frankort [6]; the edge of the piston is set into bending motion at the first mode while the centre portion remains relatively uniform in motion. As the frequency is increased the region of the cone in bending motion grows towards the centre.

The effects of adding the outer suspension to the structure can be seen in figure 5. The fundamental 'surround resonance' can be identified at a frequency below the first bending resonance of the piston. Higher order resonances in the surround material can also be seen. The resonant peaks on the piston appear smoother because of the added damping. The lower order modes are attenuated more than those of higher frequency. The greater part of the motion occurs at the edge of the diaphragm in these modes and so it is to be expected that the higher loss factor of the surround material has a greater influence in the damping.

Figures 6 and 7 present measurements made on the actual diaphragm for which the F.E. analysis was performed. These were taken using a laser vibration interferometer.

A stepper motor was used to move the diaphragm equal intervals in a path normal to the laser beam. The result is that the 'meridional coordinate' of figures 6 and 7 is actually the distance along a straight line drawn through the profile of the diaphragm (Fig 3). The measurements of motion are taken perpendicular to this line in the direction of the beam rather than being truly normal to the surface.

Fig 3. Profile of the diaphragm showing how measurements were made.



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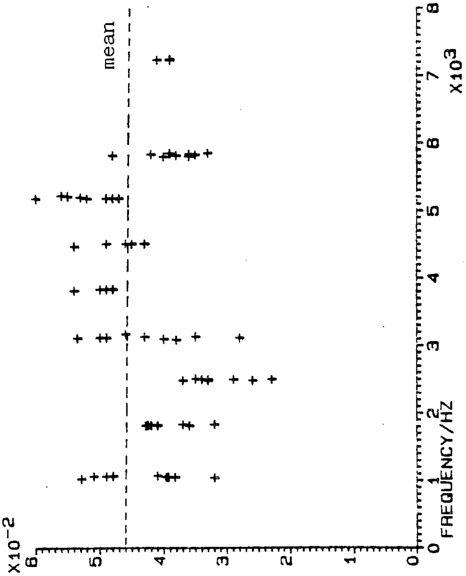

Fig 2. Loss factor measurements made on polystyrene piston.

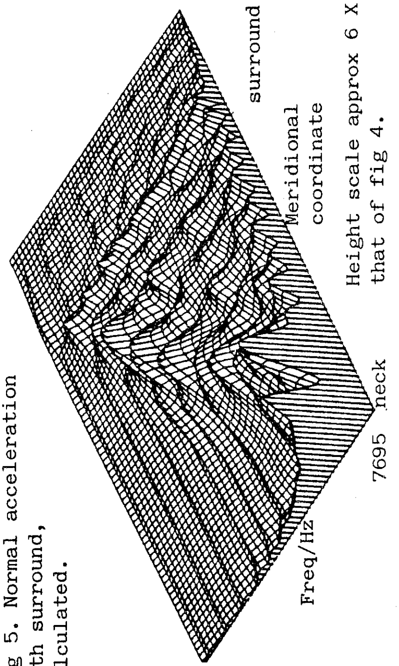

Fig 5. Normal acceleration with surround, calculated.

Height scale approx 6 X that of fig 4.

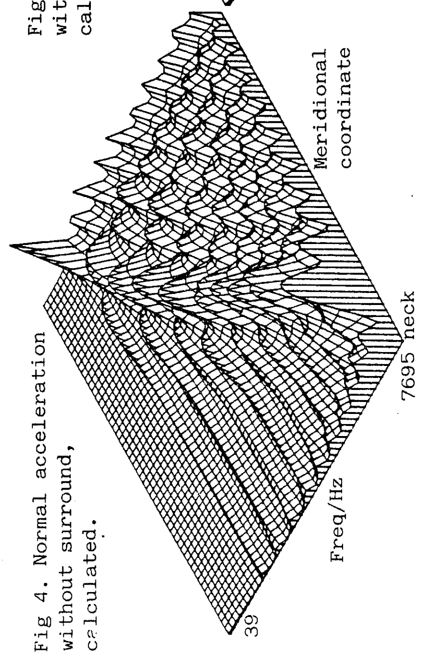

Fig 4. Normal acceleration without surround, calculated.

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A further difficulty in comparing the measured and F.E. results arises in that the driving coil mechanism did not deliver a force constant in amplitude over the frequency range.

The measured results display the same features as the calculated ones. Figure 7 shows well the attenuation at low frequencies and the surround resonance. The effect of the response of the driving coil mechanism, and the presence of asymmetric modes of vibration, can also be seen. The absence from the F.E. model of these, and the effects of the imperfections of a particular manufactured diaphragm, means that the F.E. results demonstrate the influence of adding the surround more clearly.

For the purpose of comparison, figure 8 shows the calculated results resolved in the direction of the laser beam and weighted for the response of the driving mechanism. The agreement between figures 6 and 8 is good, the major remaining differences being due to asymmetric motion and the fact that measurements could not be made at the very edge of the real piston.

