THE NUMERICAL SIMULATION OF THE PERFORMANCE OF A ROBUSTISED, BROADBAND, FREQUENCY DOMAIN LMS ADAPTIVE BEAMFORMER.

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1. INTRODUCTION

This paper is concerned with the optimal processing of data from an array of sensors/antennas. Such sensors may be sonar, radar, VHF/HF radio or 'acoustics in air'. The processing aims might be any of the following (a) Detection and identification of weak wanted signals. (b) Bearing estimation of weak wanted signals. (c) Presentation of the time series of wanted signals at maximum S/N ratio for further processing / display. (d) Accurate bearing estimation and discrimination of strong signal sources.

All ABF techniques suffer difficulties when applied in the field. Highly optimised algorithms are quickly degraded by multipathing, array deformation and by sensor errors. Time varying noise fields and finite integration time exact a further toll. Eigenvector methods such as MUSIC are rather expensive when applied to broadband environments.

In this paper we shall consider a BROAD BAND robustised LMS frequency domain adaptive algorithm as described in Nunn (1989). Its performance will be analysed in numerical simulations incorporating multipathing, array distortion, sensor errors and finite integration time.

2. THE ALGORITHM

The algorithm is basically a frequency domain LMS adaptive beamformer that is extended to bandwidths of order 1 octave. This is done by a technique described in Nunn (1987) in which the complex weight vector for each FFT cell is expanded as a function of cell no k, retaining only terms up to quadratic in k. The resulting algorithm is suboptimal with a low work load, but more robust than an assembly of single frequency processors. In addition the processor employs a technique for further robustisation related to that described by Cantoni and Er (1986). The processor forms a set of CBF beams for every desired look direction. A subset of sensors is used to form the adaptive channels which are weighted and added into the CBF beams and used to steer broadband nulls. A quantity λ controls a soft norm constraint applied to the adaptive channel weight vector. With λ set high the norm is low with the result that the CBF mainlobe response is more or less intact, and the system acts as a broadband null steerer. In this mode the system is highly robust, with same level of robustness as CBF processing.

In the robust mode of operation a significant level of anisotropy is required for there to be any performance gains over CBF processing. With λ set low we secure a non-robust mode of operation suitable for arrays with few sensors, undersized arrays, or when high bearing accuracy for strong sources is required. The algorithm described here is very flexible and by suitable choice of λ can cope with any situation. However it is particularly well suited to arrays with large numbers of sensors where array deformation is likely - ie. the naval towed array problem.

3. MATHEMATICAL DETAILS OF THE ALGORITHM

We first define the 'main array' consisting of N isotropic elements located at (Xi, Yi, Zi) with unknown amplitude responses $\varepsilon_i(f)$ and phase errors $\zeta_i(f)$. Sensor data is sampled at a rate $F_s=1/T \sim 6F_{max}$. Blocks of L data samples $\underline{X}(kT)$ are FFT'd to give a frequency domain set of array vectors $\{\underline{X}_k\}$. For a suboptimal processor L need only be large enough (512 or 1024) to keep sidelobe leakage to a minimum and provide sufficient frequency resolution for optimisation purposes.

Now optimisation takes place in a processing band (PB) $\lesssim$ 1 octave, defined by the FFT cell no set {k}. The PB may consist of separate blocks, provided the total bandwidth does not exceed 1 octave in all. The assembly of PB's may be adaptively varied during processing in order to shift optimisation effort to desired regions of the spectrum.

Formation of the beams

A set of output CBF beams $\{Y_o^k\}$ are formed from the main array of N elements. The main array is normally the entire array or that subset that gives minimum beamwidth and lowest sidelobe level. Unreliable sensors should be excluded from the main array. The CBF beams must cover the entire set of desired look directions, and with a density appropriate to the bearing resolution of the optimised output. So we have

$$Y_o^k = W_k^T X_k$$

where,

$$W_k = \exp\left[\frac{-2\pi ikF_s}{L}\,\tau_j(\phi,\,\theta)\right]\frac{\bar{U}_j^k}{N}$$

The $\bar{U}_j^k$ are suitable shading functions and the τ are the usual CBF time delays. Note that the τ_j will in practice be quantised to the nearest multiple of T, though this will not be considered in the present situation.

