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BISPECTRAL ANALYSIS AS A MEANS TO IDENTIFY THE RELATIVE STRENGTHS OF TWO SOURCES USING DATA FROM ONE RECEIVER

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The techniques of spectral analysis are widely used to transform a time history into its constituent frequency components. In particular, the power spectrum allows the energy in a given frequency band to be estimated. Apart from the power spectrum, there are cross-spectral functions that enable the contribution to a total signal from a particular source to be calculated. However, two transducers whose outputs are recorded simultaneously are needed for this application and there can be problmes in data acquisition. This paper presents a different approach in which the contributions from an harmonic train can be distinguished from background noise using data sampled from just one transducer. In addition, this technique can, in certain circumstances, seperate the contributions from two differenct harmonic series.

The method is based on the work of other authors who have studied the bispectrum [1-5] and the bicoherence [1,6], or normalised bispectrum. The first treatment of bispectral analysis seems to be that of Brillinger and Rosenblatt [2,3] who define the bispectrum as a two dimensionsal generalisation of the ordinary power spectrum.

The phase of the signal, which is expected to vary in time, is discarded when computing the ordinary power spectrum since the Fast Fourier Transform (FFT) $X(\omega)$ of a time history $x(t)$ is multiplied by its complex conjugate when estimates for the power spectrum $P(\omega)$ are made from the equation

$$P(\omega)=\frac{1}{N}\sum_{j=1}^{N} X_j(\omega)\, X_j^*(\omega)\, .T \qquad (1)$$

where the summation is over a number of records and T is the time length of each record. The bispectrum, as defined by Brillinger and Rosenblatt, utilises the phase information of a signal by using the phase relationships between different frequencies. Instead of depending on the FFT at just one frequency the bispectrum is two dimensional and an estimator [1] is:

$$b(\omega_1,\omega_2)=\frac{1}{N}\sum_{j=1}^{N} X_j(\omega_1)\, X_j(\omega_2)\, X_j^*(\omega_1+\omega_2).T^2 \qquad (2)$$

There is ambiguity about the phase of the FFT at a single frequency (ω) since changing the time origin shifts the FFT by a factor $e^{-i\omega T}$ However, this ambiguity is non-existent for the bispectrum since $e^{-i\omega_1 T} e^{-i\omega_2 T} e^{i(\omega_1+\omega_2)T}=1$. Thus, the bispectrum is a complex quantity and therefore depends on the phase relationships between the FFT at two frequencies and at the sum of those frequencies.

It is a widely quoted result [1,2,4] that the bispectrum of Gaussian noise is zero. This a theoretical result in the limit as the number of samples becomes infinite and is obviously true from equation 2. If $x(t)$ is Gaussian noise then the phases of $X(\omega_1)$, $X(\omega_2)$ and $X(\omega_1+\omega_2)$ will be statistically independent, therefore the terms in the summation will cancel even though the spectral contributions remain finite.

The bicoherence is defined as the bispectrum normalised with respect to the power spectrum [1,6] but there is some discrepancy between the estimator used by Huber et al [1] and that used by Sato et al. [6]. Huber et al's estimator gives bicoherence values that are not normalised to any particular value whilst Sato et al's real time analyser gives bicoherence values whose modulus lies strictly between 0 and 1. In this paper the lead of Sato et al is taken and the bicoherence estimator of Huber er al is modified to be:

$$\mathrm{bic}(\omega_1,\omega_2) = \frac{\frac{1}{N}\sum_{j=1}^{N} X_j(\omega_1)\,X_j(\omega_2)\,X_j^{*}(\omega_1+\omega_2)}{\left\{\frac{1}{N}\sum_{i=1}^{N}\left[|X_i(\omega_1)|^2\,|X_i(\omega_2)|^2\,|X_i(\omega_1+\omega_2)^2|\right]\right\}^{1/2}} \quad (3)$$

The problem with previous analyses is that no direct physical interpretation has been given for the bispectrum, largely because the dimensions of the bispectrum are not directly attributable to any particular physical quantity. For example, if $x(t)$ is an input force signal the bispectrum has dimensions of (Newtons)^3 (Seconds)^2. The major advantage of the bicoherence estimator that has been introduced in equation (3) is that it is demensionless and so can be applied directly to physical processes. The remainder of this paper will deal with the interpretation of the estimates given by equation (3).

