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ROBUST BROADBAND LMS ABF ALGORITHM

A NOVEL ROBUST BROADBAND FREQUENCY DOMAIN LMS ALGORITHM FOR ADAPTIVE BEAMFORMING.

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1. Introduction

This paper is concerned with the optimal processing of data from an array of sensors/antennas. Such sensors may be sonar, radar, VHF or HF radio or 'acoustics in air'. The aims of the processing are

(a) The detection of weak wanted signals

(b) The bearing estimation of weak wanted signals

(c) Presentation of the wanted signal with maximum S/N ratio, by steering nulls at interfering signals

(d) Accurate bearing estimation and discrimination of strong sources.

2. Existing Techniques

These divide into a number of categories. LMS/Maximum Likelihood methods present the wanted signal time series with a maximum S/N ratio, thereby enhancing identification and detection. The LMS technique may be narrow band and frequency domain or broadband and time domain (Frost 72, Nunn 83).

More recent methods present signal power as a function of bearing, e.g. Eigenvector methods (MUSIC), Maximum Entropy, Kalman filtering and Singular Value Decomposition (Farrier, This Volume). All these methods are primarily narrow band.

All ABF techniques suffer from certain difficulties which make application in the field not easy. Highly optimised solutions are rapidly degraded by sensor errors, array deformation and multipathing. Time varying noise environments exact a further toll. MUSIC methods are particularly sensitive to errors in general. Another problem is that narrow band techniques such as MUSIC, and frequency domain LMS, involve a heavy workload when applied to broadband problems.

3. Present proposal

In this paper we present a broadband frequency domain LMS algorithm that is specially robustised against sensor errors, array deformation and multipathing. The technique for extending frequency domain LMS ABF to the broadband case is as in Nunn 87, and this gives a suboptimal broadband processor with a low workload. Robustness is imparted in a manner shown in Honarmand and Nunn (This Volume) and also in Er and Cantoni (1986). The processor forms a set of CBF beams or similar using the whole array. A subset of sensors is used to form adaptive channels, which are added into the CBF beams. By limiting the weight vector norm these channels act as broadband null steerers, leaving the CBF mainlobe response intact. The system thus forms a fixed set of overlapping modified CBF beams, the degree of robustness being as for CBF.

An interesting feature of the processor is that a norm limiter parameter λ controls the manner of operation. If λ is set high the processor acts as broadband null steerer and is robust. With λ set low we secure a non robust operation suitable for small arrays, arrays not matched to wavelength or when high bearing accuracy for strong sources is required.

This algorithm is primarily intended for broadband passive sonar or acoustics in air. It is particularly good for large arrays and where sizeable array errors or multipathing occur. However by a suitable choice of λ any problem may be dealt with. Note that the robust case requires significant anisotropy otherwise no gain results.

4. Algorithm Description

The main array consists of N_o isotropic elements co-ordinates (X_i, Y_i, Z_i), with unknown amplitude responses $\varepsilon_i(f)$ and phase errors $\xi_i(f)$.

Sensor data is sampled at a rate $F_s = 1/T \sim 6$ Fmax. Blocks of $\mathcal{L}$ data samples X(kT) are FFT'd to give a frequency domain array data set {Xk}. For a suboptimal broadband processor $\mathcal{L}$ need only be large enough (512 or 1024) to keep sidelobe leakage to a minimum and provide adequate frequency resolution. Optimisation takes place separately in a processing band (PB) < 1 octave, defined by the FFT cell no set {k}. The PB can consist of several separated blocks not exceeding 1 octave in all. The full set of PB's may be adaptively varied to shift optimisation effort to desired regions of the spectrum.

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Formation of CBF beams

A set of CBF beams $Y_o^{(k)}$ are formed from the 'main' array of N elements. The main array is normally the entire array or that set of elements giving minimum beamwidth and/or lowest side lobe level, given the prevailing error level. Unreliable sensors should be excluded from the main array.

$$Y_o^{(k)} = W_k^T X_k$$

where W_k is the CBF weight vector for look direction (θ,ϕ).

$$[W_k]_j = \exp\left(-\frac{2\pi ikFs}{L}\tau_j(\phi,\theta)\right) U_j/N$$

Here U_j is a suitable shading function and $\tau_j(\phi,\theta)$ is the CBF time delay. Note that the τ_j may be quantised to the nearest T.

