DISPERSION PROPERTIES OF MAGNETOACOUSTIC WAVES IN PLASMA WITH NON-ADIABATIC PROCESSES

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Dispersion properties of the magnetoacoustic (MA) waves in the plasma with temperature and density dependent heating and cooling processes are analyzed. We show that the non-adiabatic processes result in the frequency dependence of the phase velocity and wave increment/decrement of slow and fast magnetoacoustic waves. The characteristic heating/cooling time subdivides the whole frequency spectrum of the MA waves into high- and low-frequency ranges. The values of phase velocities in the high-frequency range are equal to the values in the equilibrium plasma. However, the values of phase velocities in the low-frequency range are defined by the non-adiabatic processes. The anisotropy of MA waves phase velocities in non-adiabatic plasma is shown by plotting polar diagrams (Friedrichs diagrams). Further, we describe the dependence of the MA waves increment/decrement on the value of the plasma beta (the ratio of the plasma pressure to the magnetic pressure) and the direction of the external magnetic field. We show that amplification of the fast MA waves is greater than amplification of the slow MA waves in the high beta plasma. In the low beta plasma, opposite situation occurs.
Keywords: magnetoacoustic waves, thermal instability

1. Introduction

The investigation of stability and evolution of magnetoacoustic waves in the non-adiabatic plasma is important for understanding the formation mechanism and nonlinear evolution of the different spatio-temporal structures in these media. The non-adiabatic state of the plasma is a consequence of various possible internal processes in the medium. In this paper, we discuss the nonadiabaticity caused be the temperature and density dependent cooling and heating processes. Typical examples of the media with such processes are interstellar media and solar atmosphere. The main cooling mechanism in these media is optically thin radiation while the heating is defined by various exothermal processes. The main objective of this paper is to describe the influence of the non-adiabatic processes on dispersion properties of the magnetoacoustic waves.

2. Model system of equations and assumptions

We conduct our analysis of magnetoacoustic waves using the full system of magnetohydrodynamic (MHD) equations. The system of equations (1) is slightly different from standard system of equations for the equilibrium plasma and takes into account heating $Q(\rho,T)$ and cooling $L(\rho,T)$ processes by the introduction of so-called heat-loss function $\Im(\rho,T) = L(\rho,T) - Q(\rho,T)$]. This function is generally used for mathematical modeling of non-adiabatic processes since the first papers [1, 2] devoted to the problem of stability in these media.

$$\frac{\partial\rho}{\partial t}+div\rho\vec{V}=0\,,\;\rho\frac{d\vec{V}}{dt}=-\nabla P-\frac{1}{4\pi}\vec{B}\times rot\left[\vec{B}\right],\;\frac{\partial\vec{B}}{\partial t}=rot\left[\vec{V}\times\vec{B}\right],\;div\vec{B}=0\,,\tag{1}$$

$$C_{V\infty}\cdot\rho\frac{dT}{dt}-\frac{k_BT}{m}\cdot\frac{d\rho}{dt}=-\rho\cdot\Im(\rho,T)\,,P=\frac{k_B\cdot T\cdot\rho}{m}.$$

Here, we use notations P,ρ,T for pressure, density, and temperature, respectively. The vectors $\vec{V},\vec{B}$ correspond to the velocity vector and the magnetic field vector, respectively. We also use notations k_B and $C_{V\infty}$ for the Boltzmann constant and specific heat at constant volume. Due to the fact that the main focus of this paper is on the non-adiabatic processes, we neglect the influence of the possible dissipation.

In current research, we restrict ourselves to the analysis of the one-dimensional wave dynamics. We use Cartesian coordinate system *x*, *y*, *z* and assume that the wave propagation direction is along *z*-axis. Furthermore, we assume that the vector of equilibrium magnetic field lies in the (x,-z) – plane, i.e. $\vec{B}_0=B_0\cdot\sin\theta\cdot\vec{x}+B_0\cdot\cos\theta\cdot\vec{z}$. Here, B_0 is an absolute value for the magnetic-field vector, θ is an angle with respect to the z-axis and $\vec{x},\vec{z}$ are the unit vectors. Hereinafter, index 0 indicates the value of any variable in the equilibrium state of the medium. The dependences of variables upon *x* and *y* are ignored $(\partial/\partial x=\partial/\partial y=0)$.