<u>The Bicoherence of a Signal Containing Contributions from a single source and background noise</u>

If only one record is taken then, like the coherence, the bicoherence estimator has a modulus of one for all frequency points. If the phases of $X(\omega_1)$ + $(\omega_2)X^*(\omega_1+\omega^2)$ change between records, increasing the number of records leads to cancellation in the numerator of equation (3) and hence a decrease in the modulus of the bicoherence estimator. Ultimately, if enough records are taken, the bicoherence should tend to be a value representing the variations in relative phase between the frequencies ω_1, ω_2 and $(\omega_1+\omega_2)$. On the other hand, if the modulus of the bicoherence is one for some frequencies ω_1 and ω_2 then there is no change in the relative phases of $X(\omega_1)$, $X(\omega_2)$ and $X(\omega_1+\omega_2)$

If the Fourier transform is defined by

$$\hat{x(t)}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)\,e^{i\omega t}\,dt \quad (4)$$

then

$$\widehat{x(t+T)}(\omega) = \hat{x(t)}(\omega)\,e^{-i\omega T} \quad (5)$$

so a time shift of T leads to a frequency dependent change in phase of $e^{-i\omega T}$ in the frequency domain. Therefore, in general, the FFT of the jth time record can be written as

$$x_j\,(n\omega_0) = a_n^j\, e^{i\phi_n^j} e^{-in\omega_0 T^j} \qquad (6)$$

where ω_0 is the frequency spacing of spectral estimates, $n\omega_0$ is the frequency of interest and $a_n^j\, e^{i\psi_n^j}$ is the FFT once the effect of time shifting has been removed.

The harmonic contributions to the signal will be independent of j whilst the non-harmonic contributions will depend on the individual record number. Therefore, the case of a single harmonic train and background noise can be analysed using the form

$$x_j(n\omega_0) = \begin{cases} (b_n e^{i\psi_n} + a_n^j e^{i\phi_n^j})\, e^{-in\omega_0 T^j} & \text{if } n\omega_0 \text{ a harmonic frequency band} \\ (a_n^j\, e^{i\phi_n^j})\, e^{-in\omega_0 T^j} & \text{otherwise} \end{cases} \qquad (7)$$

where $b_n e^{i\psi_n}$ is the contribution from the harmonic train and $a_n^j e^{i\phi_n^j}$ is the background noise. Putting this form into equation (3) gives the result that in the limit as the number of records becomes large

$$|\mathrm{bic}\,(n\omega_0, m\omega_0, (n+m)\omega_0)| = \begin{cases} \{(1+r_n^2)(1+r_m^2)(1+r_{n+m}^2)\}^{1/2} & \text{, if all the frequency bands are harmonic} \\ 0 & \text{, otherwise} \end{cases} \qquad (8)$$

where $r_n = \bar{a}_n / b_n$, $\bar{a}_n$ being the averaged background noise in the nth frequency band. This result follows from the relationships

$$E\,(e^{i\phi_n^j}) = 0,\; E\,(a_n^j) = \bar{a}_n \qquad (9,10)$$

and assumes that the amplitude and phase variations of background noise are independent [6].

It is clear from equation (8) that if the level of background noise is small then r_n, r_m, r_{n+m} are small and the modulus of bicoherence is close to one. As the level of background noise increases the ratios r_n increase and the modulus of bicoherence drops.