System output

An adaptive subarray of M elements is selected, with data vector {Zk}. The subarray may consist of the whole array, any subset of sensors of the main array, or indeed could consist of a separate array colocated with the main array. For arrays with many sensors M may be much less than N in order to reduce the adaptive workload. In principle it is possible to use the set of CBF beams as adaptive channels, but this requires that the set of CBF beams be formed to cover all possible look directions. It would also fix the number of adaptive channels thus removing flexibility. Thus for a fixed look direction ϕ, θ and PB defined by {k} we have

$$Y_k = Y_o^k + U_k^T Z_k \qquad\qquad k \subset \{k\}$$

$$= W_k^T X_k + U_k^T Z_k$$

For suboptimal processing, with the PB {k}, U_k is expanded in powers of $k-\bar{k}$ as follows (Nunn, 1987)

$$U_k = V_o + \psi_1(k) V_1 + \psi_2(k) V_2$$

$$\psi_1(k) = (k-\bar{k})/\bar{k}; \ \psi_2(k) = \psi_1(k)^2$$

$$\bar{k} = \sum_{\{k\}} k/N_k \ ; \ N_k = \sum_{\{k\}} 1$$

The number of real adaptive variables in the PB is now 6M, compared to $2N_k.N$ for an assembly of N_k narrow band processors.

The set of complex weight vectors {Uk} may be derived from a 3Mx1 complex master weight vector V defined by

$$V^T = [\ V_O^T \mid V_1^T \mid V_2^T\ ]$$

using

$$U_k = B_k\ V \qquad\qquad k \subset \{k\}$$

where

$$B_k = [\ \underline{\underline{1}}_M \mid \psi_1(k)\ \underline{\underline{1}}_M \mid \psi_2(k)\ \underline{\underline{1}}_M\ ]$$

Broadband output power within the PB is given by

$$P = \sum_{\{k\}} \overline{Y_k\ Y_k^*}$$

which may be shown to be equal to

$$P = Po + V^H Q\ V + V^H Qc + Qc^H V$$

where

$$Q = \sum_{\{k\}} \left(B_k^H \, \widetilde{\widetilde{R}}_k \, B_K \right)$$

is the master correlation matrix and R_k the adaptive array correlation matrix for FFT cell k.

$$\widetilde{\widetilde{R}}_k = \overline{Z_k^* \, Z_k^T}$$

The matrix Qc is a matrix of 'broadband correlations' between the CBF beams Y_o^k and the adaptive array output

$$Qc = \sum_{\{k\}} B_k^H \, \widetilde{R}_k \, W_k$$

where

$$\widetilde{R}_k = \overline{Z_k^* \, X_k^T}$$

Here P_0 is simply the CBF power given by

$$P_0 = \sum_{\{k\}} W_k^H \, R_k \, W \quad = \quad \sum_{\{k\}} \overline{Y_0^k \, Y_0^{k*}}$$

where

$$R_k = \overline{X_k^* X_k^T}$$

<u>The optimum weight vector V_{opt}</u>

To secure the optimum weight vector V=Vopt we minimise broadband output power P subject to (a) A soft norm constraint on V and (b) A linear constraint C^T V=0 which closely ensures that a broadband signal from the look direction ϕ, θ is not contained in the adaptive channel $\{U_k Z_k\}$ - otherwise wanted signal rejection may occur. Simple matrix algebra gives

$$V_{opt} = X \left[\not{c}^* (Y \not{c}^*)^{-1} \, Y - \underline{\underline{1}}_{3M} \right] Qc$$

where

$$Y = \not{c}^T X \; ; \; X = [Q + \Lambda]^{-1}$$

and

$$\Lambda = \lambda \underline{\underline{1}}_{3M}$$

The norm limiter parameter λ is critical. If set high ~0.1 Q_{11} the CBF mainlobe is largely preserved and the system is robust. If λ is set low ~0.01 Q_{11} sharper beam resolution is permitted for strong sources but the processor will be less robust. Low λ operation is also appropriate to 'superdidrective processing' for undersized arrays and arrays with few sensors.

Note that λ is continuously variable parameter and may be varied adaptively to correspond to different environments etc.