3. Dispersion properties

Using standard linearization procedure of Eqs. (1) and substitution of the harmonic wave solution in the form $\sim\exp(-i\omega t+ikz)$, we obtain dispersion relations for the fast and slow magnetoacoustic waves in the following form:

$$\left(\frac{\omega^2}{k^2}\right)^2-\frac{\omega^2}{k^2}\left(c_a^2+\tilde{c}^{\,2}\right)+\tilde{c}^{\,2}c_a^2\cos^2\theta=0.\tag{2}$$

Here, ω,k correspond to the wave frequency and wave number, respectively. We also use the standard notation for the squared Alfven speed c_a^2 an introduce the squared complex speed $\tilde{c}^{\,2}$:

$$c_a^2=\frac{B_0^2}{4\pi\rho_0},\quad\tilde{c}^{\,2}=\frac{\left(C_{P0}-i\omega\tau_0C_{P\infty}\right)}{\left(C_{V0}-i\omega\tau_0C_{V\infty}\right)}\frac{k_BT_0}{m}.\tag{3}$$

The dispersion relation mentioned above has two roots which correspond to separated dispersion relations for fast ('+' sign) and slow ('-' sign) magnetoacoustic waves:

$$\frac{\omega^2}{k^2}=\frac{\left(c_a^2+\tilde{c}^{\,2}\right)\pm\sqrt{\left(c_a^2+\tilde{c}^{\,2}\right)^2-4\tilde{c}^{\,2}c_a^2\cos^2\theta}}{2}.\tag{4}$$

In case of infinitely small magnetic field, Eqs. (2), (4) are reduced to the dispersion relation for acoustic perturbations:

$$\frac{\omega^2}{k^2}=\tilde{c}^{\,2}.\tag{5}$$

Dispersion properties can be described in different ways, for example, by using characteristic spatial values or frequencies [3, 4]. However, such approaches result in cumbersome expressions and meaningless quantities in the most cases. For this reason, in our research we use an analogy between the non-adiabatic plasma and non-equilibrium gas. Therefore, we use thermal capacities under constant volume C_{V0} and pressure C_{P0}. in Eq. (3).

$$C_{V0}=\frac{k_B}{m}\Im_{L0T},\ C_{P0}=\frac{k_B}{m}\left(\Im_{L0T}-\Im_{L0\rho}\right),\ C_{P\infty}=C_{V\infty}+\frac{k_B}{m},\quad \Im_{L0T}=\frac{T_0\Im_{0T}}{Q_0},\ \Im_{L0\rho}=\frac{\rho_0\Im_{0\rho}}{Q_0}. \tag{6}$$

Special attention has to be paid to the characteristic heating/cooling time:

$$\tau_0=\frac{k_BT_0}{mQ(\rho_0,T_0)}=\frac{k_BT_0}{mL(\rho_0,T_0)}. \tag{7}$$

This parameter allows us to describe the frequency dependence of the wave characteristics and subdivide the whole spectrum of possible wave frequencies into two qualitatively different ranges: high-frequency range $(\omega >> 1/\tau_0)$ and low-frequency range $(\omega << 1/\tau_0)$.

Using the assumption of the weak amplification per the wavelength, the frequency dependent phase velocities of fast and slow magnetoacoustic waves are obtained in following form [5, 6]:

$$c_{f,s}(\omega)=\frac{\omega}{\mathrm{Re}(k)}=\sqrt{\frac{\left(c_a^2+c_{Snd}^2(\omega)\right)\pm\sqrt{\left(c_a^2+c_{Snd}^2(\omega)\right)^2-4c_{Snd}^2(\omega)c_a^2\cos^2\theta}}{2}} \tag{8}$$

Here, indeces " f " and " s " correspond to, fast and slow magnetoacoustic wave, respectively. In Eq. (8), we use the expression describing the phase velocity of pure acoustic waves in the non-adiabatic gas (see Eq. (9)) [7]:

$$c_{Snd}(\omega)=\frac{\omega}{\mathrm{Re}(k)}=\sqrt{\frac{\left(C_{V0}^2c_0^2+\omega^2\tau_0^2C_{V\infty}^2c_\infty^2\right)}{\left(C_{V0}^2+\omega^2\tau_0^2C_{V\infty}^2\right)}}, \tag{9}$$

$$c_0=\sqrt{\frac{C_{P0}}{C_{V0}}\frac{k_BT_0}{m}}=\sqrt{\gamma_0\frac{k_BT_0}{m}},\ c_\infty=\sqrt{\frac{C_{P\infty}}{C_{V\infty}}\frac{k_BT_0}{m}}=\sqrt{\gamma_\infty\frac{k_BT_0}{m}}, \tag{10}$$

In the high-frequency limit $(\omega >> 1/\tau_0)$, the phase velocity $c_{Snd}(\omega)$ in Eq. (9) reduces to c_∞ while in the low-frequency limit $(\omega << 1/\tau_0)$, it reduces to c_0. In hydrodynamics, velocities c_0, c_∞ are called the equilibrium and frozen speeds, respectively. Quantities γ_0, γ_∞ are the equilibrium and frozen adiabatic indeces.

