THE AP2 NORMAL MODE PROGRAM

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ABSTRACT

AP2 is a normal mode computer program which predicts acoustic propagation loss in a horizontally stratified ocean with a flat bottom. Since it is based on an exact solution of the wave equation, AP2 is used primarily as a check on the predictions of ray programs which, though more economical and therefore more widely used, frequently yield erroneous results. In order that it may serve in such a capacity, it is designed to accept the same environmental inputs as ray programs. Being dimensioned for 500 modes, AP2 is useful in typical deep-water environments at frequencies up to about 100 Hz.

Routine theoretical predictions of acoustic propagation loss are usually made on the basis of ray theory because of the high computational efficiency of ray-type computer programs. Ray programs are especially efficient for predictions at short to moderate ranges where the assumption of a horizontally stratified ocean is usually acceptable. However, ray theory is subject to a number of well know limitations, and even modern ray programs such as the FACT Model [1], which incorporate certain wave corrections, frequently run into difficulties. When questionable results are obtained, it is highly desirable to have available a program which provides an exact solution of the wave equation for the same environmental inputs. AP2 is a normal mode program which serves this purpose.

AP2 is designed to accept an arbitrary velocity profile (figure 1) consisting of as many as 26 points. The bottom may be specified in either of two ways, (1) as a physical bottom consisting of up to 10 homogeneous, nonelastic, attenuating layers, or (2) as an empirical bottom characterized by a set of curves of bottom reflection loss versus grazing angle. The physical bottom has not proved particularly useful, partly because of the paucity of suitable oceanographic data and partly because of the difficulty of attempting to synthesize layered structures which simulate measured bottom loss curves. The empirical bottom, though it involves some approximations, can be specified with precisely the same inputs as are entered into the ray programs.

AP2 is dimensioned for 500 modes and is useful in typical deep-water environments at frequencies up to about 100 Hz. The number of modes required to generate a complete propagation loss curve is roughly the number of half wavelengths in the ocean depth. The program may be used with caution at somewhat higher frequencies if the user is willing to ignore erroneous predictions in the bottom bounce region at short ranges.

AP2 is completely automatic and does not require of the user any special knowledge of normal mode theory or of the internal details of the program.

The solution of the wave equation embodied in AP2 is a steady-state CW solution. The wave function $\Psi\ (r,z,z_s,t)$, representing the acoustic pressure, is written in equation (1) of figure 2 as the product of a spatial function $\Phi\ (r,z,z_s)$ and the time factor $\exp(i\omega t)$, where z_s is the source depth, z the receiver depth, and r the receiver range in a cylindrical coordinate system. Separation of variables leads to the integral (2) over the horizontal wave number k, which is evaluated by contour integration in the complex k-plane. In order that the solution may represent the field of an omnidirectional point source, the Green's function $U(z,z_s,k)$ is expressed in the form of equation (3), where u and v are the two solutions of the separated depth equation and W is their Wronskian. The contour of integration, shown in figure 3, is based on the Pekeris branch cut [2] and leads to a solution consisting of a finite set of modes whose eigenvalues lie to the right of the branch point k_B, an infinite set of so-called "improper" modes whose eigenvalues lie to the left of k_B, and a branch line integral whose contribution in most cases of practical interest is negligible. In general the mode eigenvalues are complex.

The program is similar in many respects to Stickler's program [3]. However, Stickler employs a different branch cut [4] which leads to a solution consisting of the same finite set of modes plus a branch line integral over the continuous spectrum covering bottom bounce propagation at the steeper angles. It has been demonstrated by numerical calculations that both approaches lead to equivalent results [3][5].

In applying the normal mode solution to AP2 the velocity profile is fitted with segments in which the reciprocal of the square of the sound speed c(z) varies linearly with depth, as indicated in equation (6), figure 4. The resulting depth functions are expressed in terms of the Airy functions $Ai(Z_n)$ and $Bi(Z_n)$, as indicated in equations (7) and (8). The function u satisfies the pressure release boundary condition at the surface and is evaluated by working downward through the layers to the source (or receiver) depth with the aid of recursion formulas (figure 5). The function v satisfies the boundary conditions at the bottom interface, which can be expressed in the form of equation (11), and is evaluated by working upward with similar recursion formulas.