The value of the r_n can be estimated if the bicoherence can be measured at four frequencies. Suppose that $n\omega_0$ is a harmonic frequency and let

$$R_1 = |\mathrm{bic}\,(n\omega_0, n\omega_0)|, R_2 = |\mathrm{bic}\,(n\omega_0, 2n\omega_0)|, R_3 = |\mathrm{bic}\,(n\omega_0, 3n\omega_0)|, R_4 = |\mathrm{bic}\,(2n\omega_0, 2n\omega_0)|$$

Then simple manipulation shows that estimates for r_n, r_{2n}, r_{3n} and r_{4n}

(11)

BISPECTRAL ANALYSIS TO IDENTIFY THE RELATIVE STRENGTHS OF TWO SOURCES

$$r_n=\left[(R_2 R_4 / R_1^3 R_3)^{2/3}-1\right]^{1/2} \qquad r_{3n}=\left[(R_1^3 R_4 / R_2^5 R_3)^{2/3}-1\right]^{1/2}$$
$$r_{2n}=\left[(R_3/R_2 R_4)^{2/3}-1\right]^{1/2} \qquad r_{4n}=\left[(R_2^2 R_4^2 / R_3^2 R_4^3)^{2/3}-1\right]^{1/2} \tag{11}$$

The use of bispectral analysis in determing harmonic relationships is demonstrated by the following example. Vibration data from a diesel engine was recorded and subjected to a familiar spectral analysis, the result of which is shown in Figure 1. The fundamental of the diesel was 8.75 Hz whilst there was another series based on 12.2 Hz that was due to the capstan 'tick' of the tape recorder. It is to be expected that the harmonics of 8.75 Hz should have constant magnitudes and relative phases whilst the multiples of 12.2 Hz should have phases that vary in relation to each other due to the more random nature of the capstan tick.

The bicoherence takes a long time to compute in full and it is difficult to display in its whole form because it is two dimensional. Only 'slices' through the bicoherence are presented here. These slices are made by fixing the first frequency (ω_1) and then allowing the second frequency (ω_2) to vary across its range until $\omega_1 + \omega_2$ equals the Nyquist frequency (the cut-off frequency of the FFT)

Figure 2 shows the bicoherence for a slice frequency of 8.75 Hz. It is noticeable that the harmonics of 8.75 Hz stand out with bicoherence modulus values close to 1, since the background noise level is much lower than the tonal levels. At non-harmonic frequencies the values of bicoherence should tend to zero, but finite values were obtained since only 50 time records were used. In contrast, Figure 3 shows the bicoherence at a slice frequency of 12.25 Hz, which is the centre frequency of the band containing 12.2 Hz. The multiples of 12.2 Hz are not 'phase locked' to the phase of 12.2 Hz therefore the values of bicoherence are low, and the capstan tick acts as a background noise. The power spectrum itself is misleading since the capstan tick has a significant contribution to the recorded signal even though it is not part of the vibration on the diesel. In this way bispectral analysis can be used to filter out non-harmonic contributions in a signal.

The Bicoherence of a Signal containing Contributions from Two Sources

Assuming that the sources are incoherent, a general way to write the FFT of the signal is:

$$X_j(n\omega_0)=\begin{cases}\left[a_n e^{i\phi_n}+b_n e^{i(\phi_n\, n\omega_0\, \chi^j)}\right]e^{-in\omega_0 T^j} & \text{, if } n\omega_0 \text{ is a harmonic frequency}\\ c_n^j\, e^{i\theta_n^j}\, e^{-in\omega_0 T^j} & \text{, otherwise}\end{cases} \tag{12}$$

where (a_n, ϕ_n) is the magnitude and phase information for the contribution from one source and (b_n, ϕ_n) is the corresponding information for the second source. The angle χ^j measures the incoherent nature of the sources.

