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SUBOPTIMAL FREQUENCY DOMAIN ADAPTIVE ARRAY PROCESSING IN BROADBAND ENVIRONMENTS

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INTRODUCTION

This paper deals with the problem of an adaptive antenna array in a broadband environment. We consider a three dimensional array of omnidirectional sensors or monopoles. The array sits in a medium containing a three dimensional noise field, and a wanted signal coming from a particular far field noise source. The noise field consists of two parts. One part, self noise, originates at or near the array, or in the processing electronics. The other part is assumed to originate in a number of broadband interfering sources (IS) located in the far field.

The task of the processor is to make a decision on whether the wanted signal is present or not. To do this an 'output beam' is formed, pointing in the direction of the wanted signal. The aim of the adaptive processor is to maximise the signal to noise ratio in the output beam.

THE FREQUENCY DOMAIN APPROACH

One approach to the broadband antenna problem is to do a full frequency domain optimisation (Hudson, 1981). Assuming M sensors, each sensor output stream is subjected to an N point FFT, giving complex sensor vectors X_k. The sampling frequency is assumed to be Fs. The optimum complex weight vector W_k is computed for each FFT cell k and for each chosen look direction. Full adaptation will require the inversion of N/2 complex MxM matrices, at least as frequently as the characteristic time describing the variation of the ambient noise field. Obviously the option exists not to optimise all cells and all look directions simultaneously, but nevertheless the computational load of a full frequency domain adaptive system remains enormous.

Another difficulty with F domain techniques is as follows. Because of the high level of optimisation, the system is very sensitive to sensor errors. Adaptation time in a single FFT cell is very long, and a fully optimised processor would be rapidly degraded by time variations in the noise field or in the array itself.

THE TIME DOMAIN METHOD

The time domain method developed by Frost (1972) and extended by Vural (1978, 1979), Ko (1981) Hudson (1982) and Nunn (1983), represents a highly suboptimal, robust approach to the broadband antenna problem. Real sensor data is passed down a tapped delay line of J taps, with a time separation $\sim 1/4f$. Output beams are formed from a weighted sum of the JM tap signals. Time domain processors have been found to give reasonable values of array gain in cases where the anisotropy is strong, and due to a small number of interfering sources. In more complex noise environments there is a substantial loss of gain as the time domain processor only has JM real weights per beam to cover the whole spectrum, as opposed to NM/2 complex weights for the F domain processor. However, being highly suboptimal, the time domain processor is more robust against sensor errors and time variations in the noise field.

The time domain processor of Frost also has some other snags. The architecture of the processor is awkward and not easily implemented in hardware. The system lacks flexibility and has a level of optimisation that is actually too low for many purposes. Accordingly we here design a suboptimal adaptive antenna processor based upon F domain techniques and the FFT algorithm.

THE SUBOPTIMAL FREQUENCY DOMAIN PROCESSOR SFDP

The SFDP resembles the frequency domain processor in many ways. Sensor data is FFT'd to give complex sensor vectors X_k at frequency 'k'. The output beams $y_k^{(\ell)}$ are derived from

$$y_k^{(\ell)} = W_k^{(\ell)^T} X_k$$

The FFT frequency cells k are arranged in groups, and the optimisation is done completely separately for each group. Each group of cells would normally form a contiguous block, but strictly speaking this is not necessary. The GROUP of cells could consist of a number of separated contiguous blocks.

For each look direction, the complex weight vector W_k for each cell within the specified group is given by an expression of the form below

$$W_k = U_o + \sum_{j=1}^{I} \phi_j(k) U_j$$

where normally I is expected to be a fairly small number. Thus the W_k are an analytic function of cell no k within each group, this function being uniquely determined by the complex supervector V

$$V^T = | \; U_o^T | U_1^T | U_2^T | \; \ldots . \; U_I^T \; |$$

If the numbers of cells in the group is N_{cell}, then the number of free weights is reduced by a factor of $(I+1)/N_{cell}$. Thus small I represents a low optimisation level and $I+1=N_{cell}$ represents full optimisation. The choice of the real functions ϕ_j is rather arbitrary, and more research is needed to determine the best choice. An orthonormal set of real functions would seem to make sense.

