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THE PARAMETRIC RECEIVING ARRAY IN AN INHOMOGENEOUS MEDIUM

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Introduction

Studies of parametric receiving arrays[1,2] have, to date, assumed the acoustic medium to be homogeneous. The effects on parametric reception of medium inhomogeneities caused by turbulence and thermal microstructure are considered in this paper. Theoretical results for a "nearfield" parametric receiver and some related model tank experiments are discussed below.

The operation of a parametric receiver in an inhomogeneous medium can be described using Fig. 1. A parametric receiver, formed by a pump transducer and an axially aligned hydrophone, is used to detect radiation from some distant signal source. Both pump and signal source radiation are scattered by medium inhomogeneities such as turbulent eddies and thermal "patches". As these inhomogeneities move about with time, the changes of phase between direct and scattered first-order pressure will cause the source density function, q, to vary randomly. Similarly, the second-order field radiated by the virtual sources will be scattered, resulting in further fluctuations of the pressure detected by the hydrophone. The electrical output of the hydrophone will therefore fluctuate randomly in both amplitude and phase in a manner determined by the spatial distribution and strength of the scatterers.

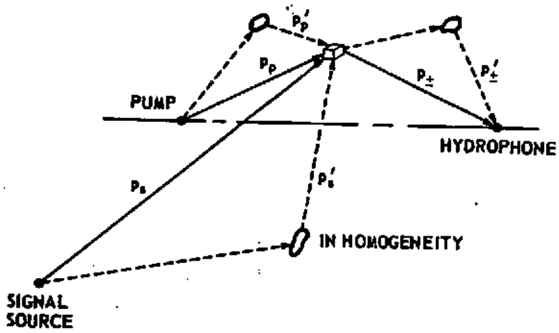

FIGURE 1
SCATTERING GEOMETRY

Theoretical Predictions

The parameter used in this study to describe the amplitude fluctuations of a pressure wave is the coefficient of amplitude variation, C. It is defined as

$$C^2 = \frac{\langle P^2\rangle - \langle P\rangle^2}{\langle P\rangle^2} , \qquad (1)$$

where P is the pressure amplitude, and the brackets denote time averaging. This parameter may be shown to be the normalized RMS variation of the pressure amplitude about its mean value.

If a parametric receiver is used to detect sound from a source located on the axis of the receiver (see Fig. 2), then the coefficient of amplitude variation for the second-order pressure field will be

$$C^2 \approx \frac{1}{L^2}\int_0^L\!\!\int \Big[\langle B_s(x)B_s(x')\rangle + \langle B_p(x)B_p(x')\rangle + \langle B_p(x)B_\pm(x')\rangle + \langle B_\pm(x)B_p(x')\rangle + \langle B_\pm(x)B_\pm(x')\rangle \Big]\,dx\,dx' \qquad (2)$$

where L is the array length, the brackets now denote ensemble averaging (the ergodic hypothesis is assumed to apply), and B_s, B_p, and $B_\pm$ are logarithmic amplitude fluctuations of the signal, pump, and second-order waves, respectively.

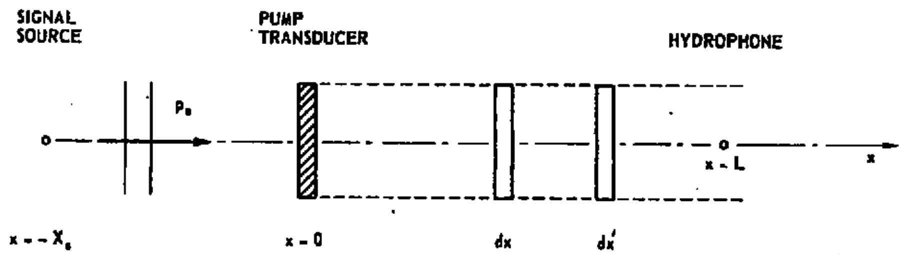

FIGURE 2
THEORETICAL MODEL

Terms of the type, <B(x)B(x')>, are spatial correlation functions of the amplitude fluctuations at x and x'. The principal assumptions made in deriving Eq. (2) are: (1) the hydrophone is located in the nearfield of the pump transducer so that the pump radiation may be approximated by plane waves; (2) the medium is assumed to be weakly inhomogeneous so that the amplitude and phase fluctuations for the signal, pump, and second-order pressure waves are very small; and (3) there is complete transverse correlation of the amplitude fluctuations in the interaction region of the parametric receiver. The derivation of this result follows similar lines to that used by Smith[3], and Chotiros and Smith[4], in their analysis of the parametric transmitting array in a random medium. Evaluation of the correlation terms in Eq. (2) can be accomplished using theoretical expressions developed by Chernov[5] and Tatarski[6]. These expressions require certain knowledge of the refractive index spectrum for the acoustic medium in order to carry out their evaluation. The pertinent description of the medium used in the present study has been developed by Chotiros and Smith[7].