The frequency dependence of magnetoacoustic wave phase velocities is analogous to the mentioned above and investigated previously frequency dependence of acoustic wave phase velocity in non-adiabatic gas[7]. This would imply that in the high-frequency limit $(\omega\tau_0 >> 1)$ the magnetoacoustic wave phase velocities are determined by the expression $c_{\infty f,s}$ which equals to phase velocities expressions for equilibrium media. However, low-frequency phase velocities for magnetoacoustic waves are equal to $c_{0f,s}$ and fully defined by the non-adiabatic processes.

$$c_{0f,s}=\sqrt{0.5\cdot\left(c_a^2+c_0^2\pm\sqrt{\left(c_a^2+c_0^2\right)^2-4c_a^2c_0^2\cos^2\theta}\right)}, \tag{11}$$

$$c_{\infty f,s}=\sqrt{0.5\cdot\left(c_a^2+c_\infty^2\pm\sqrt{\left(c_a^2+c_\infty^2\right)^2-4c_a^2c_\infty^2\cos^2\theta}\right)}. \tag{12}$$

In order to describe the influence of the external magnetic field, we use the beta of a plasma:

$$\beta=\frac{8\pi P_0}{B_0^2}=\frac{2}{\gamma_\infty}\frac{c_\infty^2}{c_a^2}. \tag{13}$$

This parameter is the ratio of the plasma pressure to the magnetic pressure. The term 'beta' is commonly used in astrophysics and in the field of controlled thermonuclear fusion. Here, we take the opportunity to call your attention to the difference in characteristic domains of plasma beta in equilibrium and non-adiabatic plasma.

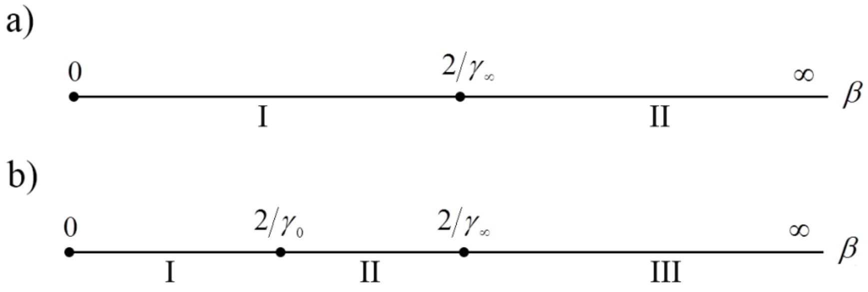

Figure 1: Plasma beta domain in the medium with a) $(c_0 > c_\infty)$, b) $(c_0 < c_\infty)$.

The possible values of beta in equilibrium plasma can be divided into two domains: (I) $\beta > 2/\gamma_\infty$ where $c_\infty > c_a$ and (II) $\beta < 2/\gamma_\infty$ where $c_\infty < c_a$ (see Fig. 1a). In case of $\beta = 2/\gamma_\infty$, the phase velocity for acoustic waves with an possible arbitrary frequency equals the Alfven speed $c_\infty = c_a$.

However, in non-adiabatic plasma the possible values of beta should be divided into three domains. For simplicity, we would describe the non-adiabatic plasma where the low-frequency phase velocity c_0 is greater than high-frequency phase velocity c_∞ (see Fig. 1b). In domain I, the following inequalities are satisfied ($\beta < 2/\gamma_0$ or $c_0 < c_a$) and the whole spectrum of acoustic waves propagates slower than Alfven waves. In domain II the inequalities ($\beta > 2/\gamma_\infty$ or $c_\infty > c_a$) are satisfied and the opposite situation takes place, i.e. the whole spectrum of acoustic waves propagates faster than Alfven waves. The most interesting domain is domain II where ($2/\gamma_0 < \beta < 2/\gamma_\infty$ and $c_0 > c_a > c_\infty$). In this domain, the part of acoustic waves spectrum propagates faster than Alfven wave and another part of acoustic wave spectrum propagates slower than Alfven wave. The only frequency ω_a (see Eq. (14)) from the whole spectrum of acoustic waves propagates with the Alfven wave speed $c_a = c_{Snd}(\omega_a)$.

