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BEARING ESTIMATION IN THE PRESENCE OF ARRAY CALIBRATION ERRORS

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1. INTRODUCTION

In this paper the problem of how to correct direction-of-arrival estimates from an array which has unknown phase errors is addressed. The standard MUSIC algorithm is discussed as a starting point, but in terms of the signal rather than noise subspace. The effect of array perturbations on this spectrum result in a loss of resolution and small deviations in the peaks from their true angles. A method is presented which uses the initial direction-of-arrival estimates from the standard MUSIC spectrum and finds the projection of these into the perturbed signal space. An algorithm which derives the corrected signal space is then given. This also finds the phase deviations on each sensor and can be used to improve the output of MUSIC or other spectral estimators such as Capon's minimum variance, or indeed of a conventional beamformer.

Current approaches to spatial power spectrum estimation include the minimum variance (MV) method due to Capon [1], Burg's maximum entropy (MEM) [2], the minimum norm method (TK) proposed by Tufts and Kumaresan [3], and MUSIC due to Schmidt [4]. In terms of resolution the minimum norm is accredited with being the best, while MUSIC and MEM come in a close second and third, leaving Capon a poor fourth. It has been shown that the minimum variance spectra is the average of a number of MEM spectra of increasing order, hence its resolution will always be less than that of MEM. Now, if the criteria adopted is robustness then the ordering of these techniques reverses, the minimum variance method makes no assumptions about the noise spatial correlation and is applicable to arbitrary, but known array shapes. Since TK is derived from autoregressive theory it is strictly applicable to linear equispaced line arrays only and like MEM and other AR methods it incorrectly models an additive noise process (MA) and is only an approximation to the true ARMA case.

Recently other methods have been proposed which iteratively rotate vectors in the signal space [5], [6]. These will cope with severe array miss-calibration, but in some circumstances they are slow to converge and the procedure followed here is a fast sub-optimum approach to the rotation.

Schmidt correctly used the measurement model which includes an additive noise process in the MUSIC method. It also has the added advantage of being applicable to arrays of arbitrary geometry. Just as for Capon's MV, it is necessary to store an array calibration table in memory (often referred to as the *array manifold*), which enables a search over all possible source bearings. It has been shown that the MUSIC algorithm is sensitive to a miss-match between the modelled and true array geometries, especially for closely spaced sources. It can, in such circumstances, fail to resolve them [7]. But MUSIC may be made more robust to array calibration and positioning errors by accounting for this discrepancy in the array manifold.

When using a towed array, for example, phase and amplitude errors can arise due to a number of causes:

(i) Poor matching of sensors and manufacturing tolerance
(ii) Drift in amplifier circuits
(iii) Ageing of components
(iv) Snakeing of the array following tow speed variations [8]

This paper addresses one way in which calibration errors may be measured from the acoustic data and then used to enhance the resulting MUSIC or Capon spectra. In the context of bearing estimation, the only assumption it makes is that the received source signals are wide sense stationary, while each source wavefront is perturbed identically by the errors in sensor positions and calibration.

The notation is introduced first, then the MUSIC algorithm is derived using a signal space approach which gives a better insight to its nature. Section 4 derives the effect of array calibration errors on the signal subspace and an algorithm to measure them is presented. Finally the performance is evaluated using simulated data in Section 5.

2. DATA MODEL AND NOTATION

Lowercase underlined letters refer to vectors, uppercase letters indicate matrices. For complex valued matrices $(*)^H$ is the Hermitian transpose or complex conjugate transpose operator and a tilde (~) above a variable implies it has been perturbed. At all times a working knowledge of linear algebra is assumed and the reader is referred to Strang [9] or another suitable text if in difficulty.

Data from an array of sensors is divided into narrow frequency bands and one of these is processed to find the source directions-of-arrival. The transmission medium is taken as isotropic and non-dispersive so radiation propagates in a straight line. Furthermore the sources are assumed to be in the far field so they appear as plane waves at the array. For simplicity the case of a line array is examined here but the method can also be extended to planar arrays. Following this the signal on the p^{th} sensor can be written as

$$x(p) = \sum_{m=1}^{M} s(m) e^{j(p-1)\phi(m)} + e(p), \qquad p = 1, \ldots, P \tag{1}$$

where $s(m)$ is the complex envelope of the m^{th} source at the first (reference) sensor, and $\phi(m)$ is the electrical phase shift between adjacent sensors due to the m^{th} plane wave source.