Experimental Results

The experimental work was done in a model tank with dimensions of 0.9m x 1.8m, and a depth of 0.8 m. Inhomogeneities were produced in the medium by an array of immersion heaters located along the bottom of the tank, as shown in Fig. 3. The flow of rising heated water is broken up into "patches" by a perforated aluminium sheet mounted about 5 cm above the heated array. A parametric receiver was formed in the tank using a 1 cm circular pump transducer

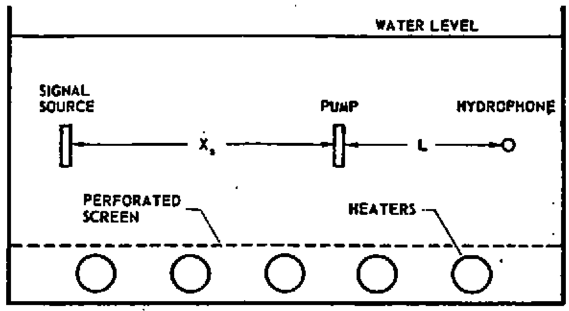

FIGURE 3
MODEL TANK

operating at 10 MHz, and a 2 mm square hydrophone placed on the acoustic axis and in the nearfield of the pump transducer. The "signal source" was a 2 cm square transducer operating at 1 MHz and located in the main beam of the parametric receiver at a distance, X_s, from the pump.

A block diagram of the apparatus used to generate and detect the signal, pump, and upper sideband pressure waves is shown in Fig. 4. Both signal and pump waves were pulsed to minimize feedover and multipath problems. The pulses were approximately 100 μsec in duration and were synchronized so that they occurred simultaneously in the interaction region of the array. The pressure detected by the hydrophone was filtered to separate the various waves. A 10 μsec segment of each received pulse was sampled, peak-detected, and recorded on paper tape for later off-line computer processing.

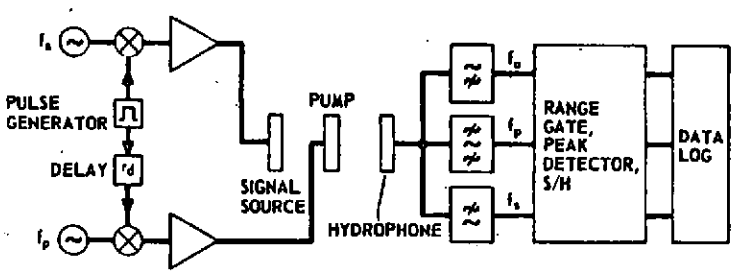

FIGURE 4
EXPERIMENTAL APPARATUS

The procedure used to conduct the experiments was as follows. The transducers were aligned in the desired geometry at mid-depth in the tank. Prior to turning on the tank heaters, 3 sets of data were recorded, with approximately 150 samples per data set, and at a sample rate of about 0.25 Hz. After recording these data, the heaters were turned on and allowed to warm up for 1 hour, so that the turbulent mixing could reach steady-state. Then an additional 5 sets of data were recorded as previously described, and the heaters were turned off until the following day.

Calculations of the coefficient of amplitude variation for the signal, pump, and upper sideband waves were made from the recorded data using Eq. (1). Results for the 1 MHz signal wave are shown in Fig. 5. Here the range parameter is the separation, X_s + L, between the signal source and the hydrophone (see

Fig. 3). Values of the coefficient of amplitude variation, C_s, are shown both with the heaters on (the circled data) and with the heaters off (the x's). The measurements obtained with the heaters off represent amplitude fluctuations produced by the electronic apparatus and by any "ambient" inhomogeneities in the tank, such as those due to microbubbles, biological matter, or residual thermal patches. These "ambient" fluctuations may be assumed to be uncorrelated

