Proceedings of the Institute of Acoustics

A 2-DIMENSIONAL FOURIER TRANSFORM METHOD FOR THE QUANTITATIVE MEASUREMENT OF LAMB MODES

David Alleyne and Peter Cawley

Department of Mechanical Engineering, Imperial College, London SW7 2BX, UK

1. INTRODUCTION

Since Worlton [1] used Lamb waves to non-destructively test plates there has been a great deal of interest in the application of plate waves in NDT. Lamb waves may be used in localised, detailed NDT applications, where the detectability of a given defect may be optimised by choosing the most suitable modes at the appropriate frequency-thickness product. Also, since they are essentially 2-dimensional, Lamb waves are attenuated less rapidly than 3-dimensional bulk waves, and hence may be propagated over considerable distances. Therefore, Lamb waves may be used in long range NDT applications, where a coarse and quick inspection of large plate-like structures may be carried out. However, more than one propagating Lamb wave always exists and velocity dispersion is usually evident. The time history of the response of the plate to an imposed excitation can therefore only be used to measure Lamb wave amplitudes and velocities approximately, because the shape of the response signal will be different at different positions along the surface of the plate. Also, if the group velocities of the Lamb modes excited are similar, a considerable propagation distance is required before they can be resolved in the time domain.

The key problem associated with the quantitative measurement of the characteristics of propagating Lamb waves is that more than one wave mode can exist at any given frequency. Therefore, standard techniques for the measurement of the velocity of dispersive waves such as the phase spectrum method developed by Sachse and Pao [2] cannot reliably be applied since they implicitly assume that only a single mode is present. This paper presents an extension of the phase spectrum method to the case of multi-mode propagation. Instead of carrying out a single Fourier transformation from the time to the frequency domain, a two dimensional Fourier transform is used to convert the data from the time-space plane to the frequency-wavenumber plane. The result is a plot of amplitude against frequency and wavenumber so the amplitude of a propagating Lamb mode at any given frequency and wavenumber can be determined. The method is demonstrated on both numerical and experimental data; further details of the technique can be found in Reference 3.

2. ANALYSIS

In the analysis that follows, the material is assumed to be linear elastic, isotropic, homogeneous, non-piezoelectric and non-absorbing. Assuming a harmonic wave propagating along a plate in the x direction, the displacement on the surface $\underline{u}(x,t)$, may be described by a general analytic expression given by Brekhovskikh [4] as,

$$\underline{u}(x,t) = \underline{A}(\omega)\, e^{i(\omega t - kx - \theta)} \qquad (1)$$

where $\underline{A}(\omega)$ is a frequency dependent amplitude constant, $\omega=2\pi f$ is the angular frequency, the wavenumber, $k=\omega/c$, c is the phase velocity and θ denotes the phase. Taking a two dimensional Fourier transform of equation (1) gives the wave amplitude, H, as a function of wavenumber and frequency:

$$H(k,f) = \int\int \underline{u}(x,t)\, e^{-i(kx+\omega t)}\, dx\, dt \qquad (2)$$

A discrete 2-dimensional Fourier transform may be defined in a similar way to the one-dimensional DFT given in, for example, Newland [5]. The result of this transformation will be a 2-dimensional array of amplitudes at discrete frequencies and wavenumbers. As in the one-dimensional case, aliasing must be avoided by sampling the data at a sufficiently high frequency in time, and wavenumber in space. Usually the signal will not be periodic within the temporal and spatial sampling windows and leakage will occur. Window functions such as the Hanning window may be used to reduce this leakage, and zeroes may be padded to the end of the signal to enable the frequency and wavenumber of the maximum amplitude to be determined more accurately. Details of the fast Fourier transform algorithm, aliasing, leakage, zero padding and other effects associated with discrete Fourier transforms may be found in standard texts, for example, Randall [6].

The 2-dimensional Fourier transform method is applied by carrying out a Fourier transform of the time history of the response at each position monitored to obtain a spectrum for each position. At this stage, an array with the spectral information for each position in its respective column is obtained. A spatial Fourier transform of the vector (row) formed by the components at a given frequency then gives the amplitude-wavenumber-frequency information. In practice a 2-dimensional fast Fourier transform algorithm (2D-FFT) may be used. In contrast to the phase spectrum technique of Reference 2, this method enables the amplitude and velocity of different modes propagating at the same frequency to be determined.