System Output

An adaptive subarray is selected, with M elements, data vectors $\{Z_k\}$. The subarray may partially overlap the main array, but must be colocated with it. For large arrays M<N to reduce workload. For look direction θ,ϕ system output Y_k is

$$Y_k = Y_o^{(k)} + U_k^T Z_k \quad ; \quad k \subset \{k\}$$

For suboptimal broadband processing, within the PB,U_k is taken to be a quadratic function of FFT cell no k (NUNN, 87).

$$U_k = V_o + \gamma_1(k)\, V_1 + \gamma_2(k)\, V_2$$

$$\text{where } \gamma_1(k) = (k-\bar{k})/\bar{k} \quad ; \quad \gamma_2(k) = \gamma_1(k)^2$$

The complex set of $\{U_k\}$ is fixed by the 3MX1 master weight vector V

$$V^T = [V_o^T : V_1^T : V_2^T]$$

Each U_k is derived from V through

$$U_k = B_k V$$

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where

$$B_k = [\underline{1}_M : \psi_1(k)\ \underline{1}_M : \psi_2(k)\ \underline{1}_M]$$

Broadband Output Power P

Broadband output power P <u>within the PB</u> is given by

$$P = \sum_{\{k\}} Y_k \quad Y_k{}^*$$

which may be shown to be equal to

$$P = P_o + V^H QV + V^H Q_c + Q_c{}^H V$$

where

$$Q = \sum_{\{k\}} B_k{}^H \approx{R}_k B_k$$

is the 'master' broadband correlation matrix and $\approx{R}_k$ is the adaptive array correlation matrix

$$\approx{R}_k = \overline{Z_k{}^* Z_k{}^T}$$

The matrix Q is more succinctly expressed as

$$Q = \sum_{\{k\}} Ⓗ_k \quad (\approx{R}_k)$$

where $Ⓗ_k$ ($\underline{b}$) is the matrix function

$$Ⓗ_k (\underline{b}) = \begin{bmatrix} b & \psi_1 b & \psi_2 b \\ \psi_1 b & \psi_1{}^2 b & \psi_1\ \psi_2 b \\ \psi_2 b & \psi_1\ \psi_2 b & \psi_2{}^2 b \end{bmatrix}$$

The correlation vector Q_c is given by

$$Q_c = \sum_{\{k\}} B_k{}^H \tilde{R}_k W_k$$

where $\tilde{R}_k = \overline{Z_k{}^* X_k{}^T}$

This is better written as

$$Q_c^T = \sum_{\{k\}} [d_k^T : \gamma_1 d_k^T : \gamma_2 d_k^T]$$

where d_k is (MX1) and is the correlation between the CBF output $Y_o^{(k)}$ and Z_k^*

$$d_k = \tilde{R}_k W_k = \overline{Z_k * X_k^T W_k}$$

The quantity P_o is CBF power, given by

$$P_o = \sum_{\{k\}} W_k^H R_k W$$

where $R_k = \overline{X_k * X_k^T}$

$$\text{or} \quad P_o = \sum_{\{k\}} \overline{Y_o^{(k)} Y_o^{(k)*}}$$

Optimum Weight Vector

To secure the optimum weight vector V_{OPT} we minimise broadband output power P subject to a) A soft norm constraint on V and b) A linear constraint $\phi^T V = 0$ which closely ensures that a broadband signal from look direction θ,ϕ is not contained in the adaptive channel $\{U_k^T Z_k\}$ - otherwise wanted signal rejection would occur.

Matrix algebra gives

$$V_{OPT} = X [\phi^* (Y\phi^*)^{-1} Y - 1_{3m}] Q_c$$

where $Y = \phi^T X$; $X = [Q + \Lambda]^{-1}$; $\Lambda = \lambda \mathbf{1}_{3M}$

The norm limiter parameter λ is critical. If set fairly high $\lambda \sim 0.1 Q_{11}$ the CBF mainlobe is largely preserved and the system is robust. If λ is set low $\lambda \sim 0.02 Q_{11}$ bearing resolution much sharper than CBF will result, but some robustness will be lost.