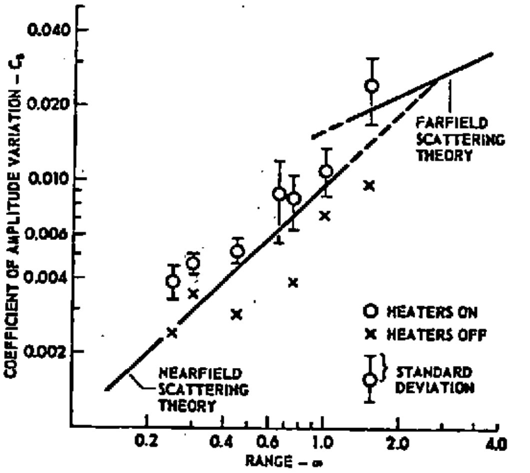

FIGURE 5
UNCOMPENSATED SIGNAL WAVE AMPLITUDE FLUCTUATIONS

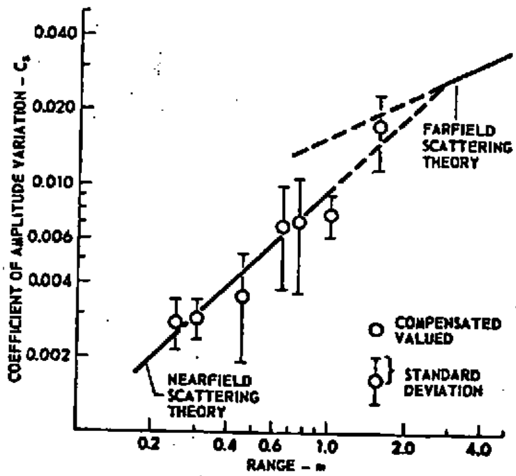

FIGURE 6
SIGNAL WAVE AMPLITUDE FLUCTUATIONS

with fluctuations produced with the tank heaters turned on. Consequently it is possible to compensate for these ambient fluctuations by subtracting the values of C measured with the heaters off (C_{OFF}) from the values of C measured with the heaters on (C_{ON}), the subtraction being done for each data set on a mean-square basis. In other words, $C_s = \sqrt{C_{ON}^2 - C_{OFF}^2}$, where C_s is the coefficient of amplitude variation due to the heaters being on. The results of this calculation, normalized to 30°C to account for variations in C with temperature[7], are shown in Fig. 6. The data are in reasonable agreement with Tatarski's theory[6] which is used in connection with Chotiros and Smith's[7] description of the medium to obtain the theoretical curves of Figs. 5 and 6. Similar results for the 10 MHz pump wave are shown in Fig. 7.

Two different kinds of measurement of C were made for the upper sideband wave. For one set of measurements, the array length was held constant at L = 0.10 m, and the range, $X_s + L$, of the signal source from the parametric receiver, was varied. Results of this experiment are shown in Fig. 8, where the same compensation for

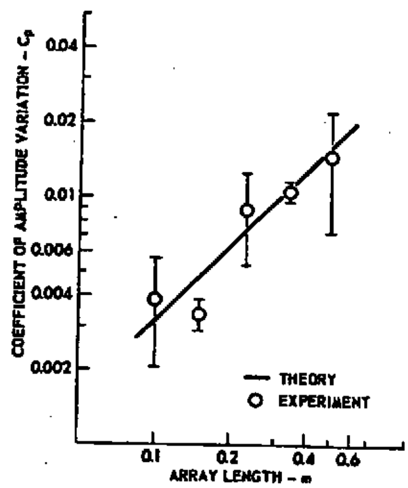

FIGURE 7
PUMP WAVE AMPLITUDE FLUCTUATIONS

ambient noise and temperature have been made as described above. A second experiment was conducted in which the range was held constant at $X_s + L = 0.75$m, and the array length was varied. Results from this experiment, compensated for ambient fluctuations and temperature, are shown in Fig. 9. In both Figs. 8 and 9, a theoretical curve is plotted which has been obtained from Eq. (2). The theoretical result appears to successfully predict the approximate level and general trend of the data.

