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A THREE-DIMENSIONAL PROPAGATION MODEL-FOR3D

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ABSTRACT

Three-dimensional variability occurs in the ocean volume and bottom on length scales that are important for acoustic propagation. Therefore, there are situations when it is necessary to use 3-dimensional propagation models for accurate predictions. A recent propagation model (called FOR3D) that has three-dimensional capability is reviewed. A summary of the theoretical development, numerical solution procedures, and computer implementation of this model will be presented. An application of the model to propagation through the sound speed field from a mesoscale ocean prediction model will be discussed. It is important to describe both the capabilities of FOR3D (what it can do) and its limitations (what it cannot do). Its current limitations suggest some enhancements which can improve its capabilities and performance. These enhancements will be discussed along with a number of research topics in the area of three-dimensional propagation modeling.

1. INTRODUCTION

In reality, three-dimensional variability occurs in the ocean bottom on a variety of length scales. These three-dimensional variabilities can be important for acoustic propagation. In such case, to obtain accurate predictions, an adequate three-dimensional acoustic propagation model must be used. The production of three-dimensional acoustic propagation models had been on the rise in the past 5 years. There exist a number of three-dimensional acoustic models; each has its advantages and limitations. An updated literature listing can be found in Ref.1. Additional references can be found in Refs. 2-4. A useful acoustic propagation model with azimuthal coupling capabilities, theoretically developed by Lee et al. [5], and used for many realistic applications, is reviewed in this paper. Details of this model are described in Refs. 5-10. This 3-dimensional acoustic propagation model was implemented into a research computer code with the acronym FOR3D, which stands for the use of a Finite difference scheme in conjunction with a numerical Ordinary-differential-equation method, and a Rational function approximation for matrix exponentials for solving 3-Dimensional acoustic propagation problems. Although the model itself has some important features, it still requires improvement both in capabilities and in performance. Enhancements of more desirable capabilities can make the model widely useful for research studies as well as applications. A section is devoted briefly to outline these enhancements. This paper begins with a brief history of the theoretical development involving the mathematical formulation and the introduction of an efficient numerical solution. Following that is the computer implementation. A section will illustrate the occurrence of true 3D effects whereas many other 3D models without adequate capability of handling the azimuthal coupling cannot detect such phenomena. Attention will be called to note an important feature of the FOR3D model upon the capability to detect 3D effects. Recent advances of the FOR3D model will be outlined.

2. BACKGROUND

2.1. Mathematical Model

The 3-dimensional (depth z, range r, and azimuth θ) FOR3D mathematical model was derived from an elliptic equation, Eq.(2.1), below,

$$\frac{\partial^2 u}{\partial r^2} + 2ik_o \frac{\partial u}{\partial r} + k_o^2(X + Y)u = 0. \tag{2.1}$$

where $u = u(r,\theta,z)$ the wave field,
k = reference wavenumber,

$X = n^2(r,\theta,z) - 1 + \frac{1}{k_o^2}\frac{\partial^2}{\partial z^2}$,

$Y = \frac{1}{k_o^2 r^2}\frac{\partial^2}{\partial \theta^2}$, and

$n = n(r,\theta,z)$ the index of refraction.

An outgoing wave equation can be obtained by factoring Eq.(2.1) into a product of outgoing and incoming wave equations [5]. The wave equation representing one-way outgoing wave propagation is identified by

$$u_r = ik_o(-1 + \sqrt{1+X+Y})u. \tag{2.2}$$

Lee et al. [5] treated the square-root operator $\sqrt{1+X+Y}$ by:

$$\sqrt{1+X+Y} \cong 1 + \frac{1}{2}X - \frac{1}{8}X^2 + \frac{1}{2}Y. \tag{2.3}$$

to derive the 3-dimensional wide angle wave equation,

$$u_r = ik_o(-1 + [1 + (1/2)X - (1/8)X^2 + (1/2)Y])u \qquad (2.4)$$

where the wide angle capability is in the vertical rz-plane, and with narrow angle capability in the azimuthal direction.

2.2. A Numerical Solution

An implicit finite difference scheme of the Alternating-Direction-Implicit (ADI) type was also introduced by Lee et al.[5] to solve Eq.(2.4) which takes this expression:

$$[\ 1 + (\tfrac{1}{4} - \tfrac{\delta}{4})X\ ]\ [\ 1 - \tfrac{\delta}{4}Y\ ]u^{j+1} = [\ 1 + (\tfrac{1}{4} + \tfrac{\delta}{4})X\ ]\ [\ 1 + \tfrac{\delta}{4}Y\ ]\ u^j. \qquad (2.5)$$

where $\delta = ik_o \Delta r$.