$$\sum_k \phi_j(k)\phi_{j'}(k) = \delta_{jj'}$$

with a set of cos/sin functions being an obvious choice

$$\phi_{2j}(k) = \cos \left| \frac{\pi . j . (k-\bar{k})}{N_{cell}} \right| \quad ; j=1, I/_2$$

$$\phi_{2j-1}(k) = \sin \left| \frac{\pi j(k-\bar{k})}{N_{cell}} \right| \; ; \pi j=1, I/_2$$

where

$$\bar{k} = \sum_k k \;/N_{cell}$$

The output power P for a specific beam, integrated over all cells k in the group, is

$$P = \sum_k \overline{y_k y_k^*} = \sum_k W_k^T \overline{X_k X_k^H} W_k^*$$

$$= \sum_k W_k^H R_k W_k \text{ where } R_k = \overline{X_k^* X_k^T}^t$$

By way of illustration take I = 2. Then

$$P = \sum_k (U_0+\phi_1(k)U_1+\phi_2(k)U_2)^H R_k (U_0+\phi_1(k)U_1+\phi_2(k)U_2)$$

In terms of V this may be written

$$P = (V^H QV) \qquad \text{where}$$

$$Q = \begin{vmatrix} \sum_k R_k & \sum_k \phi_1 R_k & \sum_k \phi_2 R_k \\ \sum_k \phi_1 R_k & \sum_k \phi_1^2 R_k & \sum_k \phi_2\phi_1 R_k \\ \sum_k \phi_2 R_k & \sum_k \phi_2\phi_1 R_k & \sum_k R_k \phi_2^2 \end{vmatrix}$$

<u>The Constraint System</u>

For a specific output beam or look direction the array response may be constrained to unity in any direction ϕ, Θ, and in frequency cell k. A monochromatic signal coming from a far field source at bearing ϕ, Θ, and with frequency $f = kFs/N$, will produce a signal vector X_k, with the ℓth component given by

$$(X_k)_\ell = \exp\left\{\frac{2\pi ikFs}{N} \; T_\ell(\phi,\Theta) + i2\pi ft\right\} \equiv s(k,\phi,\Theta)e^{i2\pi ft}$$

$$\text{where } T_\ell(\phi,\Theta) = \left\{\frac{X_\ell}{c}\cos\Theta\cos\phi + \frac{Y_\ell}{c}\cos\Theta\sin\phi + \frac{Z_\ell}{c}\sin\Theta\right\}$$

and x_ℓ y_ℓ and z_ℓ are the sensor coordinates.

For unit response at ϕ, Θ, k we must have

$$S^T(k,\phi,\Theta).W_k = 1$$

In terms of V, a single point constraint may be written

$$C^T(k,\phi,\Theta)V = 1$$

where $C^T = |\ S^T(k,\phi,\Theta)\,|\phi_1(k)S^T| \ldots\ldots |\phi_I(k)S^T\ |$

Normally several point constraints will be needed since unity response will be required over the frequency band defined by the cell group in question. Also the main beam may have to be broadened in ϕ, Θ by imposition of further constraints. As in normal F-domain ABF, gradient constraints may be usefully employed. The most obvious constraints, in order of usefulness, are

$$\frac{\partial c^T}{\partial \ell} V=0 \ ; \quad \frac{\partial c^T}{\partial \phi} V=0; \quad \frac{\partial^2 c^T}{\partial \ell^2} V=0; \frac{\partial c^T}{\partial \Theta}.V=0;$$

The complete set of K point and gradient constraints may be collected up into a single matrix constraint

$$\mathcal{C}^T.V=F$$

where for example, in the case of 4 point and 4 gradient constraints we have

$$= \left| C_1 | C_2 | C_3 | C_4 | \frac{\partial C_5}{\partial \phi} | \frac{\partial C_6}{\partial \ell} | \frac{\partial C_7}{\partial \ell^2} | \frac{\partial C_8}{\partial \Theta} \right|$$

$$F^T = |\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ |$$

<u>Norm Constraint On V</u>

Under certain conditions the matrix Q may be very ill-conditioned or even singular. In such circumstances the norm of V will be very large, and the optimal solution unstable and prone to sensor errors. A soft norm constraint may be applied to V by adding a well conditioned positive Hermitian matrix η to Q.