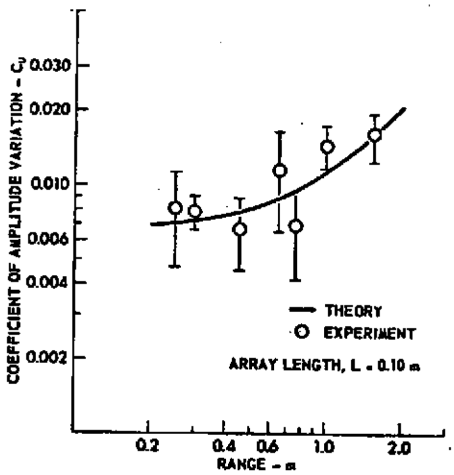

FIGURE 8
C_u FOR FIXED ARRAY LENGTH AND VARIABLE RANGE

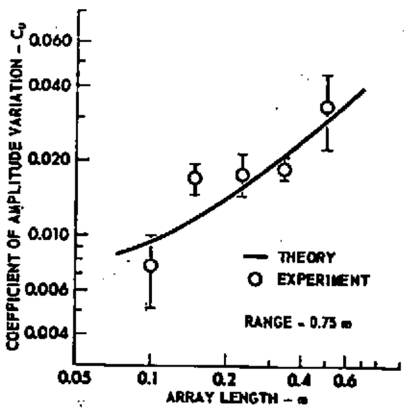

FIGURE 9
C_u FOR FIXED RANGE AND VARIABLE ARRAY LENGTH

A direct comparison of the performance of a parametric receiver and a "point" hydrophone was made by simultaneously measuring C_u and C_s, and then calculating the ratio, C_u/C_s. The experiment was conducted for two array lengths and a variety of ranges. The results of these measurements, having been compensated for ambient fluctuations and temperature, are shown in Fig. 10. Somewhat better agreement between theory and experiment is evident at the longer array length (L = 0.50); this is probably because the fluctuation-to-noise ratio, C_{ON}/C_{OFF}, was higher at the longer array lengths, thus enabling more accurate measurements of C_u. The theoretical curves, obtained from Eq. (2), agree reasonably well with the measured results.

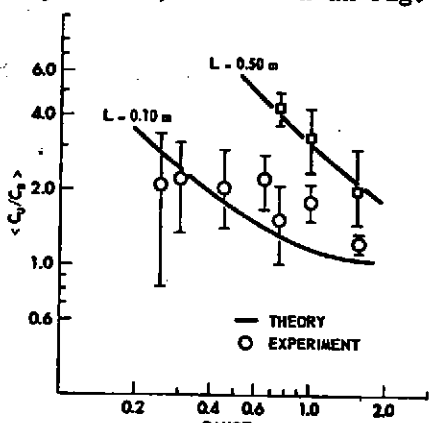

FIGURE 10
COMPARISON OF UPPER SIDEBAND AND SIGNAL WAVE FLUCTUATIONS

Discussion

There are a few simple, but useful, conclusions that can be drawn from the data presented in Figs. 6 through 10.

The agreement with theory of measurements of signal and pump wave amplitude fluctuations (Figs. 6 and 7) act as evidence that the acoustic medium is correctly described by the theoretical model[7] used.

Examination of Figs. 8 and 9 shows that the level of the upper sideband amplitude fluctuations tends to increase with either increasing range or increasing array length. This result seems reasonable if we recall the physical considerations discussed in connection with Fig. 1. It was noted there that fluctuations in the second-order pressure wave depends on fluctuations of the signal and pump waves at the interaction point. As the range between signal source and parametric receiver increases, the signal wave fluctuations at the interaction point increase (Fig. 6), and due to its dependence on signal wave fluctuations, C_u increases. Similarly, as the array length increases, the pump fluctuations at the interaction point increase (Fig. 7), causing C_u to increase.

Simple physical considerations can also be used to discuss the results of Fig. 10. It may be seen from the results shown in this figure that, for a 0.1 m long array, the upper sideband fluctuations are approximately equal to the signal wave fluctuations at the longer ranges. A similar theoretical result can be obtained for the 0.5 m array length. This result is reasonable, because as the range of the signal source from an array is increased, the fluctuations in the signal wave will increase while the high frequency fluctuations in the interaction region remain constant. At sufficiently long ranges, the fluctuations produced by scatterers in the interaction region will become negligible, and only the signal wave component will effectively contribute to the upper sideband fluctuations. For array lengths greater than the spatial correlation distance of the signal wave fluctuations, it may be expected that C_u would become less than C_s at sufficiently long ranges. This effect is due to the fact that the parametric receiver would effectively be summing uncorrelated "noise" throughout the interaction volume, thus "averaging out" some of the fluctuations. The size limitation of the model tank and choice of signal frequency prevented a demonstration of this effect; it would be an interesting point to pursue in future research.

Summary

This study investigated some aspects of parametric reception in an inhomogeneous medium. Theoretical predictions for C_u were compared to results obtained from model tank experiments. Some simple conclusions were drawn from the results and from physical considerations: C_u increases with range and with array length; and for sufficiently large ranges, signal wave fluctuations measured by a point hydrophone will approximately equal the sideband fluctuations measured by the parametric receiver.

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