The stability of this marching scheme is assured by the fact that the norms of the operators $[1+(1/4-\delta/4)X]^{-1}[1+(1/4+\delta/4)X]$ and $[1-(\delta/4)Y]^{-1}[1+(\delta/4)Y]$ are both unitary.

2.3. Computer Implementation

The symbolic form of Eq.(2.5) can be written as

$$ABu^{j+1} = A^*B^*u^j. \qquad (2.6)$$

The computer code solves Eq.(2.6) in two steps:

1. $Aw^{j+1} = A^*B^*u^j$. (2.7)

2. $Bu^{j+1} = w^{j+1}$. (2.8)

In each step, the code solves a symmetric tridiagonal system of equations which can be computed economically.

3. DETECTING 3-DIMENSIONAL EFFECTS

When real data are available, it is not clearly known a priori whether 3-dimensional effects will occur. Therefore, it is preferable to use an acoustic propagation model wih azimuth-coupling capability that will detect any 3-dimensional effects that may be present. This capability is an important feature of the FOR3D model. An analysis of this capability would help the reader to understand how the FOR3D code was constructed. Based on azimuthal variations, a 3-dimensional problem can be classified into 3 categories according to physical effects [6]: (1) true 3-dimensional effects with the presence of azimuthal coupling, (2) 3-dimensional problem without azimuthal coupling, and (3) true 2-dimensional problems.

3.1. Mathmatical Classification [6]

Equation (2.1) can be written as

$$u_{rr} + 2ik_o u_r + u_{zz} + (1/r^2)u_{\theta\theta} + k_o^2(n^2(r,\theta,z)-1)u = 0. \qquad (3.1)$$

A special case of Eq.(3.1) can be considered to be

$$u_{rr} + 2ik_o u_r + u_{zz} + k_o^2(n^2(r,\theta,z)-1)u = 0. \qquad (3.2)$$

A special case of Eq.(3.2) can be written as

$$u_{rr} + 2ik_o u_r + u_{zz} + k_o^2(n^2(r,z) -1)u = 0. \qquad (3.3)$$

Equation (3.1) accommodates true 3-dimensional wave propagation with azimuthal coupling because of the presence of both terms $(1/r^2)u_{\theta\theta}$ and $n(r,\theta,z)$ in the equation. Equation (3.2) is a special case of Eq.(3.1) because of the absence of the term $(1/r^2)u_{\theta\theta}$; however, it is still considered 3-dimensional even though the azimuthal coupling is absent, because the 3-dimensional index of refraction $n(r,\theta,z)$ still appears in the equation. Equation (3.3) represents a truly 2-dimensional model because of the absence of the term, $(1/r^2)u_{\theta\theta}$.

In addition, the index of refraction is 2-dimensional, i.e. $n=n(r,z)$. Thus, 2-dimensional models used to solve Eq.(3.3) (recognized as "2D" solutions) are not sufficient to solve Eqs.(3.1) and (3.2). Again, incomplete 3-dimensional models used to solve Eq.(3.2) (recognized as "N x 2D" solutions) are not sufficient to solve Eq.(3.1). On the other hand, the models with azimuthal coupling capability used to solve Eq.(3.1) (recognized as "3D" solutions) are also sufficient for solving Eqs.(3.2) and (3.3).

From the above discussion, it follows that an accurate and efficient 3D model with azimuthal coupling capability should be a desirable one. In fact, the FOR3D model has this advantage.

3.2. Computational Discussion

The numerical marching procedure to compute scheme (2.5) deals with the solution of 2 symmetric tridiagonal systems, (2.7) and (2.8). In the ADI numerical algorithm the matrix B represents the azimuthal coupling. From the formulation [5], the matrix B has off-diagonal elements, s, in the form:

$$s = \frac{i}{4} \frac{\Delta r}{k_0 r^2 (\Delta\theta)^2} . \tag{3.4}$$

In the 2D case, one needs to solve only Eq. (2.7) because the B matrix is an Identity matrix. In the N x 2D case where only the index of refraction, $n(r,\theta,z)$, is 3-dimensional, one still needs to solve only system (2.7). In the true 3D case, one needs to solve both systems (2.7) and (2.8) where the B matrix in NOT an Identity matrix.