$$\omega_a = \frac{1}{\tau_0}\frac{C_{V0}}{C_{V\infty}}\sqrt{\frac{\left(c_0^2 - c_a^2\right)}{\left(c_a^2 - c_\infty^2\right)}} = \frac{1}{\tau_0}\frac{C_{V0}}{C_{V\infty}}\sqrt{\frac{\left(\beta\gamma_0 - 2\right)}{\left(2 - \gamma_\infty\beta\right)}}. \quad (14)$$

The effect mentioned aboveis important not only for understanding dispersion properties of magnetoacoustic waves in non-adiabatic plasma but also for the research of possible non-linear interaction between MA and Alfven waves [8 - 10].

The magnetoacoustic dispersion properties also depend on the angle between the magnetic field and propagation direction. It is worth mentioning here that the slow magnetoacoustic and Alfven waves have stronger anisotropy than fast magnetoacoustic and Alfven waves. The most common way to show this anisotropy is plotting polar diagrams (Friedrichs diagrams). The phase polar diagram represents the dependence of phase velocity $V_{ph} = \omega/\mathrm{Re}(k)$ on the angle θ between the wave vector $\vec{k}$ and magnetic field vector $\vec{B}$ in the polar coordinate system (V_{ph}, θ).

The frequency dependence of magnetoacoustic phase velocities results in the fact that each wave harmonic ω is described only by one polar diagram corresponding to this harmonic. In equilibrium media the whole wave spectrum is described by one polar diagram.

In this paper, we restrict ourselves by the description of the phase polar diagrams of the waves in non-adiabatic plasma with beta values lying in domain II (see Fig. 1b).

Thus, in Fig. 2b, we show the case mentioned above when the part of acoustic wave spectrum propagates faster and another part slower than Alfven wave ($2/\gamma_0 < \beta < 2/\gamma_\infty$ and $c_0 > c_a > c_\infty$). The frequency dependence is described by the colour scheme. The dark colours correspond to low frequencies and bright colours correspond to high frequencies.

The case of equal phase velocities of acoustic and Alfven waves in equilibrium medium is shown in Fig. 2a.