$$Q' = Q + \eta$$

A suitable choice for η is of the form

$$\eta = \begin{bmatrix} \lambda_o \underline{\underline{1}} & 0 & 0 \\ 0 & \lambda_1 \underline{\underline{1}} & 0 \\ 0 & 0 & \lambda_2 \underline{\underline{1}} \end{bmatrix}$$

whence

$$V^H Q' V = V^H Q V + \lambda_o (U_o^H U_o) + \lambda_1 (U_1^H U_1) + \lambda_2 (U_2^H U_2)$$

Addition of η will approximately limit $U_o^H U_o$ to $\frac{1}{\lambda_o} \frac{Q_{11}}{G}$, $(U_1^H U_1)$ to $\frac{1}{\lambda_1} \frac{Q_{11}}{G}$ and $(U_2^H U_2)$ to $\frac{1}{\lambda_2} \frac{Q_{11}}{G}$. Here G is an estimate of array gain.

Calculation of optimum master weight vector V_{opt}

For each look direction the optimum weight vector V_{opt} is found by minimising the equivalent power $(V^H Q' V)$, subject to the linear constraint $\mathbb{C}^T . V = F$. Simple matrix algebra yields the result that

$$V_{opt} = Q'^{-1} * (\mathbb{C}^T Q'^{-1} \mathbb{C}^*)^{-1} F$$

Orthogonalisation of the Constraint Matrix

The constraint vectors $C_1, C_2, \ldots, \frac{\partial c}{\partial \phi} \frac{\partial c}{\partial \ell}$ etc. as defined will be linearly independent but not orthogonal or normalised. It is of some use to transform the constraint vectors to an orthonormal set. This serves to (a) condition the matrix algebra; (b) simplify the realisation of V_{opt} if steepest descent methods are being used.

For sensor space $\mathbb{C}^T V = F$, where $\mathbb{C}$ has the dimensions $(I+1).M \times K$, where K is the number of constraints. Premultiplying by Γ, where Γ is $K \times K$

$$\Gamma \mathbb{C}^T V = D^T V = G = \Gamma F$$

where $D = \mathbb{C} \Gamma^T$;

We require the rows of D^T to be an orthonormal set, or

$$D^T D^* = \underline{\underline{1}}$$

therefore $\Gamma \mathbb{C}^T \mathbb{C}^* \Gamma^H = \underline{\underline{1}} = \Gamma A \Gamma^H$

where $A \equiv \mathbb{C}^T \mathbb{C}^*$

Thus the matrix Γ is given by $\Gamma = YS$

where S is the unitary matrix that diagonalises Hermitian matrix A and $Y_{ij} = \delta_{ij} \lambda_i^{-\frac{1}{2}}$, where the λ_i are the eigenvalues of A in the same order as in (SAS^{-1}) . Hence the orthogonalised constraint is

$$(YS\mathbb{C}^T).V = YSF$$

BEAMSPACE PROCESSING

The SFDP can be configured to operate with inputs which are a set of conventional (CBF) beam outputs. Assuming that the CBF beams are FFT'd we have an input vector u_k for each cell k, where

$$U_k = \mathcal{H}_k X_k$$

and

$$(\mathcal{H}_k)_{ij} = \frac{1}{M} e^{-\frac{2\pi i k F s}{N}} T_j(\Theta_i, \phi_i)$$

For a given output look direction, let us form a beam output d_k from

$$d_k = Z_k^T U_k$$

where the complex input beam weight vector Z_k is as before an analytic function of cell no k

$$Z_k = T_o + \phi_1(k)T_1 + \phi_2(k)T_2 + \dots \phi_I(k)T_I$$

We may now define a beamspace "super weight vector" E

$$E^T = \left| \; T_o^T \middle| T_1^T \middle| T_2^T \middle| \; \dots \; T_I^T \; \right|$$

The power of the output beam, integrated over the frequency band in question is

$$P_B = \sum_K \overline{d_k d_k^*} = E^H M E$$

For the case I=2, the matrix M is given by

$$M = \begin{vmatrix} \sum P_k & \sum \phi_1 P_k & \sum \phi_2 P_k \\ \sum \phi_1 P_k & \sum \phi_1^2 P_k & \sum \phi_1 \phi_2 P_k \\ \sum \phi_2 P_k & \sum \phi_1 \phi_2 P_k & \sum \phi_2^2 P_k \end{vmatrix}$$

$$\text{where } P_k = \overline{U_k^* U_k^T}^t = (\textcircled{H}_k^* R_k \textcircled{H}_k^T)$$