The contribution of the bottom is transmitted through the parameter μ_B. When a layered physical bottom is specified, μ_B is evaluated as indicated in equation (12), figure 6. When the bottom is specified by empirical bottom loss curves, μ_B is computed from the reflection coefficient R at the bottom interface, as indicated by equation (13), figure 7. Several approximations are made. (1) It is assumed that the imaginary part of the eigenvalue may be neglected in computing the bottom contribution. (2) In the immediate vicinity of the bottom interface it is assumed that the mode may be treated as a plane wave, the parameter β of equation (14) representing the vertical wave number. (3) Since only the magnitude of R is determined by the bottom loss $N_B(\theta)$, equation (15), the phase angle is unknown and is arbitrarily assumed to be zero. (4) Only those modes whose equivalent rays strike the bottom are assumed to interact with the bottom, the effects of the exponential tails of the depth functions in RSR type modes being neglected.

The mode eigenvalues are values of k which satisfy the characteristic equation (17), figure 8. They are computed by an iteration procedure which requires a fairly accurate initial estimate. For modes whose phase velocities are smaller than the maximum sound speed of the profile the initial estimates are computed by a WKB approximation. The estimates for higher order modes are obtained by extrapolation from previously computed eigenvalues.

A unique feature of the AP2 program is an algorithm which improves the computational efficiency by bypassing the calculation of weakly excited low-order modes. The concept is illustrated in figure 9. Modes whose phase velocities are appreciably smaller than the sound speeds at the source and receiver depths are excited only through the exponential tails beyond their turning points. The algorithm computes a minimum required phase velocity such that the attenuation factor between the turning point and source or receiver depth (whichever lies closer to the channel axis) is exp(-8). Only those modes whose phase velocities exceed this value need be computed, thereby conserving computation time and making space available in the computer memory arrays for modes beyond the 500th.

The last four figures illustrate some of the results obtained from the AP2 program. Figures 10 and 11 show comparisons between the physical and empirical bottoms for two different deep-water profiles. To facilitate the comparisons, smoothed curves have been plotted over the original data. In each case a layered bottom was selected, from which a bottom loss curve was computed by an auxiliary program and then inserted for the empirical bottom. Figure 10 shows almost perfect agreement, while figure 11 is representative of the poorest agreement observed in a set of test runs. It is seen that even here the discrepancies are relatively minor.

Figure 12 is an example of a run in which more than the available 500 modes are required to describe the sound field. In this case the estimated number of modes required is approximately 720. The missing modes correspond to propagation at grazing angles steeper than about 45 degrees. A simple ray calculation based on straight-line geometry reveals that the mode deficiency should result in erroneous predictions at ranges shorter than 12,000 yards. This result is clearly evident in the curves of figure 12. It should be noted, however, that in the direct propagation zone at very short ranges, where the sound field is dominated by rays which leave the source at shallow angles, the predictions of AP2 are valid.

A comparison between AP2 and the semi-coherent predictions of the FACT ray model is shown on figure 13. Aside from the remarkable agreement in the two interference patterns, the feature of primary interest in these plots is the convergence zone which appears in the FACT curve at 75 kyd, but is totally absent from the AP2 curve. An investigation of this case has revealed that the convergence zone is formed by an exceedingly narrow bundle of rays whose energy ray theory assumes to remain intact but in reality leaks away by diffraction before reaching the convergence zone range. This is an example of the value of a normal mode program as a check upon ray predictions.

REFERENCES

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2. C.L. Pekeris, "Theory of Propagation of Explosive Sound in Shallow Water," In Propagation of Sound in the Ocean, Geo. Soc. Am. Mem. 27 (1948).
3. D.C. Stickler, "Normal Mode Program with both the Discrete and Branch Line Contributions," J. Acoust. Soc. Am., 57, 856-861 (1975).
4. W.M. Ewing, W.S. Jardetzky, and F. Press, Elastic Waves in Layered Media, (McGraw-Hill, New York, 1957), p 133.
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AP2 NORMAL MODE PROGRAM