The linear null point constraint at frequency k F_s/L, bearing ϕ, θ is given by

$$C^T(k, \theta, \phi)\ V = 0$$

where

$$C = B_k^T \bar{S}_K(\phi, \theta)$$

and

$$\bar{S}_k \,\Big|_j = \exp\{2\pi i F_s \tau_j(\phi, \theta)/L\}$$

To establish a broadband constraint we apply Nc = 3-5 point constraints spaced across the processing band at frequencies k_j.

For applications not involving fine bearing resolution for strong sources it may be appropriate to stagger the point constraints in bearing. There is no point in overconstraining the system with large Nc, since the presence of sensor errors renders the constraints approximate anyway. The constraint on V thus has the form

$$\not{C}^T V = \left[C(k_1, \phi, \theta) \,|\, C(k_2, \phi, \theta) \,|\, \ldots\ldots C(k_5, \phi, \theta)\right]^T V = 0$$

In practice it is generally desirable to transform the $C(k_j, \phi, \theta)$ to an orthonormal set of vectors, and also to add a small diagonal matrix to Y C^* prior to performing the Nc x Nc inverse. These measures will improve the matrix conditioning considerably.

4. COMPUTER SIMULATIONS

All the simulations are for a 1/2 octave band from 450-600 Hz, with an FFT length L=512. Sample frequency is taken to be 4 kHz, and the linear constraint system uses 5 point constraints evenly spaced across the processing band. Three arrays will be considered. These are (a) A linear array of 20 elements, with sensor separation of 1.2 metres. (b) A similar line array of 60 elements, subject to circular deformation and (c) A circular array of 5 elements, radius 3 metres.

The noise environment contains broadband horizontally isotropic noise, white sensor self noise, and broadband interfering sources with strong spectral lines. The wanted signal may contain strong spectral lines that are multipathed with high correlations. The program can either simulate the case of infinite integration time, or assume a finite integration time of T_{int}, presumed to be about 2 seconds.

A large number of simulations have been performed with these arrays. Using the short line array in nonrobust mode the broadband bearing resolution for strong broadside sources has been investigated. Angular resolutions of order 1.5 degrees were found. This value compares very favourably with that from high resolution methods such as MUSIC.

With the long linear array the simulation employed the robust mode, with a weak wanted signal, horizontally isotropic noise and strong broadband interfering sources. The system is found to steer deep broadband nulls - 40 dbs against strong sources. Null depth depends on source strength relative to that of isotropic noise and self noise. Tests were made on the effect of multipathing wanted signals and on the effect of array bending. The system was found to be robust in these circumstances.

Using the 60 element linear array simulations were made in which the adaptive array only had 20 elements. The performance loss compared to M=60 was quite small < 1dbs.

Finally using the 20 element linear array the effect of finite integration time was investigated.

In this paper there is only space for a few sample results. Figure 1 shows the broadband bearing resolution in nonrobust mode for a 60 element linear array. Resolution ~3 degrees is obtained, far superior to CBF and at least comparable with high resolution methods. Next we take a 20 element linear array in robust mode, and assume some unknown circular deformation.

Figure 2 shows the array responses in the direction of all broadband directional sources. The array response in the direction of the wanted signal is clearly close to unity, indicating robustness. The strong unwanted signals have broadband nulls steered at them of depth -40dbs.

Figure 3 shows the optimised broadband power $P(\phi, \theta)$ for the case of an undersized linear array of 10 elements with sensor separation of 0.2 metres. The improvement in directional capability as compared to CBF will be apparent.

5. CONCLUSION

A suboptimal broadband LMS algorithm has been presented whose mode of operation may be changed by a continuously variable parameter. Detailed numerical simulations reveal a high degree of robustness and good performance relative to CBF processing.

REFERENCES

Nunn D. (1983) Proc IEE, Vol 180, Pts F+H, No 1, p139 - 144
Nunn D. (1987) Proc IEE, Vol 134, Pt F, No 4, p341 - 351
Er, M. H. and Cantoni A. (1986) Proc ICASSP, 1986. Tokyo, Japan.

