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Wavenumber Filtering of a Random Excitation by an Elastic Plate Immersed in a Fluid.

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1. INTRODUCTION

The behaviour of a thin plate immersed in a fluid is studied with regard to the transmission of arbitrary wavenumber and frequency components of both normal and tangential forces applied to one face. The behaviour is found to be markedly different for the two cases.

2. THEORY

2.1. Mathematical Formulation of the Problem

Consider an infinite flat plate immersed in an infinite fluid and lying so that its neutral plane is $z = 0$. The plate is excited into vibration by applying a random excitation (normal and tangential) over a finite area for a finite time on the face $z = -h$. This excitation may be decomposed into its space-time Fourier components to give stresses of the form $P_{zz} = N \exp[i(\omega t-kx)]$ normal and $P_{xz} = S \exp[i(\omega t-kx)]$ tangential to the face.

No damping mechanism is included for the solid, but it may still lose energy by radiation into the fluid provided $kc_o<\omega$, where c_o is the fluid sound speed. If $kc_o>\omega$ no radiation can occur and there is the possibility of undamped resonances.

The treatment here follows that of Ewing, Jardetsky and Press [1]. The compressional and shear wave velocities, α and β, are related to the Lamé elastic constants and the solid density ρ_1 by $\alpha^2 = (\lambda+2\mu)/\rho_1$ and $\beta^2 = \mu/\rho_1$. It will be assumed that all quantities depend only on x and z; the vector displacement potential $\underline{\psi}$ can then be chosen so that only its y-component, denoted by ψ, is not zero. The scalar displacement potential is denoted by ϕ.

The effect of the applied stresses is to excite multiply reflected compressive and shear waves in the solid, described by the potentials ϕ_1 and ψ_1, and compressive waves in the fluid described by ϕ_o and ϕ_2. Suppressing the time dependence the displacement potentials are given by

$$\phi_o = A_o e^{i\gamma_o z} e^{-ikx}, \quad z<-h; \quad \phi_2 = A_2 e^{-i\gamma_o z} e^{-ikx}, \quad z>h$$

$$\phi_1 = (A\sin\gamma z + B\cos\gamma z)e^{-ikx}; \quad \psi_1 = (C\sin\gamma' z + D\cos\gamma' z)e^{-ikx} \quad \ldots\ldots(2.1)$$

where $\gamma = (\omega^2/\alpha^2 - k^2)^{\frac{1}{2}}$, $\gamma' = (\omega^2/\beta^2 - k^2)^{\frac{1}{2}}$, $\gamma_o = (\omega^2/c_o^2 - k^2)^{\frac{1}{2}}$ which are the z-components of wavenumber for compressive and shear waves in the solid and compressive waves in the fluid respectively. The six unknown wave amplitudes in equations (2.1) are found by requiring continuity of P_{zz}, P_{xz} and w_z across the two interfaces $z = \pm h$ giving six equations. As a consequence of the symmetry about $z = 0$, these equations may be combined to give two uncoupled matrix equations of order three.

These matrix equations describe modes of vibration which are antisymmetric and symmetric with respect to the plane $z = 0$. The first antisymmetric mode is the bending wave of thin plate theory. The first symmetric mode represents compressional waves familiar from longitudinal rod theory. Dispersion relations for the first two symmetric and antisymmetric modes are shown in Figure 1.

2.2. Thin Plate Approximations for the Displacement

The matrix equations can be solved for the contributions of the symmetric (w_s) and antisymmetric (w_a) displacements on the face $z = +h$ of the plate giving

$$w_s = \frac{N\rho_1\omega^2\gamma + Sik\mu\,[(\gamma'^2-k^2)\cot\gamma h - 2\gamma\gamma'\cot\gamma' h]\,e^{-ikx}}{2\mu^2\left[\dfrac{i\rho_o\omega^4\gamma}{\rho_1\beta^4\gamma_o} + (\gamma'^2-k^2)^2\cot\gamma h + 4k^2\gamma\gamma'\cot\gamma' h\right]} \qquad \ldots\ldots(2.2)$$

$$w_a = \frac{N\rho_1\omega^2\gamma - Sik\mu\,[(\gamma'^2-k^2)^2\tan\gamma h - 2\gamma\gamma'\tan\gamma' h]\,e^{-ikx}}{2\mu^2\left[\dfrac{-i\rho_o\omega^4\gamma}{\rho_1\beta^4\gamma_o} + (\gamma'^2-k^2)^2\tan\gamma h + 4k^2\gamma\gamma'\tan\gamma' h\right]} \qquad \ldots\ldots(2.3)$$

The dispersion relations are obtained by equating the denominators of (2.2) and (2.3) to zero.

These solutions will be examined close to the first symmetric resonance at $k = k_s\,(=\alpha\omega/[2\beta(\alpha^2-\beta^2)^{\frac{1}{2}}])$. Making the thin plate approximation $\omega h/\alpha$, $\omega h/\beta$, $\omega h/c_o << 1$, equations (2.2) and (2.3) can be approximated by

$$w_s \approx \frac{\left[-N\gamma_s^{\,2} + \dfrac{Sik_s}{h}\left(\dfrac{\lambda}{\lambda+2\mu}\right)\right]e^{-ikx}}{\dfrac{4\rho_1 k_o\omega^2}{ik_s h\gamma_{os}}\left[k-k_s + \dfrac{i\rho_o k_s h\omega^2}{8\mu k_o}\ \dfrac{\lambda}{(\lambda+2\mu)(\lambda+\mu)}\right]} \qquad \ldots\ldots(2.4)$$

$$w_a \approx \frac{i\gamma_{os}}{2\rho_o\omega^2}\left[\frac{N + Sik_s h}{1 + ih\rho_1\gamma_{os}/\rho_o}\right]e^{-ikx} \qquad \ldots\ldots(2.5)$$

where the subscript s implies that a quantity is evaluated at $k = k_s$.