3. NUMERICAL RESULTS

A series of finite element studies has been undertaken in order to check the operation of the technique. In each case, a single Lamb mode was excited at one end of a plate and the resulting displacements at a series of points along the plate are monitored in order to study the initial propagation of the wave and its interaction with boundaries or defects. The finite element package used was FINEL which was developed at Imperial College [7]. A uniform mesh of 8 noded square elements was used, the mesh subdivision being selected to give a minimum of 10 nodes per wavelength. The input signal was a 5 cycle, 1 MHz tone burst in a Hanning window and the response of the plate was output at intervals of 100 nsec giving a 'sampling frequency' of 10 MHz. Further discussion of the numerical excitation of a single Lamb mode can be found in Reference 3.

Figs 1a and 1b show the normalised time histories of the response of the top surface of a 0.5mm thick steel plate at x=0 and 100mm respectively, where the input at x=0 was appropriate to excite only the s_0 Lamb mode. The response of the plate at x=100mm shows that the propagating wave is essentially non-dispersive and the group velocity is 5.43km/s. Fig 1c shows the result of

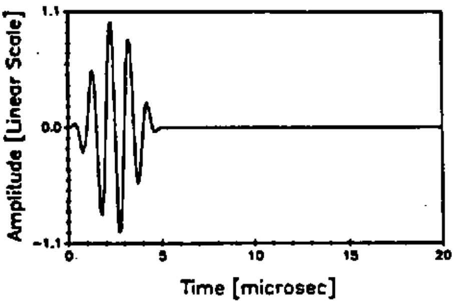

Fig. 1a Normalised time history of the z-displacement at x=0 in all the numerical tests.

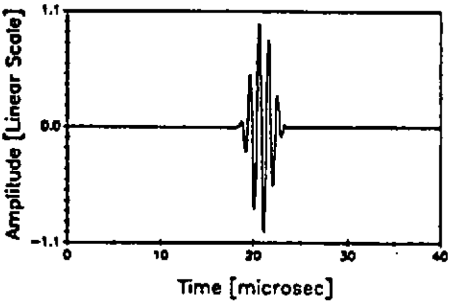

Fig. 1b Predicted normalised time history at x=100mm in a 0.5mm thick plate when the input at x=0 was designed to excite only s_0.

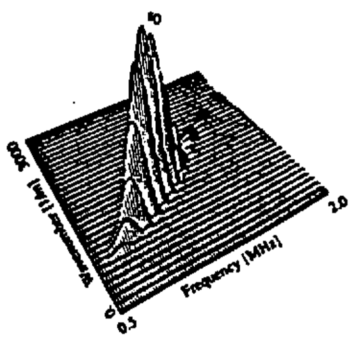

Fig. 1c Normalised 3-D plot of the 2D-FFT results of the case given in Fig 1b, showing a single propagating mode, s_0.

carrying out a 2-dimensional Fourier transform on the 64 equally spaced monitored time histories between x=100mm and x=163mm. The results are presented in the form of a 3-dimensional plot of amplitude versus frequency and wavenumber. The amplitude scale is linear and the units are arbitrary. (This scale is not shown to improve the clarity of the plot.) At each frequency, the amplitude is only significant at a single wavenumber, indicating that only one mode, in this case, s_0, is present. The amplitude reaches a maximum at a frequency of 1MHz which is the centre frequency of the excitation tone burst.

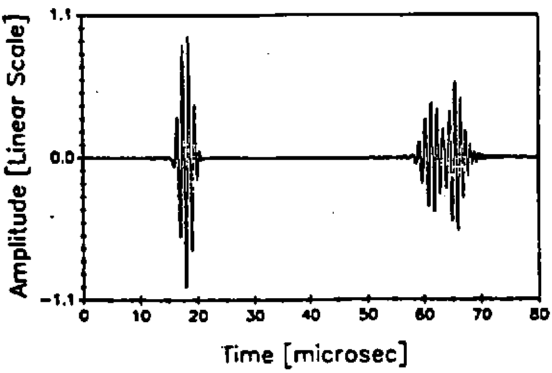

Fig. 2a Predicted normalised time history at x=50mm in a 3.0mm thick plate, showing the first passage of the a_0 wave and the reflection containing both a_0 and a_1 from the free end of the plate.

Fig. 2b Normalised 3-D plot of the 2D-FFT results from the reflected wave shown in Fig 2a.

QUANTITATIVE MEASUREMENT OF LAMB MODES

Fig 2a shows the normalised time history of the response of a 3 mm thick steel plate at x=50mm, when the input at x=0, which had the same amplitude-time relationship as that shown in Fig 1a, was appropriate to excite only a_0. The duration of the test was long enough to include the response of the plate after reflection from the free end of the plate at x=125 mm. The first wave packet seen in the response is the a_0 mode propagating along the plate; its shape has changed very little, indicating that the wave is essentially non-dispersive at the input frequency-thickness.