Noise on the p^{th} sensor is included as a zero mean complex Gaussian random variable $e(p)$ with variance σ_N^2. Let $\underline{x}_n$ be the $P \times 1$ observed signal vector from the P sensors at time instant n, and $\underline{e}_n$ be the additive noise at the array also at time instant n. The M source envelopes are grouped together as a vector $\underline{s}_n$. Furthermore each source can be expressed as its complex envelope, $s(m)$ multiplied by a direction vector $\underline{d}_m$, which defines the phase response of the array to the m^{th} plane wavefront arriving from bearing θ_m. All the direction vectors for the M plane waves present form the columns of a direction matrix D. These vectors are detailed below

$$\underline{x}_n^T = [x(1),\dots,x(P)] \qquad \underline{s}_n^T = [s(1),\dots,s(M)]$$

$$\underline{e}_n^T = [e(1),\dots,e(P)] \qquad D = [\underline{d}_1,\dots,\underline{d}_M]$$

Using these definitions Eq.(1) can be simplified to

$$\underline{x}_n = D\underline{s}_n + \underline{e}_n \tag{2}$$

The problem at hand can be stated as: knowing $\underline{x}_n$, $n=1,\dots,N$, estimate the direction matrix D. This requires the $P \times P$ spatial cross-spectral density matrix, defined as

$$R = E[\underline{x}\underline{x}^H]$$

to be formed, where $E[]$ is the expectation operator over time n. Also define the $M \times M$ source cross-power matrix as

$$C = E[\underline{s}\underline{s}^H] \tag{3}$$

Now the cross-spectral density matrix can be expressed as

$$R = DCD^H + \sigma_N^2 I \tag{4}$$

which is known as the direction-of-arrival decomposition.

3. SIGNAL SPACE SPECTRUM ANALYSIS

A set of M orthonormal vectors that are contained in the same subspace as the columns of D can be obtained by performing an eigen decomposition on R,

$$R = VZV^H \tag{5}$$

In the usual notation V is the matrix of orthonormal eigenvectors, and Z is a diagonal matrix of the eigenvalues arranged in descending order. It is known that the eigenvalues are distributed as follows [10].

$$\zeta_i = \begin{cases} \mu_i + \sigma_N^2 & i = 1,\dots,M \\ \sigma_N^2 & i = M+1,\dots,P \end{cases}$$

The smallest P-M eigenvalues are all equal to the noise power and the largest M are equal to the noise power plus the signal power referenced to the eigen-basis. The usual approach is to split the space spanned by R into a signal and noise subspace, but here a split is performed which mimics the direction-of-arrival decomposition Eq.(4).

$$R = BB^H + \sigma_N^2 I \tag{6}$$

where

$$B = V(Z - \sigma_N^2 I)^{1/2} \tag{7}$$

Notice the similarity of Eq.(4) and Eq.(6), where the columns of DCD^H and BB^H must span the same space. They are in fact related by a transformation matrix T such that

$$DCD^H = (D\Psi)(D\Psi)^H = BB^H \tag{8}$$

let

$$T = \Psi^{-1}$$

therefore

$$D = BT$$

The problem has been reduced to finding an arbitrary invertable matrix T which operates on columns of B, to give the columns of D. In practice however the exact covariance matrix R is unknown, but is approximated by averaging together a number of array data snapshots, consequently the columns of B and D will not span exactly the same space. If the number of sensors, P, is very much greater than the number of sources, M, then a solution is available in the least squares sense for T, and is given by

$$T = B^{\#} D \tag{9}$$

where $(*)^{\#}$ is the pseudo-inverse operator. This least squares solution is in effect equivalent to minimizing the following Frobenius norm

$$\underset{T}{\text{minimize}} \quad J = |BT - D|^2 \tag{10}$$

This would be fine if D were known completely, but since it is not then a search vector $\underline{q}(\theta)$ can be used to search over the stored array manifold to find the corresponding minimum in this reduced dimension Frobenius norm

$$J(\theta) = |B\underline{t} - \underline{q}(\theta)|^2 \tag{11}$$

where

$$\underline{t} = B^{\#}\underline{q}(\theta) \tag{12}$$

The columns of T can be approximated by the M vectors $\underline{t}$ corresponding to the M minima of $J(\theta)$. This forms the optimum transformation in the range space of B to give estimated source direction vectors lying as close as possible in a least squares sense to the true direction vectors.