Thus
$$\gamma_s^{\,2} = \frac{-\rho_o\lambda^2\omega^2}{4\mu(\lambda+2\mu)(\lambda+\mu)} \quad \text{and} \quad \gamma_s'^{\,2} = \frac{\rho_1\omega^2(3\lambda+2\mu)}{4\mu(\lambda+\mu)}$$

Equation (2.4) is clearly a resonance term of width, at half height,

$$\Delta k = \frac{\sqrt{3}\ \rho_o\rho_1^{\frac{1}{2}}c_o\omega^2 h\lambda^2}{8\mu^{3/2}(\lambda+\mu)^{3/2}(\lambda+2\mu)^{\frac{1}{2}}} \qquad \ldots\ldots(2.6)$$

and of height $$\hat{w}_s = \frac{-i\gamma_{os}}{2\rho_o\omega^2}\left[N + \frac{Sik_s}{\gamma_s^2 h}\frac{\lambda}{\lambda+2\mu}\right]e^{-ikx} \qquad \text{......(2.7)}$$

The main features of the formulae (2.5) and (2.7) are (i) for pure normal excitation (S=0), the symmetric and antisymmetric modes are equal in amplitude and opposite in phase to lowest order in h and therefore cancel to that order. This gives a sharp dip in the displacement (or radiated pressure) versus wavenumber spectrum at $k = k_s$. (ii) for pure shear excitation (N=0), the symmetric resonance peak displacement is proportional to ω^2/h and the antisymmetric displacement to h. Their ratio is, to lowest order in h, $\lambda/[(\lambda+2\mu)\gamma_s^2h^2]$. Hence, in the thin plate approximation, the symmetric mode is dominant close to $k = k_s$. (iii) It can be seen from Figure 1 that the first symmetric mode is non-dispersive for kh<1. Hence this mode propagates at constant speed c_s ($= \frac{2\beta}{\alpha}(\alpha^2-\beta^2)^{\frac{1}{2}}$ the thin plate compressional wave speed) if its wavenumber is such that kh<1 so that radiation at the particular angle ($\sin^{-1}c_o/c_s$) is reinforced from each Fourier component having $\omega = kc_s$.

3. NUMERICAL RESULTS

The exact equations (2.2) and (2.3) have been used to calculate the pressure per unit exciting stress (N=1, S=0 or N=0, S=1) for a range of frequencies. A typical set of results is displayed in Figures 2-4 with a frequency of 3kHz, a plate thickness of 5cm and the material constants of GRP. Figure 4 shows that for normal forces the symmetric resonance at $k=6.64\text{m}^{-1}$ is cancelled by the antisymmetric mode resulting in a dip. The antisymmetric resonance at $k=28.6\text{m}^{-1}$ is not radiation damped but is, however, dispersive and therefore cannot lead to any concentration of radiation at a particular angle. There is another non-radiating resonance close to k_o corresponding to the surface wave discussed, for example, by Morse and Ingård [2].

4. CONCLUSIONS

In the presence of a random shear force, it has been demonstrated that a thin plate will act selectively to transmit those components of the excitation force which propagate with speeds close to the solid compressive wave speed. This results in radiation from the far face concentrated in a particular direction of angle $\sin^{-1}(c_o/c_s)$ to the plate normal.

For a random normal excitation, however, this particular component is suppressed by the plate.

5. REFERENCES

1. W.M.Ewing, W.S.Jardetsky and F.Press, Elastic Waves in Layered Media, McGraw Hill (New York 1957).
2. P.M.Morse and K.U.Ingard, Theoretical Acoustics, McGraw Hill (New York 1968).

6. ACKNOWLEDGEMENTS

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FIGURE 1.

DISPERSION CURVES FOR FIRST AND SECOND SYMMETRIC AND ANTISYMMETRIC MODES FOR A FREE ELASTIC PLATE WITH σ = 0.25 (AFTER TOLSTOY AND USDIN)

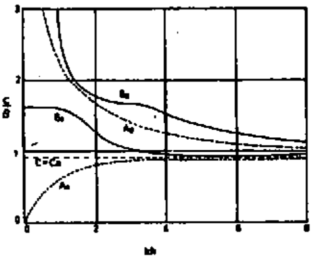

FIGURE 2.

COMPONENT PRESSURE WAVENUMBER SPECTRA FOR SHEAR EXCITATION AT 3 kHz (2h = 5cm)

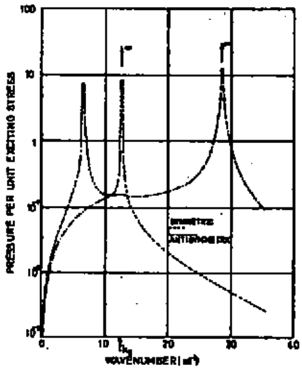

FIGURE 3.

COMPONENT PRESSURE-WAVE NUMBER SPECTRA FOR NORMAL EXCITATION AT 3kHz (2h = 5cm)

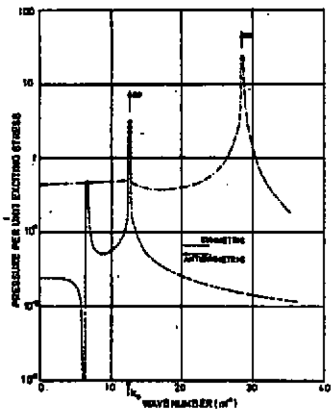

FIGURE 4.

TOTAL PRESSURE WAVENUMBER SPECTRUM FOR NORMAL EXCITATION AT 3kHz (2h = 5cm)