On reflection from the end of the plate ($t \geq 40\mu s$) more then one propagating mode is present (a_0 is mode converted), but the modes are superimposed and their amplitudes cannot be determined from the time history of the response of the plate. Fig 2b shows the result of carrying out a 2-dimensional Fourier transform on the time histories of 64 equally spaced positions from x=30mm to x=67.8mm, when the incident signal ($t \leq 40\mu s$) was gated out. The maximum amplitude of the response of the plate is at 1MHz, which is the centre frequency of the excitation tone burst. However, at each discrete frequency in Fig 2b there are two distinct wavenumbers at which the amplitude is a maximum. These correspond to a_0 and a_1. Hence, there are two propagating modes and the amplitudes of the two modes are a function of frequency.

4. EXPERIMENTAL RESULTS

The aim of the experimental investigation was to excite a single Lamb wave and to study its propagation. Two grease coupled variable angle transducers mounted in a perspex body were used as the transmitter and receiver, the transmitter being driven by a 5 cycle tone burst at a pre-selected frequency. The signal from the receiver was captured by a Le Croy 9400 digital oscilloscope and was then passed to a micro-computer for storage and processing. The receiving transducer was indexed along the plate to obtain measurements at 64 equally spaced positions.

The accuracy of positioning the transducer in these experiments was about 2% of the spatial sampling interval. However, the quality of the coupling between the transducer and the plate probably varied as the transducer was indexed, which introduced errors. It should also be remembered that the measured response is integrated over the area of the transducer element. These problems could be reduced by using a transducer consisting of an array of small elements to capture the time histories.

The plates used in the experimental investigations were approximately 300mm wide and 1m long, and the average propagation distance was restricted to 200mm in order to keep the signal-to-noise ratio high. Individual Lamb waves were selectively excited by applying the coincidence principle (see, for example, Worlton [1]). The angle of incidence θ required for the excitation of the desired mode is calculated from $\theta = \sin^{-1}(c_L/c)$, where c_L is the phase velocity of a compression wave incident on the surface of the plate and c is the phase velocity of the Lamb wave to be excited. Since the transducers were grease coupled, only motion normal to the plate surface could be detected. In all the experiments the sampling frequency was 8MHz and a 1024 (1k) point temporal FFT was used.

Fig 3a shows the response of a 0.5mm thick steel plate at 200mm from the transmitter, where the incident angle was appropriate to excite and receive s_0. The frequency of the excitation tone burst

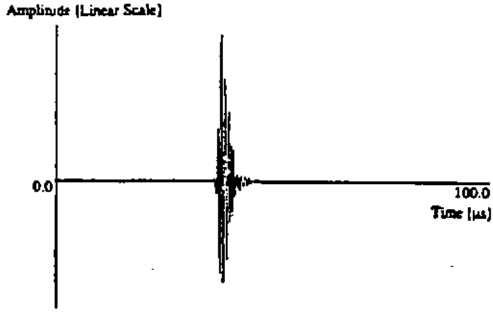

Fig. 3a Normalised time history of the measured response at x=200mm in a 0.5mm thick plate when the excitation was appropriate for s_0.

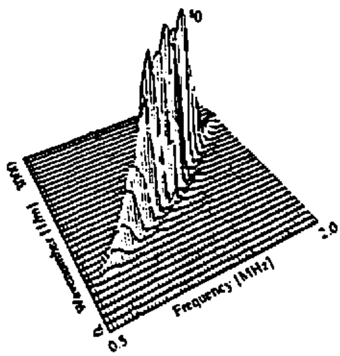

Fig. 3b Normalised 3-D plot of the 2D-FFT results for the case of Fig 3a.

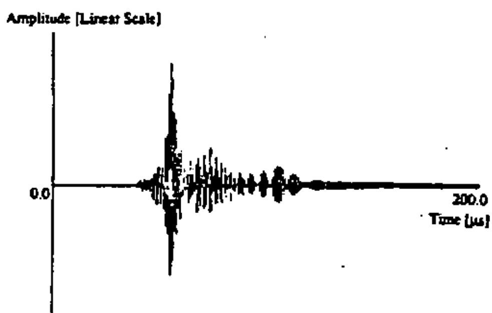

Fig. 4a Normalised time history of the measured response at x=200mm in a 3.0mm thick plate when the excitation was appropriate for a_1.

