BRITISH ACOUSTICAL SOCIETY

71SC8

BAS Spring Meeting 1971
Underwater Acoustics

TRANSDUCER BANDWIDTH

D. Stansfield

The transfer of power from a generator to a load has been discussed by various writers, and, in particular, Fano (Ref 1) considered matching with particular reference to aerials. This present paper discusses the application of similar methods to low frequency sonar transmitting transducers. The problem to be considered is that of maximising the frequency band over which the power delivered from a generator into its load lies within specified limits.

Fig 1 represents an idealised network for a piezoelectric transducer. The transducer itself is represented as usual by C_o, C_m, L_m, R_m, the resistance R_m representing the radiation resistance. The generator is assumed to be a voltage source E in series with a resistance, R_o. Between the generator and the transducer is inserted a loss-less matching network, the design of which has to be determined to optimise the power transfer over the band into the load R_m. Fano reduced this band-pass filter problem to the corresponding low-pass problem of matching to a load consisting of a series L, R combination, the capacitor C_o then being included within his coupling network. His approach used as a parameter the reflection coefficient, ρ, defined by

$$\rho = \frac{Z - R_o}{Z + R_o}$$

where Z is the impedance presented to the generator, as indicated in Fig 1. The fractional power reflected back into the generator is then given by $|\rho|^2$, and the coupling network is designed so that the reflection coefficient does not exceed some specified value $|\rho|_{max}$ at all frequencies within a specified band. Fano showed, following Bode, that a limit exists to the value of the integral

$\int_o^\infty \ln \frac{1}{|\rho|} d\omega$. The widest bandwidth is thus obtained by making

$|\rho| = |\rho|_{max}$ throughout the band, and unity at all other frequencies. Making $|\rho| < |\rho|_{max}$ at any frequency within the band, by improving the matching at a particular frequency, involves a reduction in the overall bandwidth, since an extra large contribution to the integral is made at that frequency. The limit on the integral shows also that the bandwidth can be increased only by reducing the value of $\ln(1/|\rho|_{max})$; ie by increasing $|\rho|_{max}$. Details of the calculations involved are given in Fano's papers, and in later notes by McCullagh (of ASWE) and Dearlove (of AUWE) (Ref 2 and 3).

An alternative interpretation of the results may be presented by considering the admittance of the load seen by the generator.

This presentation is useful, since this range of loads is an important factor in the design of the driving amplifier in cases where size and power are limited. If a generator (E) of source resistance $R_o = 1/G_o$ works into a complex admittance $Y = G + jB$, the power (P) delivered into the load is given by the equation

$$G^2 + B^2 + 2G_o(1 - 2P_o/P)G + G_o^2 = 0$$

where P_o is the maximum power obtainable from the source, ie $P_o = E^2G_o/4$. For a given value of P/P_o, this represents a circle, having its centre at $G_o(2P_o/P - 1)$, 0 and radius $2G_o(P_o/P)(1 - P/P_o)^{\frac{1}{2}}$. Since P/P_o is related to ρ by $P/P_o = 1 - |\rho|^2$, the co-ordinates of the centre of the circle are $\left\{G_o\left(\frac{1 + |\rho|^2}{1 - |\rho|^2}\right), 0\right\}$, and the radius is $\frac{2G_o|\rho|}{1-|\rho|^2}$. For maximum power transfer, ($|\rho| = 0$,), the permissible load shrinks to a point at G_o, 0; as the value of $|\rho|_{max}$ is increased, the circle enclosing the permissible range of admittances increases in radius and moves its centre outwards along the G-axis. The Fano criterion for bandwidth is then that frequency range within which all admittance values (seen by the generator) lie within the admittance circle for the specified value of $|\rho|_{max}$. Further, the integral limitation shows that the bandwidth is maximised by making all the admittance values lie as near as possible to the $|\rho|_{max}$ circle.