STRATIFIED OCEAN WITH FLAT BOTTOM

ARBITRARY VELOCITY PROFILE

ALTERNATE BOTTOM SPECIFICATIONS

(1) PHYSICAL BOTTOM – NON-ELASTIC ATTENUATING HOMOGENEOUS LAYERS

(2) EMPIRICAL BOTTOM – CURVES OF BOTTOM LOSS VS. GRAZING ANGLE

MODE CAPACITY – 900 MODES

FIGURE 1 - CHARACTERISTICS OF THE AP2 NORMAL MODE PROGRAM

APPLICATION TO AP2

PROFILE SEGMENTS

$$\frac{1}{C(z)^2} = \frac{1}{C_n^2} + a_n (z - z_n) \qquad (6)$$

DEPTH FUNCTIONS

$$u(z) = A_n Ai(Z_n) + B_n Bi(Z_n) \qquad (7)$$

$$v(z) = C_n Ai(Z_n) + D_n Bi(Z_n) \qquad (8)$$

WHERE

$$Z_n(z) = \alpha_n (z - z_n) - \frac{1}{\alpha_n^2}\left(\frac{\omega^2}{C_n^2} - k^2\right) \qquad (9)$$

$$\alpha_n = (\omega^2 a_n)^{1/3} \qquad (10)$$

FIGURE 4 - APPLICATION OF THE AP2 NORMAL MODE PROGRAM

FORMAL SOLUTION

$$\Psi(r, z, z_0, t) = \phi(r, z, z_0)\, e^{j\omega t} \qquad (1)$$

$$\phi(r, z, z_0) = \int_0^\infty U(z, z_0, k)\, J_0(kr)\, k\, dk \qquad (2)$$

$$U(z, z_0, k) = \begin{cases} -\dfrac{2u(z, k)\, v(z_0, k)}{W(k)} & z < z_0 \\ -\dfrac{2u(z_0, k)\, v(z, k)}{W(k)} & z > z_0 \end{cases} \qquad (3)$$

$$W(k) = u\frac{dv}{dz} - v\frac{du}{dz} \qquad (4)$$

FIGURE 2 - FORMAL SOLUTION OF THE WAVE EQUATION

EVALUATION OF u(z) AND v(z)

AT SURFACE:

$u = 0 \qquad \frac{du}{dz}$ = ARBITRARY

AT BOTTOM INTERFACE

v = ARBITRARY $\qquad \frac{dv}{dz} = \rho_B v \qquad (11)$

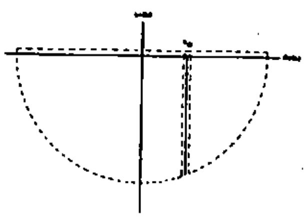
ρ_B = BOTTOM PARAMETER

COMPUTATION OF u AND $\frac{du}{dz}$ IN SUCCESSIVE LAYERS ACCOMPLISHED WITH RECURSION FORMULAS

SIMILAR RECURSION FORMULAS USED ALSO FOR v AND $\frac{dv}{dz}$

FIGURE 5 - EVALUATION OF THE DEPTH FUNCTIONS

INTEGRATION IN THE COMPLEX PLANE



[illegible] (5)

FIGURE 3 - CONTOUR OF INTEGRATION

PHYSICAL BOTTOM

$$\rho_B = \frac{\rho_W}{\rho_B}\, \frac{1}{u_B}\, \frac{du_B}{dz}\bigg|_{z_N} \qquad (12)$$

z_N = DEPTH OF OCEAN

ρ_W = DENSITY OF WATER

ρ_B = DENSITY OF ADJACENT BOTTOM LAYER

u_B = BOTTOM DEPTH FUNCTION

u_B AND $\frac{du_B}{dz}$ EVALUATED BY RECURSION FORMULAS, BEGINNING AT SEMI-INFINITE BASEMENT LAYER

FIGURE 6 - EVALUATION OF BOTTOM PARAMETER FOR PHYSICAL BOTTOM

EMPIRICAL BOTTOM

$$\rho_B = -i\beta \; \frac{1-R}{1+R} \qquad (13)$$

$$\beta = \sqrt{\frac{\omega^2}{C_W^2} - \mathrm{Re}(\lambda)^2} \qquad (14)$$

$$R = 10^{-0.05 N_B(\theta)} \qquad (15)$$

$$\theta = \sin^{-1}\left(\frac{\beta}{\omega / C_W}\right) \qquad (16)$$

C_W = SOUND SPEED AT BOTTOM OF WATER

θ = GRAZING ANGLE AT BOTTOM OF WATER

$N_B(\theta)$ = BOTTOM LOSS (dB)