As before $bic\,(n\omega_0, m\omega_0)$ tends to zero unless $n\omega_0, m\omega_0$ and $(n+m)\ \omega_0$ are all harmonic frequency bands, when the bicoherence has the value:

$$bic\,(n\omega_0 m\omega_0) = \frac{e^{i(\phi_n+\phi_m-\phi_{n+m})} + r_n\, r_m\, r_{n+m}\, e^{i(\phi_n+\phi_m-\phi_{n+m})}}{\left\{(1+r_n^2)(1+r_m^2)(1+r_{n+m}^2) + 2\, r_n\, r_m\, r_{n+m}\right\}^{1/2}} \quad \text{if } n \neq m \tag{13}$$

$$= \frac{e^{i(2\phi_n-\phi_{2n})} + r_n^2\, r_{2n}}{\left\{(1+r_n^2)^2(1+r_{2n}^2) + 2r_n^2\,(1+r_{2n}+r_{2n}^2)\right\}^{1/2}} \quad \text{if } n = m$$

where

$r_n \quad b_n/a_n$

Equation 13 has been tested by computing the bicoherence of the sum of two square wave generators; one with a fundamental frequency of 70 Hz and the other with a 210 Hz fundamental. Time histories were stored simultaneously from a dual-channel FFT analyser and then digitally summed to give the overall signal. The spectrum of the combined signal is shown in Figure 4 and the idividual spectra were also computed to give direct measure of the relative proportions of each in the harmonic frequency bands. The computed bicoherence for a slice frequency of 70 Hz is shown in Figure 5.

The bicoherence at (70,70) is close to 1, which demonstrates that there is little contribution to the total signal from the 210 Hz generator at 70 and 140 Hz. Theoretically the bicoherence should have a value of 1 at (70,70) since $r_{70} = r_{140} = 0$ but the slight drop in the modulus can be explained by the fact that the second generator will have some contribution to all frequency bands and that background noise will also affect the result.

The seperate spectra of the two generators give the following values for some of the 'r_n' values in various frequency bands:

Freq	70	140	210	250	420
	$.15\times10^{-5}$	3.9×10^{-2}	0.98	4.8×10^{-4}	2.43

As an input data are square waves the relative phase between any two harmonics is zero and the values given in the table above can be used to predict bicoherence values using equation (13).

Freq	(ω_1, ω_2) :	(70,70)	(70,140)	(70,350)
$\lvert bic$	$(\omega_1,\omega_2)\rvert$:	1.0	0.70	0.36

The computed values, shown in Figure 5, are slightly lower than the theoretical values. This is due to background noise effects and to the assumption that the relative phase of all the harmonics is zero.

This example highlights some of the problems with the technique. In general, the phase angles between the harmonics of the signal will be unknown and these phase angles can have a significant variation in the computed modulus of the bicoherence. For example if $r_n = r_m = r_{n+m} = 1$, the modulus of bicoherence can take any value between 0 and $2/\sqrt{10}$. The phase of the bicoherence cannot be used to derive any useful information about these relative phases and there can be may values of the r_n each of which give the same bicoherence value.

Another problem is that large numbers of records need to be taken for bicoherence values at non-harmonic peaks to tend to zero. In the example above, where 25 records were taken, the peak at (70,350) is barely distinguishable from the random background noise levels.

However, the first problem can sometimes be overcome. If the fundamental frequencies of the two sources are not too close there will be harmonic frequency bands where only one source contributes and the ' r_n ' value for this frequency will be zero. When this happens there is no ambiguity in the phase of the bicoherence at frequencies where this harmonic contributes and the technique can be used to calculate other ' r_n ' values by taking the slice frequency to be that of this harmonic band.

In conclusion, when a signal has contributions from two sources the relative magnitudes and phases of the two sources can be measured by the bicoherence estimator defined in this paper. If one of the sources is background noise or if the fundamental frequecies of the two sources are not too close then measurements of the modulus of bicoherence can be used to estimate the relative strengths of the two sources.

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BISPECTRAL ANALYSIS TO IDENTIFY TO RELATIVE STRENGTHS OF TWO SOURCES

References

[1] P.J. Huber, B. Kleiner, T. Gasser, G. Dumermuth, "Statistical Methods for Investigating Phase Relations in Stationary Stochastic Processes" (IEEE Transactions on Audio and Electroacoustics Vol AU-19 No.1, March 1971)

[2] D.R. Brillinger,. M. Rosenblatt, "Computation and Interpretation of k'th Order Spectra (In Advanced Seminar on Spectral Analysis of Time Series. B. Harris, Ed. New York: Wiley, 1967 pp. 189-232).