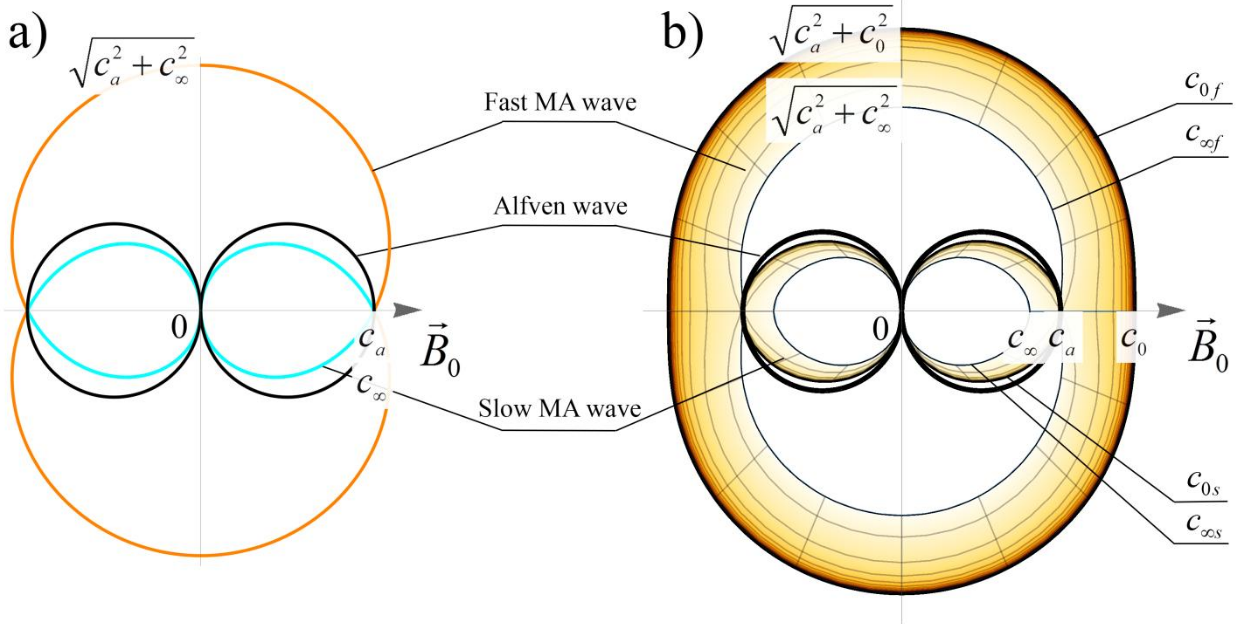

Figure 2: Phase polar diagrams of fast and slow magnetoacoustic waves and Alfven waves in a) equilibrium plasma b) non-adiabatic plasma with $(c_0 > c_\infty)$.

The non-adiabatic processes cause not only frequency dependence of phase velocities but also frequency dependence of wave increments. Using the assumption of the weak amplification on the wavelength, we get the expressions for magnetoacoustic wave increments in the form:

$$\alpha_{f,s} = \mathrm{Im}\,k = \frac{\omega^2 \xi}{4\rho_0 c_{f,s}^3} \Xi_{Snd}, \tag{15}$$

where

$$\Xi_{Snd}(\omega) = 2\frac{c_{f,s}^2 - c_a^2 \cos^2\theta}{2c_{f,s}^2 - \left[c_a^2 + c_{Snd}^2\right]} = \left(1 \pm \frac{c_{Snd}^2 - c_a^2 \cos 2\theta}{\sqrt{c_{Snd}^4 + c_a^4 - 2c_{Snd}^2 c_a^2 \cos 2\theta}}\right). \tag{16}$$

In Eq. (15), we use the notation for the frequency dependent bulk viscosity coefficient ξ.

$$\xi = \frac{\xi_0 C_{V0}^2}{C_{V0}^2 + \omega^2 \tau_0^2 C_{V\infty}^2}, \quad \xi_0 = \frac{\rho_0 \tau_0 C_{V\infty}(c_\infty^2 - c_0^2)}{C_{V0}} = \frac{P_0 \tau_0 \left(\Im_{L0\rho}/(\gamma_\infty - 1) + \Im_{L0T}\right)}{\Im_{L0T}^2}. \tag{17}$$

where ξ_0 is the low-frequency bulk viscosity coefficient.

It can be easily seen that both fast and slow magnetoacoustic waves become unstable in case of the negative bulk viscosity coefficient. This amplification condition is coincides with the condition of isentropic instability.

In low-frequency and high-frequency limits, Eq. (14) transforms into following forms:

$$\alpha_{0f,s} = \frac{\omega^2 \xi_0}{4\rho_0 c_{0f,s}^3} \Xi_0, \qquad \alpha_{\infty f,s} = \frac{\xi_0 C_{V0}^2}{4\rho_0 c_{\infty f,s}^3 \tau_0^2 C_{V\infty}^2} \Xi_\infty \tag{18}$$

$$\Xi_0 = 2\frac{c_{0f,s}^2 - c_a^2 \cos^2\theta}{2c_{0f,s}^2 - \left[c_a^2 + c_0^2\right]}; \qquad \Xi_\infty = 2\frac{c_{\infty f,s}^2 - c_a^2 \cos^2\theta}{2c_{\infty f,s}^2 - \left[c_a^2 + c_\infty^2\right]}. \tag{19}$$

In order to describe the dependence of MA waves increment on plasma beta and the direction of the external magnetic field, we plot the dimensionless quantities $\alpha_{\infty f} c_{\infty f} / \alpha_\infty c_\infty, \alpha_{\infty s} c_{\infty s} / \alpha_\infty c_\infty$ (see Fig. 3). Using these quantities, we exclude the ambiguity in some limit cases. Here, we use the high-frequency increment of pure acoustic waves in the following form:

$$\alpha_\infty = \frac{\xi_0 C_{V0}^2}{2\rho_0 c_\infty^3 \tau_0^2 C_{V\infty}^2}, \tag{20}$$

The plot represents the surface of revolution with height $\alpha_{\infty f} c_{\infty f} / \alpha_\infty c_\infty$ or $\alpha_{\infty s} c_{\infty s} / \alpha_\infty c_\infty$ at radius $\lg\beta$. The azimuthal angle θ is varied between 0 and $\pi/2$.