The amplifier design depends largely on the range of maximum to minimum admittances and maximum phase angles presented to it. The ratio of $|Y|_{max}/|Y|_{min}$ for admittances on or within the circle for a given $|\rho|_{max}$ is readily shown to be

$$\frac{|Y|_{max}}{|Y|_{min}} \leqslant \frac{(1 + |\rho|)^2}{(1 - |\rho|)^2}$$

and the maximum phase angle θ is given by

$$\sin \theta_{max} \leqslant \frac{2|\rho|}{1 + |\rho|^2} .$$

Typical ranges generally regarded as applicable to high power-density amplifiers would be $Y_{max}/Y_{min} = 2$, corresponding to $|\rho|_{max} = 0.172$ and $\theta_{max} = 19.5^o$, and $Y_{max}/Y_{min} = 4$, corresponding to $|\rho|_{max} = 0.33$ and $\theta_{max} = 36^o$. Note that for $Y_{max}/Y_{min} = 4$, the power into the load varies by only 0.5dB over the band, for a suitably designed amplifier.

The networks derived from Fano's work are of the type shown in Figs 2 and 3. In these circuits, all the impedances are normalised to make $R_o = 1$ ohm. C_2, C_1, L_1, and R_1 represent the basic transducer, and these components are often combined with a parallel tuning coil L_2 to form a network with two resonant elements (ie $n = 2$). Fig 2 shows the addition of one series resonant $L_3 - C_3$ combination, to form the $n = 3$ case, and fig 3 shows the network for $n = 4$. All the combinations L_1C_1, L_2C_2, etc resonate at the centre-band frequency, and the ratio C_1/C_2 is a parameter of the transducer, related to its coupling coefficient. It must therefore satisfy the condition $C_1/C_2 \leqslant W^2 = k^2/(1 - k^2)$, where k is the coupling coefficient of the transducer. The other basic parameter of the transducer is its mechanical Q-factor, $Q_m = \omega_1 L_1/R_1$, where $\omega_1^2 = 1/L_1C_1$.

Fano considered the problem of designing a network to maximise the band for a given load. In the transducer case, a maximum value of W^2 is assumed, and a value chosen for $|\rho|_{max}$; with these parameters, the element values in Figs 2 and 3 are calculated to maximise the bandwidth, and both electrical and mechanical elements are realised accordingly, thus determining the value of Q_m.

In this paper, a value of $W^2 = 0.30$ is used for the examples, since it is a typical maximum value for a lead zirconate titanate transducer, and the circle for $|\rho|_{max} = 0.172$ is used to illustrate the results. Details of the methods used to derive the network elements are given in the references. Taking the limit as $|\rho|_{max} = 0.172$ the maximum fractional band for a transducer with parallel tuning coil (and $W^2 = 0.3$) is 0.38; for the n = 3 case (as in Fig2), the band is 0.57; and for n = 4 (Fig 3), the band is 0.61. The admittance curve corresponding to the n = 3 case is shown in Fig 4, where the dashed circle indicates the limiting circle for $|\rho|_{max} = 0.172$. Most of the improvement in bandwidth is achieved by using n = 3, and little extra is obtained by using the extra elements of n = 4.

The networks discussed so far are realisable by adding electrical elements to the basic piezoelectric transducer. It is reasonable to ask whether the networks could instead be realised by mechanical elements. This question has been considered in detail by Redwood and Rodrigo (Ref 4). Many electrical filter networks are not applicable, because an inductance having one terminal at ground potential is not mechanically realisable (in the "impedance analogue" system of equivalents). Thus, inductors such as L_2 in Fig 3 cannot be simulated mechanically. Following a study of those circuits which can be constructed mainly by mechanical elements, Rodrigo concluded that the most suitable filter network was a coupled resonator network having a low-ripple Tchebyshev transmission response. The basic network is shown in Fig 5. For the mainly mechanical version of this network, the piezoelectric ceramic is represented by the capacitors C_2, C_{23}, and thus $C_2/C_{23} \leq W^2$. Further, the widest bandwidth is achieved when C_1 and C_3 both become infinitely large - ie are short-circuited. The resonant frequencies of the meshes must be equal, when the other meshes are open-circuited, and the design considered is also symmetrical, having $L_1 = L_3$ and $C_{12} = C_{23}$. Thus $L_1C_{12} = L_3C_{23} = 1/\omega_1^2$ and $L_2C_2 = (1 + 2W^2)/\omega_1^2$. Scaling all the components in terms of $\omega_1 C_{23}$, the terminating resistor is expressed as $\omega_1 C_{23} R_1 = d = R_1/\omega_1 L_1$. The transducer is then designed to obtain the best combination of d and W^2, and figure 6 shows the admittance curve for a transducer having $W^2 = 0.3$ and d = 0.45 (corresponding to $Q_m = 2.22$). This curve just fits inside a circle for $|\rho| = 0.172$, and has limiting frequencies on this criterion of x = 1.15 and 0.68. Its mid-band frequency is therefore $0.88\,\omega_1$, and its fractional band 0.53. A transducer based on these design methods has been made and tested, and shows satisfactory agreement with theory, which has to take into account various complications such as the details of the structure itself and the variation of radiation loading with frequency. All the elements except L_3 are realised mechanically.