FIG 1

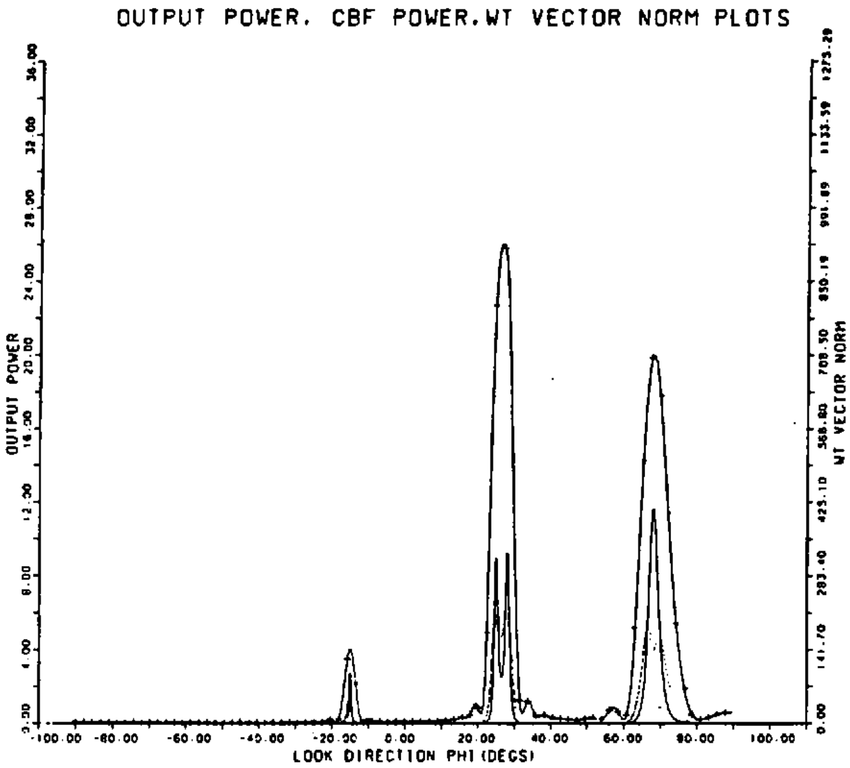

OUTPUT POWER. CBF POWER.WT VECTOR NORM PLOTS
OUTPUT POWER
WT VECTOR NORM
LOOK DIRECTION PHI (DEGS)