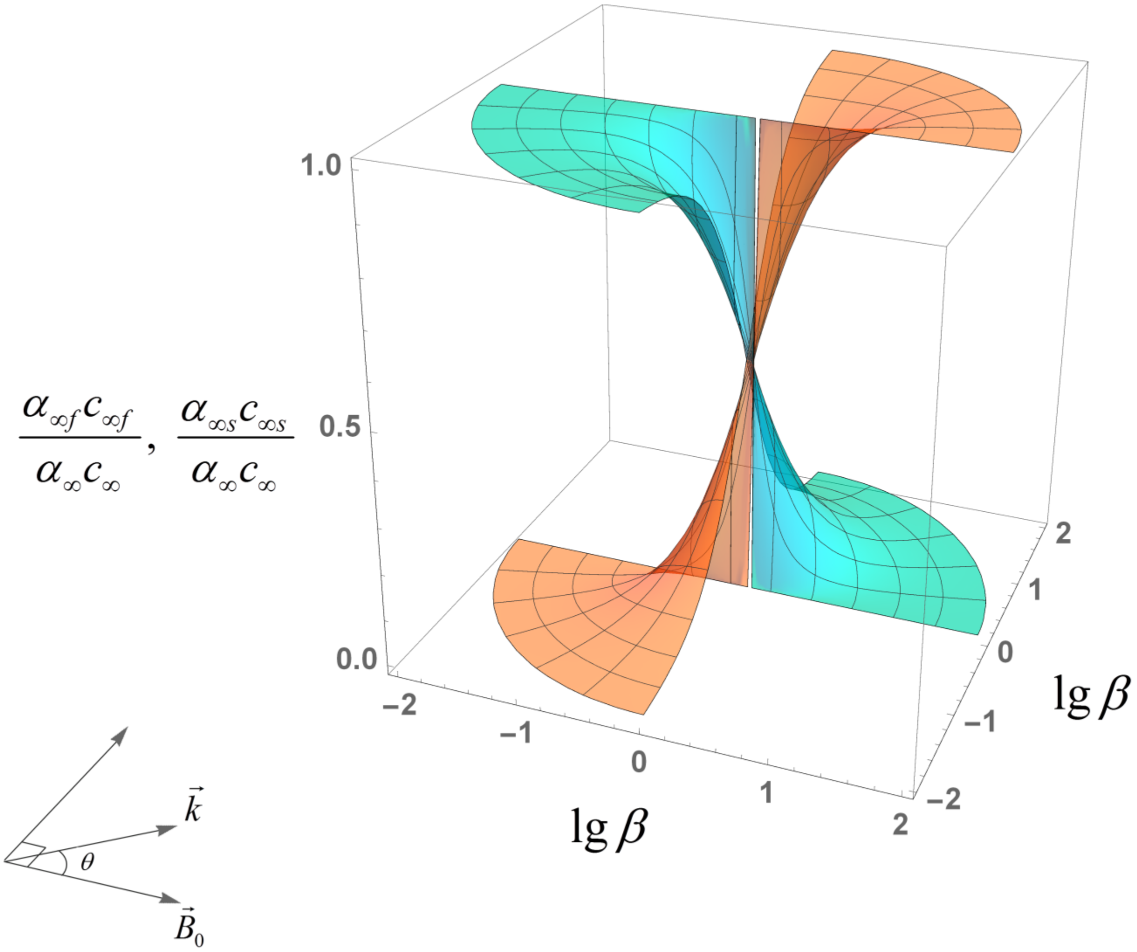

Figure 3: The plot of increment dependence on the value of the plasma beta and the direction of the external magnetic field for fast (orange) and slow (cyan) magnetoacoustic waves.

The plots in Fig. 3 show that amplification of the fast MA waves is greater than amplification of the slow MA waves in the high beta plasma. In the low beta plasma, the opposite situation occurs. They also show that in case of $\beta = 2/\gamma_\infty$ both incitements for fast and slow MA waves equals to half of the pure acoustic wave increment.

4. Conclusion

In this paper, we have shown that non-adiabatic processes significantly effect on the dispersion properties of magnetoacoustic waves. They cause frequency dependences of wave increment, phase velocity, and also group velocity (this problem will be described in further papers). Moreover, in case of the isentropic instability, non-adiabatic processes result in positive the feedback between the medium and magnetoacoustic perturbation. It causes their amplification. The discussed results are important for diagnostics and analysing spatio-temporal structures in the natural and laboratory non-adiabatic plasma.

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