Both of these methods give about 50% improvement in bandwidth, and work is still in progress to establish design techniques more precisely and to assess their particular advantages and limitations.

References

1. Fano, R M; J Franklin Inst. Jan & Feb 1950.

2. McCullagh, W J; ASWE Laboratory Notes XRA-68-11 and XRA-69-1.

3. Dearlove, P C; AUWE Tech Note to be published.

4. Redwood M and Rodrigo G C; Ph D Thesis. Analysis and Design of Piezoelectric Sonar Transducers. London 1970.

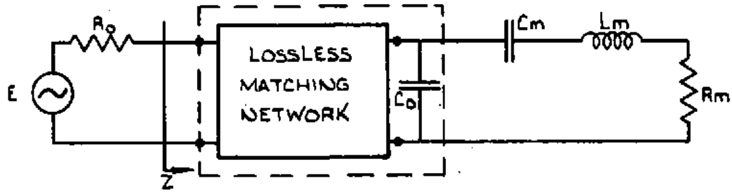

Fig 1 Basic network

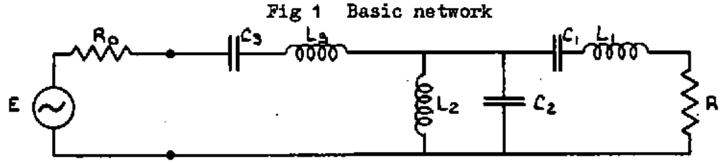

Fig 2 Matched network n = 3

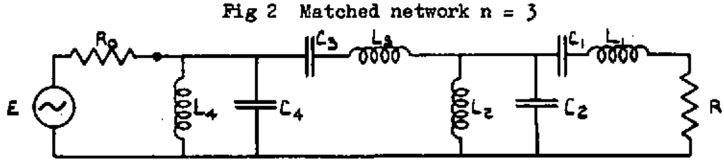

Fig 3 Matched network n = 4

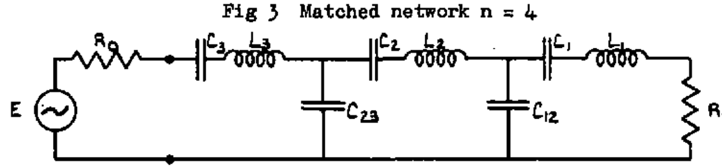

Fig 5 Coupled resonator network

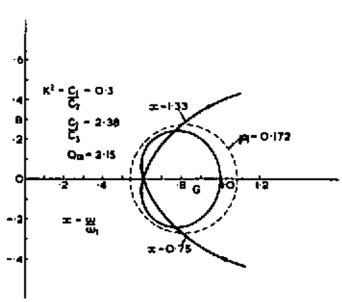

FIG. 4. ADMITTANCE CURVE FOR "FANO" NETWORK (n=3)

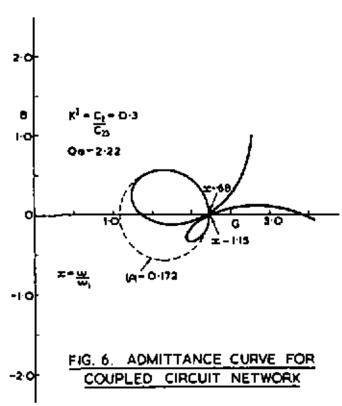

FIG. 6. ADMITTANCE CURVE FOR COUPLED CIRCUIT NETWORK