Now it is possible to simplify the cost function by expanding the expression for the pseudo-inverse, whereupon J becomes

$$J(\theta) = \|BB^{\#}\underline{q}(\theta) - \underline{q}(\theta)\|^2 = |B(B^H B)^{-1} B^H \underline{q}(\theta) - \underline{q}(\theta)|^2$$
$$= \|V_s V_s^H \underline{q}(\theta) - \underline{q}(\theta)|^2 \tag{13}$$

This is simply the squared length of the error vector between $\underline{q}(\theta)$ and its projection into the signal space V_S. Forming the reciprocal of J results in the conventional MUSIC spectrum but expressed in terms of the signal space.

$$\frac{1}{J(\theta)} = \frac{1}{P - \underline{q}^H(\theta) V_s V_s^H \underline{q}(\theta)} \tag{14}$$

The source cross power matrix may now be estimated by

$$C = (T^{-1})(T^{-1})^H \tag{15}$$

4. SPECTRUM ANALYSIS IN THE PRESENCE OF CALIBRATION ERRORS

The previous model for the sensor covariance matrix made the assumption that D had a Vandermonde form. This can be shown to have full rank M [10], provided the source angle-of-arrivals are all different. This section shows how to correctly use the MUSIC algorithm in the presence of perturbations from an ideal Vandermonde structure. One must still assume however that D maintains full rank in its new form, or else the signal subspace spanned by the columns of DCD^H will have dimension less than the number of sources present. This has the same effect as having fully correlated sources which will also degenerate the rank of DCD^H.

For an imperfect array, D can be replaced by an array manifold which is the product of a phase perturbation matrix, detailing the array phase mismatch, and a Vandermonde matrix which is the ideal response.

$$\tilde{D} = \Delta D \tag{16}$$

where

$$\Delta = \mathrm{diag}[1 \quad e^{j\delta_2}, \ldots, e^{j\delta_P}] \tag{17}$$

A perturbed covariance matrix can be formed using the information about array phase characteristics.

$$\tilde{R} = \Delta D C D^H \Delta^H + \sigma_N^2 I \tag{18}$$

$$= (\Delta B)(\Delta B)^H + \sigma_N^2 I \tag{19}$$

As a phase shift applied to the data does not affect its variance, then the covariance matrix will maintain the same eigenvalues as before, but the eigenvectors will have a simple phase shift equal to the phase perturbation added to each element. Therefore the array phase deviation from linear is imbedded equally in all the signal eigenvectors, each eigenvector coefficient carrying the corresponding phase perturbation in that sensor.

Now following a similar argument to the one above, the correct cost function to be minimized is

$$\underset{T}{\text{minimize}} \quad J = |\tilde{B}T - \Delta D|^2$$

However Δ is unknown, so an initial spectral estimate is formed using a search vector $\underline{q}(\theta)$ over the array manifold in the same way as before.

$$J(\theta) = \left|\tilde{V}_s \tilde{V}_s^H \underline{q}(\theta) - \underline{q}(\theta)\right|^2 \tag{20}$$

At each of the M minima of this function then J is the squared length of the error vector $\underline{\epsilon}_m$ between $\underline{d}_m$ and the space of $\tilde{V}_s$, therefore

$$\tilde{V}_s \tilde{V}_s^H \underline{d}_m = \underline{d}_m + \underline{\epsilon}_m \qquad m = 1, \ldots, M \tag{21}$$

Also of course $\underline{d}_m + \underline{\epsilon}_m$ is in the range space of $\tilde{V}_s$, such that $\tilde{V}_s \tilde{V}_s^H (\underline{d}_m + \underline{\epsilon}_m) = \underline{d}_m + \underline{\epsilon}_m$, but at each minima, $\Delta \underline{d}_m$ corresponds to the way the m^{th} source was sampled by the array, so it is also true to say that

$$\tilde{V}_s \tilde{V}_s^H \Delta \underline{d}_m = \Delta \underline{d}_m$$

therefore

$$\Delta \underline{d}_m = \underline{d}_m + \underline{\epsilon}_m \qquad m = 1, \ldots, M \tag{22}$$

This allows Δ to be found when the M minima search vectors are collected together as the columns of Q, and the M projected search vectors are the columns of $\breve{Q}$. The first element of each projected vector should be normalized to 1 and subsequent elements should have the same absolute value as the corresponding element in the search vectors.

$$\text{diag}(\Delta) = \text{diag}(\breve{Q} Q^*) \tag{23}$$

The pseudo-inverse is a convenient mathematical tool at this point but Δ may equally be found by phase unwrapping the columns of $\breve{Q}$ [5], and performing a least squares fit to these.