FIG 2

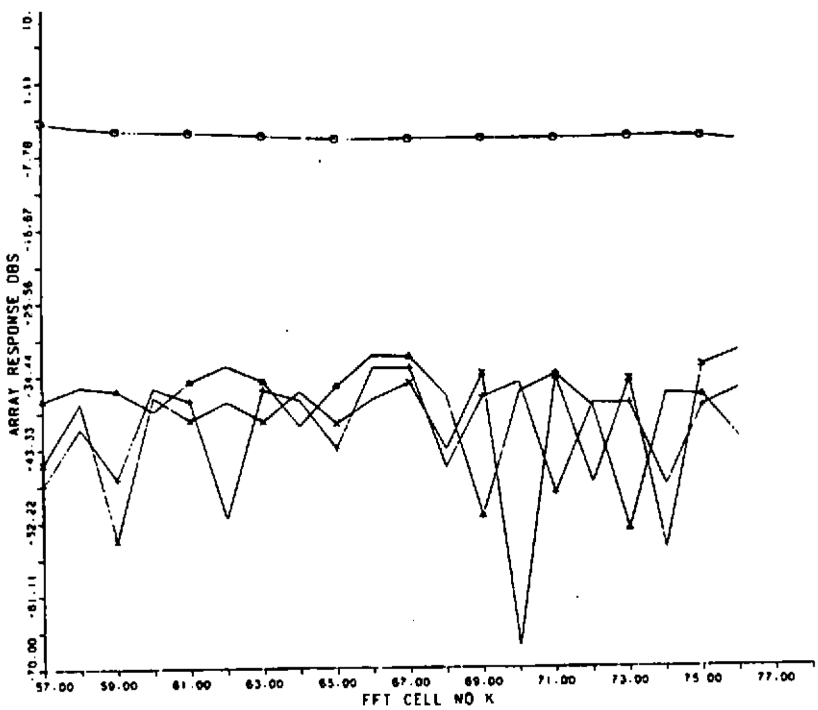
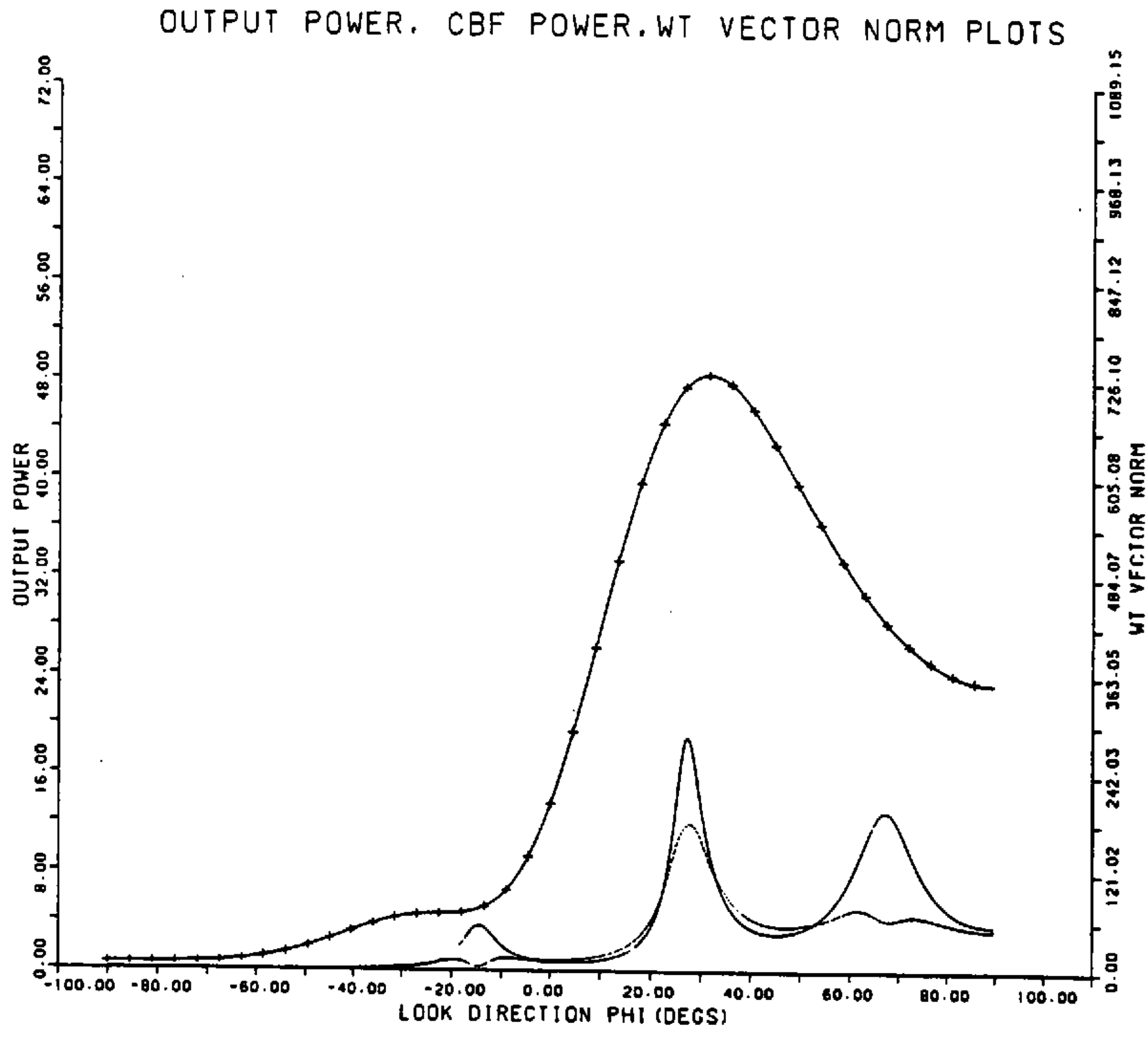

ARRAY RESPONSE DBS
FFT CELL NO K

FIG 3


OUTPUT POWER, CBF POWER, WT VECTOR NORM PLOTS
OUTPUT POWER
WT VECTOR NORM
LOOK DIRECTION PHI (DEGS)
0.00
8.00
16.00
24.00
32.00
40.00
48.00
56.00
64.00
72.00
121.02
242.03
363.05
484.07
605.08
726.10
847.12
968.13
1089.15
-100.00
-80.00
-60.00
-40.00
-20.00
0.00
20.00
40.00
60.00
80.00
100.00