R = REFLECTION COEFFICIENT, ASSUMED TO BE REAL

FIGURE 7 - EVALUATION OF BOTTOM PARAMETER FOR EMPIRICAL BOTTOM

DETERMINATION OF EIGENVALUES

CHARACTERISTIC EQUATION

$$W(\lambda) = \rho_B \, u(z_W) - \frac{du}{dz}\Big|_{z_W} = 0 \qquad (17)$$

$W(\lambda)$ SOLVED FOR λ_m BY ITERATION PROCEDURE

INITIAL ESTIMATES OF λ_m FOR ITERATIONS:

(1) WKB APPROXIMATION FOR MODES WITH PHASE VELOCITIES $< C_{MAX}$

(2) EXTRAPOLATION FROM PREVIOUS MODES FOR PHASE VELOCITIES $> C_{MAX}$

WHERE C_{MAX} = MAX. SOUND SPEED OF PROFILE

FIGURE 8 - DETERMINATION OF EIGENVALUES

MINIMUM REQUIRED PHASE VELOCITY

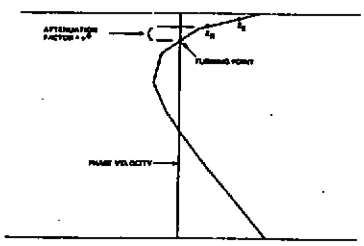


FIGURE 9 - CONCEPT OF THE MINIMUM REQUIRED PHASE VELOCITY

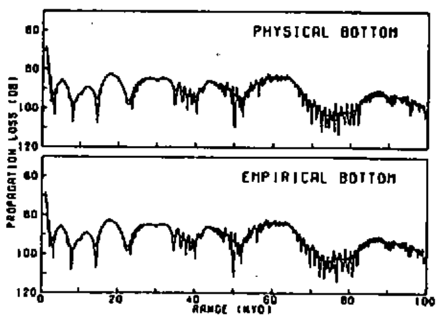


FIGURE 10 - COMPARISON OF RESULTS FOR PHYSICAL AND EMPIRICAL BOTTOMS, CASE 1

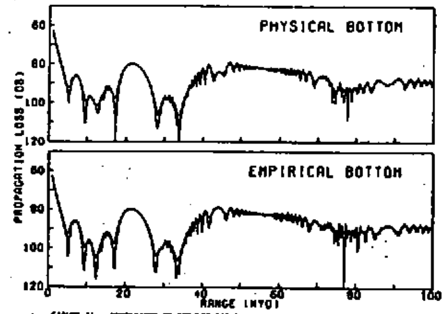


FIGURE 11 - COMPARISON OF RESULTS FOR PHYSICAL AND EMPIRICAL BOTTOMS, CASE 2

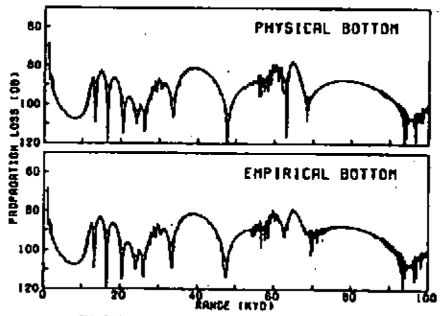


FIGURE 12 - ILLUSTRATION OF THE EFFECT OF MODE DEFICIENCY

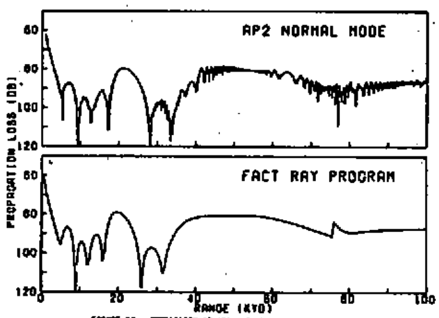


FIGURE 13 - COMPARISON OF AP2 WITH FACT RAY MODEL