In order to quantify the effect of the surround damping on the motion at different frequencies, modal damping ratios were estimated from the data of figure 5. This was done by calculating the Q factor of the resonance peaks in amplitude of displacement.

This measurement is expected to yield high values because of the large interval between points in the frequency domain. Calculations were made at several points on the meridian in order to find resonance peaks across the frequency range. These results are presented as twice the modal damping ratio in figure 9 so that they may be compared with the loss factors of the individual materials, using (5). The graph shows the transition between the major influence of the loss factor of the surround at low frequencies to that of the piston material at higher frequencies. This shows that in investigating materials from which to construct the piston part of the diaphragm, the loss factor must be measured in the higher frequency range where its influence is most important.

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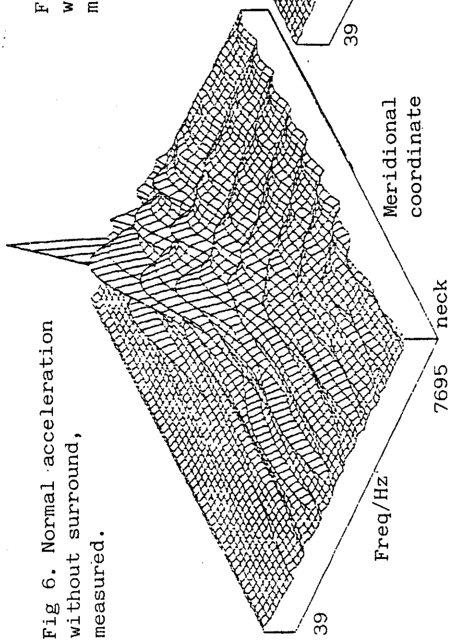

Fig 6. Normal acceleration without surround, measured.

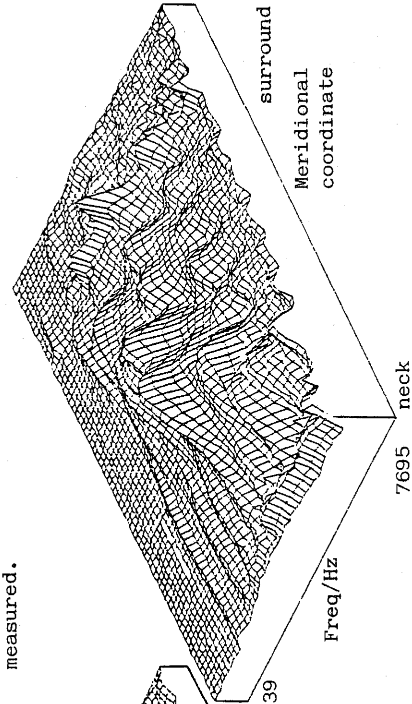

Fig 7. Normal acceleration with surround measured.

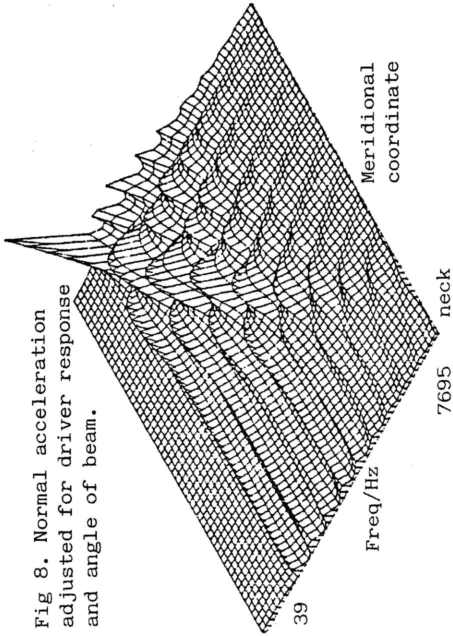

Fig 8. Normal acceleration adjusted for driver response and angle of beam.

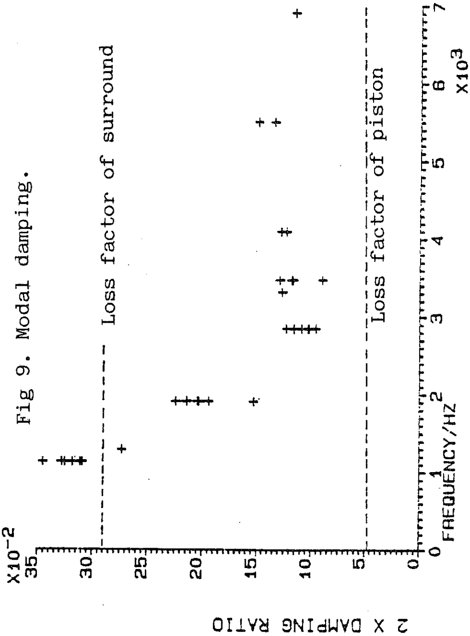

Fig 9. Modal damping.

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