<u>Beamspace Mainbeam constraints</u>

In the beamspace case, it is easily shown that a point constraint at k, ϕ, Θ takes the following form

$$H^T(k,\Phi,\Theta).E = 1$$

where

$$H^T = \left[S^T(k,\phi,\Theta).\textcircled{H}_k^T \middle| \phi_1(k)S^T \textcircled{H}_k^T \middle| \dots \phi_I(k)S^T.\textcircled{H}_k^T \right]$$

A suitable choice of point and gradient constraints may be collected up into an overall constraint matrix

$$\mathcal{H}^T E = F_B$$

$$\mathcal{H} = \left[H_1 \middle| H_2 \middle| \dots \middle| \frac{\partial H}{\partial \ell} \middle| \frac{\partial H}{\partial \phi} \middle| \frac{\partial^2 H}{\partial \ell^2} \middle| \frac{\partial H}{\partial \Theta} \right]$$

$$F_B^T = | \; 1 \; 1 \; 1 \dots 0 \; 0 \; 0 \; 0 \; |$$

The optimum beamspace master weight vector E_{opt} is obtained by minimising P_B subject to the look direction constraint $\mathcal{H}^T E = F_B$. The result is

$$E_{opt} = \left| M + \eta'' \right|^{-1} \mathcal{H}^{*} \left| \mathcal{H}^{T} (M + \eta'')^{-1} \mathcal{H}^{*} \right|^{-1} F_B$$

where a diagonal matrix η'' serves to limit the norm of E and provide robustness and stability.

REALISATION OF THE OPTIMUM WEIGHT VECTORS

The optimum weight vectors E_{opt}, V_{opt} may be realised by a number of techniques, provided digital processing is assumed.

1. Estimates of R_k or P_k are obtained by taking N_s successive samples of input data

$$\langle R_k \rangle = \frac{1}{N_s} \sum_{m=1}^{N_s} (X_k^{*} X_k^{T})_{t_m}$$

$$\langle P_k \rangle = \frac{1}{N_s} \sum_{m=1}^{N_s} (U_k^{*} U_k^{T})_{t_m}$$

The optimum weight vectors may be computed directly with software which evaluates the matrix algebra.

2. A running estimate of R_k or P_k is taken, using a deweighting factor α.

$$\langle R_k \rangle_{t_{m+1}} = (1-\alpha) \langle R_k \rangle_{t_m} + \alpha (X_k^{*} X_k^{T})_{t_m}$$

$$\langle P_k \rangle_{t_{m-1}} = (1-\alpha) \langle P_k \rangle_{t_m} + \alpha (U_k^{*} U_k^{T})_{t_m}$$

Again the optimum weight vectors may be computed with software as often as desired.

3. The inverses of M and Q are updated recursively, using the algorithm of Bartlett. For the sensor space case, define first a 'super input vector' $\bar{x}_k$.

$$\bar{x}_k^{T} = \left| X_k^{*T} \middle| \phi_1(k) X_k^{*T} \middle| \ldots \middle| \phi_I(k) X_k^{*T} \right|$$

The matrix Q is successively updated by dyads of the form $(\bar{x}_k \bar{x}_k^{H})$. Applying the Bartlett algorithm we then get

$$\left| Q^{-1} \right|_{n+1} = \left| Q^{-1} \right|_{n} + \frac{\left| Q^{-1} \middle|_n \bar{x}_k \bar{x}_k^{H} \middle| Q^{-1} \right|_n}{(1 + \bar{x}_k^{H} \left| Q^{-1} \right|_n \bar{x}_k)}$$

The stabilising matrix η'' *is incorporated as an initial condition on* Q^{-1}. *The update must be performed successively for all cells k in the block before moving on to the next time sample.*

4. If the constraint matrix is orthogonalised as indicated previously, steepest descent methods may be employed usefully. These methods have the advantage of simplicity and possible implementation by hardware.

ALGORITHM TRIALS

The sensor space algorithm has been extensively tested for the case I=2, for a linear array. Full adaptation and time stationary data were assumed and a variety of sensor error conditions were simulated.

It was found that the performance of the system was very good. In the presence of strong broadband interfering sources deep broadband nulls were formed. Details of the trial results will be presented at Loughborough.

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