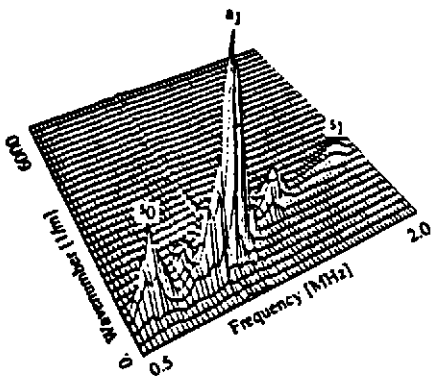

Fig. 4b Normalised 3-D plot of the 2D-FFT results for the case of Fig 4a.

was 1.2MHz and the spatial sampling interval was 1mm. Fig 3b shows the normalised 3-dimensional view of the amplitude-wavenumber-frequency information, which was obtained by carrying out a 2-dimensional Fourier transformation of the time histories from 64 equally spaced positions between x=175mm and x=238mm. The maximum amplitude of the response of the plate is between 0.8MHz and 1.8MHz. At each discrete frequency in Fig 3b the wavenumber at which the amplitude is a maximum corresponds to the s_0 mode, and, as in Fig 1c, no other peak is evident confirming that only the s_0 mode is observed. The dips in the amplitude of the experimental results seen in Fig 3b correspond to those in the spectrum of a five cycle tone burst in a rectangular window [3]; these are not seen in the numerical results because the numerical input was a tone burst in a Hanning window which leads to a smoother spectrum.

Fig 4a shows the time history of the response of a 3mm thick steel plate 200mm from the transmitter, when the input signal was intended to excite only a_1, the frequency of the excitation tone burst was 1MHz and the spatial sampling interval was 0.75mm. Dispersion is present, but a discrete wave packet with large amplitude is present and its group velocity indicates that it is a_1. Fig 4b shows the normalised 3-D view of the amplitude-wavenumber-frequency information, which was obtained by carrying out a 2-dimensional Fourier transformation of the time histories of 64 equally spaced positions between x=175mm and x=222.25mm. Three modes, s_0, s_1, and a_1 are present. The different modes are observed in the frequency-thickness regions where their wavenumber and mode shape are appropriate for their excitation and reception; a_1 dominates the response of the plate, its maximum amplitude being measured at 1MHz the centre frequency of the excitation. s_0 is seen only at frequencies below 1MHz while s_1 is seen at higher frequencies and has been excited by the sidelobes of the excitation signal.

The experimental results show that it is very difficult to obtain a pure mode using variable angle transducers. The plate is excited over a range of frequencies and wavenumbers, the frequency range being mainly dependent on the number of cycles in the excitation tone burst and whether a smoothing window is applied. The wavenumber range is mainly dependent on the incident angle and on the size and shape of the transducers.

The effective frequency range of the excitation may be reduced by using tone bursts modulated by an appropriate weighting function to reduce the amplitude of the high and low frequency components. Experimentally, this can be achieved by using an arbitrary function generator to produce the excitation tone burst.

5. CONCLUSIONS

The 2D-FFT method has been used to measure the amplitudes and velocities of propagating Lamb waves over a range of frequencies and phase velocities in a single test. It has been shown that this technique may be used when there is multi-mode propagation and/or dispersion.

The 2D-FFT method is not affected by the Lamb wave propagation distance, indeed the propagation distance is limited only by the signal-to-noise ratio. This makes the method potentially attractive in NDT applications. Multi-element transducers are now readily available, thus making the implementation of the 2D-FFT method commercially attractive. The computational requirements of the 2D-FFT are fairly modest and it is anticipated that the method could readily be implemented on a micro-computer interfaced to a data capture system.

6. REFERENCES

1. D.C. Worlton, Non-destructive testing, 15, 218 (1957).
2. W. Sachse and Y-H. Pao, J. Appl. Phys., 49, 4320 (1978).
3. D.N. Alleyne and P. Cawley, 'A 2-dimensional Fourier transform method for the measurement of propagating multi-mode signals', J. Acoust Soc Am (in press).
4. L.M. Brekhovskikh, Waves in layered media, (Academic Press, New York, 1960).
5. D.E. Newland, Random vibrations and spectral analysis (Longman Inc., New York, 1984).
6. R.B. Randall, Frequency analysis (Bruel and Kjaer, 1987).
7. D. Hitchings, FE77 user manual, Imperial College internal report (1987).