An important feature of the FOR3D model is that one can select a small θ-increment in Eq.(3.4) such that the B matrix is generated to be different from the Identity. This feature helps the user to ask the code explicitly to try to allow for 3-dimensional effects whether they are present or not. Consequently, if 3-dimensional effects are not physically present, the computation would automatically generate an Identity matrix B. This can be seen in the discussion given next.

In the numerical formulation of the system (2.8), each row of Bu^{j+1}, excluding those from boundary terms, has the form

$$(Bu^{j+1})_{m,\ell} = -su^{j+1}_{m,\ell+1} + (1 + 2s)u^{j+1}_{m,\ell} - su^{j+1}_{m,\ell-1} \tag{3.5}$$

where the superscript j is the range index, m the depth index, and ℓ the azimuth index. In the event that there is no true azimuthal coupling, then

$$u^{j+1}_{m,\ell+1} = u^{j+1}_{m,\ell} = u^{j+1}_{m,\ell-1} . \tag{3.6}$$

Consequently, Eq.(3.5) will become

$$(Bu^{j+1})_{m,\ell} = Iu^{j+1}_{m,\ell} . \tag{3.7}$$

Therefore, non-azimuthal coupling would consistently occur in the numerical computation. More details of this discussion can be found in

Ref. 6.

4. A NUMERICAL APPLICATION

True 3-dimensional effects have been demonstarted by examples in Refs. 6-8 . As supporting evidence, an example from Ref.8 is shown next.

This example describes propagtion confined in a sector 80 degrees wide, a variable water depth up to 2500 meters deep, and a propagation range up to 100 kilometers. Other inputs for this application consist of: source location at 39°N and 72.7°W, source depth=50 m, source frequency=50 Hz, left sector boundary at 140° true, right sector boundary at 220° true, density in the water=1.0 g/cm^3 , density at bottom= 1.7 g/cm^3 , and the receiver depth=100 m.

As the figure below shows an 2D or (N x 2D) models do not adequately detect true 3-dimensional effects.

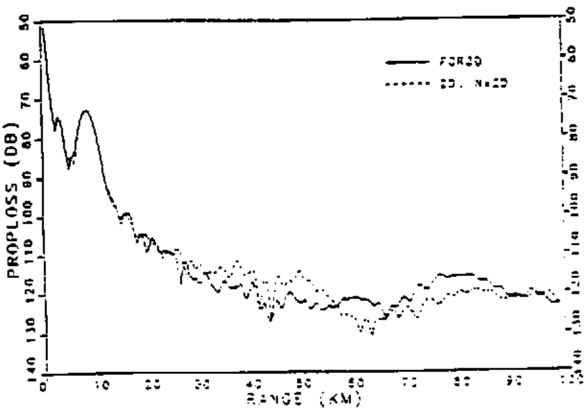

Figure 4.1: Propagtion Loss vs RANGE for 2D, N x 2D, and FOR3D

5. ENHANCEMENTS

In its current version, the FOR3D model can predict wave propagation in 3 dimensions. The prediction covers problems to fairly long ranges, for shallow and/or deep water. The model treats the density variation in the medium numerically by handling an irregular bottom interface. Most importantly, the model allows the user to call for detection of 3D effects. The model can be made more useful if some practical capabilities can be incorporated. These additional capabilities are considered to be the enhancements. Detailed discussions of these enhancements would be very lengthy but can be found in the three references 11,12, and 13. Briefly, these enhancements include:

(1) The development of a capability to handle the interface between fluid and elastic media in 2 dimensions [11]. This capability has been extended to handle wide angle propagation in an elastic medium. In addition, the capability to handle shear wave propagation in three dimensions is also under development.

(2) The development of a high-order numerical finite difference scheme to improve the computation speed for long range propagation problems was published in Ref. 12.

(3) A recent useful enhancement of this model is the development of a mathematical model for 3-dimensional backscattering. Similar numerical techniques, used for the FOR3D model, were used for 3-dimensional backscattering computations. Basic progress has been made in this topic, as is reported in an article cited at Ref. 13.

6. CONCLUSION

It is understood that 3-dimensional propagation effects exist in the naturral ocean. When this situation occurs, using 2-dimensional (2D) or incomplete 3-dimensional (N x 2D) models is not adequate to solve true 3-dimensional problems. In this case, a 3-dimensional model with azimuthal coupling capability is preferable. The FOR3D model can be used for this case. Further improvements to the FOR3D model with respect to capability and performance will be very valuable.

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