The Linear Constraint System

A linear null point constraint at frequency kFs/L, bearing θ,ϕ is of the form

$$C^T (k,\phi,\theta)V = 0$$

where $C = B_k{}^T \tilde{S}_k (\phi,\theta)$

and $\tilde{S}_k / _j = \exp\{2\pi ikFs\tau j(\phi,\theta)/L\}$

A broadband constraint consists of about 5 point constraints spaced across the PB. For applications not involving fine bearing resolution it may be useful to stagger the point constraints in bearing.

$$\mathcal{C}^T V = [C(k_1,\phi,\theta) : C(k_2,\phi,\theta) : ....C(k_5,\phi,\theta)]^T V = 0$$

It is generally desirable to transform the $C(k_j,\phi,\theta)$ to an orthonormal set of vectors and also to add a small diagonal matrix to YC^* prior to performing the 5 x 5 inverse. These measures ensure matrix conditioning and enable 32 bit precision to be used.

5. **Online Implementation**

The scalar $<P_o>$ and vector $<Q_c>$ are accumulated for every look direction ϕ, θ using a deweighting factor α~0.2 say.

$$<P_o>_{(n+1)LT} = (1-\alpha) <P_o>_{nLT} + \alpha \sum_{\{k\}} Y_o{}^{(k)} Y_o{}^{(k)*} / _{nLT}$$

$$<Q_c>_{(n+1)LT} = (1-\alpha) <Q_c>_{nLT} + \alpha \sum_{\{k\}} B_k{}^H Z_k{}^* Y_o{}^{(k)} / _{nLT}$$

The matrix <Q> is accumulated for the whole PB

$$<Q>_{(n+1)LT} = (1-\alpha) <Q>_{nLT} + \alpha \sum_{\{k\}} B_k{}^H Z_k{}^* Z_k{}^T B_k / _{nLT}$$

and $Y_o{}^{(k)} /_{nLT} = W_k{}^T X_k / _{nLT}$

The current optimum weight vector is given by

$$\langle X\rangle = [\langle Q\rangle + \Lambda]^{-1}$$

$$\langle Y\rangle = \Phi^{T}\langle X\rangle$$

$$\langle V_{OPT}\rangle = \langle X\rangle\ [\Phi^{*}(\langle Y\rangle\Phi^{*})^{-1}\ \langle Y\rangle - 1]\langle Q_c\rangle$$

Broadband output power is given by

$$\langle P(\phi,\theta)\rangle = \langle P_o\rangle + \langle V_{OPT}\rangle^{H}\ \langle Q_c\rangle + \langle Q_c\rangle^{H}\ \langle V_{OPT}\rangle + \langle V_{OPT}\rangle^{H}\ \langle Q\rangle\ \langle V_{OPT}\rangle$$

where all quantities are at $(n+1)\ \angle T$.

The FFT coefficients of the wanted signal for direction ϕ,θ are given by

$$Y_k = (W_k{}^{T}X_k) + \langle V_{OPT}\rangle^{T} B_k{}^{T} Z_k .$$

In so far as storage capabilities permit the vector $\langle V_{OPT}\rangle$ should be applied to the data used to calculate that vector.

The data stream Y_k may be subjected to zooming, and time integration can displayed in the usual way.

6. Algorithm Test

The algorithm has been tested for the time stationary, fully adapted case. A linear array of 20 elements was selected, with constant random sensor errors and also array deformation. (The program has a multipath capability). For strong sources broadband bearing resolution was about 2° for this array - a figure comparable with narrow band techniques. Simulations with weak wanted signals revealed an impressive null steering capability. Up to 4 broadband nulls could be steered with depths ~ 50 dbs.

System robustness is as expected. For high resolution operation with low λ sensor errors ~ 0.3 dbs, 7° are tolerated. With weak wanted signals and high λ, levels of robustness commensurate with CBF are secured.

7. Conclusion

A highly robust broadband adaptive beamformer has been developed. The algorithm has a good deal of flexibility in how the adaptive effort is distributed spectrally. The manner of operation of the algorithm is continuously variable - by varying a single parameter λ the level of robustness may be altered and either bearing resolution or S/N ratio may be emphasised. It is anticipated the algorithm will be highly suitable for realistic passive sonar processing problems.

References

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