[3] D.R. Brillinger, M. Rosenblatt, "Asymptotic Theory of Estimate of K'th Order Spectra" (In Advances Seminar On Spectral Analysis of Time Series.
B. Harris, Ed. New York: Wiley, 1967 pp. 153-188.

[4] J.B. Perrochaud, "Bispectral Analysis of Nonlinear Systems" (M.Sc Thesis, Institute of Sound and Vibration Research, University of Southampton (1982)

[5] M. Ren Hoopen, P.A. Zandt, "2nd Order Correlation FUnctions and Bi-spectra in Biological - Rythm Research" (Mathematical Biosciences, 33, pp. 193-212 (1977)

[6] T. Sato, K. Sasaki, Y. Nakamura, "Real-time Bispectral Analysis of Gear Noise and its Application to Contactless Diagnosis" (J, Acoust. Sox. Am., Vol 62, No 2, August 1977).

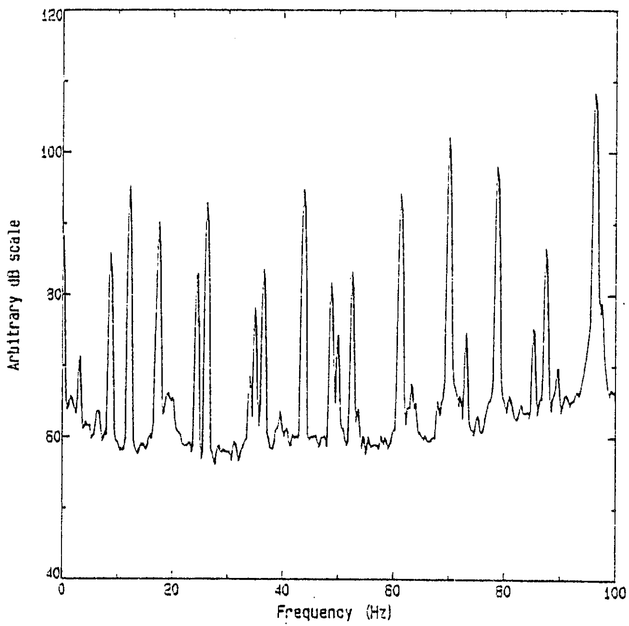

Fig 1: Spectrum of diesel data

BISPECTRAL ANALYSIS TO IDENTIFY THE RELATIVE STRENGTHS OF TWO SOURCES

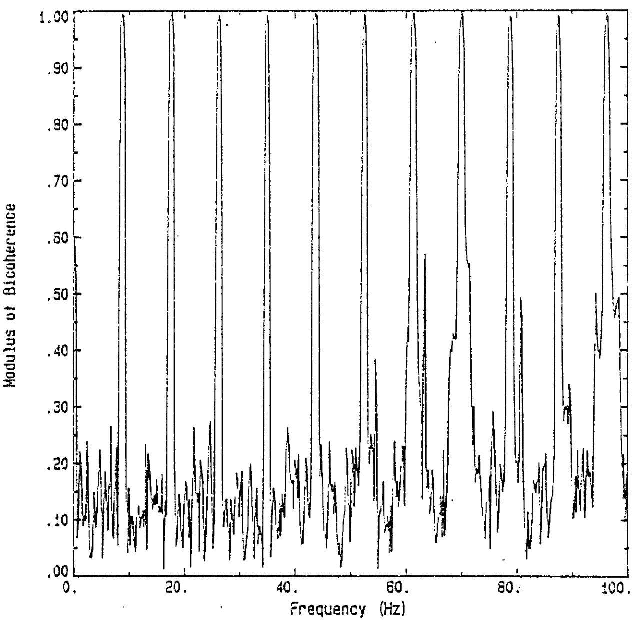

Fig 2: Bicoherence of diesel data.
Slice frequency 8.75 Hz.

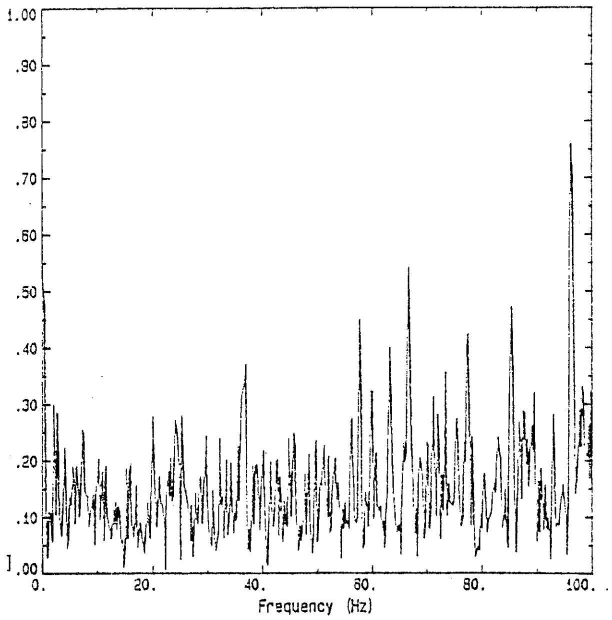

Fig 3: Bicoherence of diesel data.
Slice frequency 12.25 Hz.

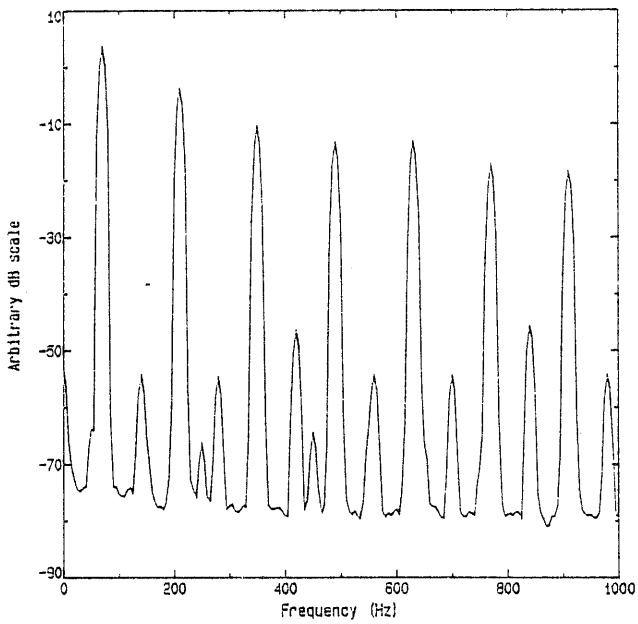

Fig 4: Spectrum of the sum of two square wave generators.

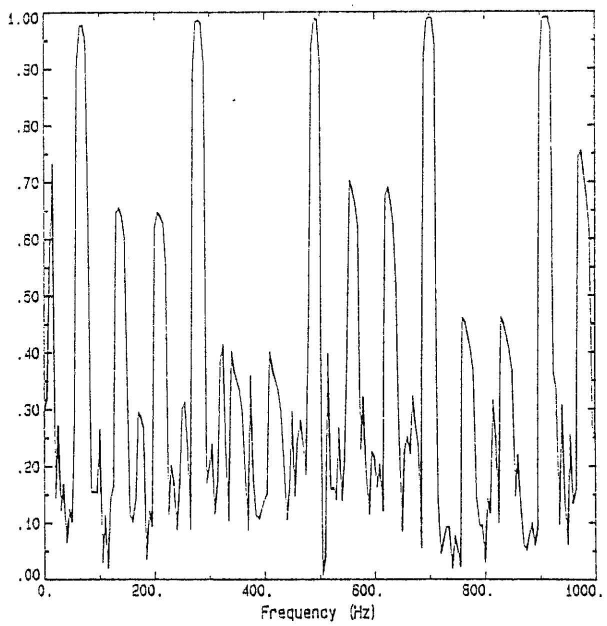

Fig 5: Bicoherence of the sum of two square waves. Slice Frequency 70 Hz.