An enhanced MUSIC spectrum can now be formed, still using the stored array manifold, by correcting the signal space with the estimated perturbation matrix.

$$\tilde{V}_s \tilde{V}_s^H = \Delta^H \tilde{V}_s \tilde{V}_s^H \Delta \tag{24}$$

5. SIMULATION RESULTS

To show the accuracy of phase angle estimates that may be attained, then a simulation has been performed using an 8 element array of sensors with half wavelength spacing. Each sensor has a phase perturbation applied which varies between -0.2 and +0.2 rads. The perturbation is held constant over the covariance matrix integration time.

In the first simulation 2 sources, each of 10dB SNR, are present with wavenumbers 1 and 1.4 rads, which ensures MUSIC may easily resolve them. The covariance matrix is integrated over 100 snapshots. Fig.(1) shows the phase angle perturbation estimates obtained from each source for 5 runs. These estimates have a mean equal to the true values and exhibit low variance. As the performance of MUSIC is known to degenerate for fewer snapshots, then in Fig.(2) the same two sources and SNRs are used but now the number of snapshots is reduced to 20. The mean estimate once again converges to the true value, but more variance is present.

If the MUSIC spectrum is corrected by using Eq.(24) above then an improved determination of the peak positions can be attained. In the final simulation 10 normal MUSIC spectra are plotted and using the same sets of data 10 enhanced spectra are also plotted. For each run the phase perturbations on each sensor are drawn from a Normal distribution with zero mean and variance of $\pi/5$ rads. The source wavenumbers are 1 and 1.25, both of 10dB SNR with 100 snapshots.

The MUSIC spectra Fig.(3a) reveal that two sources are present but grossly over-estimates their separation and exhibit high sidelobe levels. After projection, a least squares fit and noise space correction the two sources are easily discriminated Fig.(3b) and at the correct bearings. Clearly even when the initial bearing estimates from MUSIC are poor (in this case out by 20%), then the method still shows good performance.

6. CONCLUSIONS

Information regarding the spatial sampling process can be extracted by considering a signal space approach to high resolution bearing estimation. Having formed a MUSIC spectrum, the estimated source direction vectors can be examined to determine how closely they match the steering vectors. This reveals whether the array manifold needs to be re-calibrated. If the perturbations are small then the signal (or noise) space can be corrected to improve resolution at little extra computational cost.

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7. REFERENCES

[1] Capon.J., "High-resolution frequency wavenumber spectrum analysis", *Proc. IEEE*, Vol.57, pp.1408-1418, August 1969.

[2] Burg.J., "Maximum entropy spectral analysis", *Proc. 37th Meeting Soc. Exploration Geophysics.*, Oklahoma City, October 1967.

[3] Kumaresan.R., Tufts.D.W., "Estimating the frequencies of multiple sinusoids: Making linear prediction perform like maximum likelihood", *Proc. IEEE* Vol.70 1982, pp.975-984.

[4] Schmidt.R.O.,"Multiple emitter location and signal parameter estimation", *IEEE Trans.*, Vol.AP-34, No.5 1986, pp.276-280.

[5] Farrier.D.R., Jeffries.D.J., "Robust, high resolution array beamforming", *Proc. IOA*, Vol.7, Pt.4, 1985, pp.17-24.

[6] Vezzosi.G., "Estimation of phase angles from the cross spectral matrix", *IEEE Trans*, Vol.ASSP-34, 1986, pp.405-422.

[7] Friedlander.B., "A sensitivity analysis of the MUSIC algorithm", *Proc. ICASSP* 1989, pp.2811-2814

[8] Van Ballegooijen.E.C., "Shape and attitude measurements in towed arrays", *Proc. Undersea Defence Technology* 1988, pp.27-32.

[9] Strang.G., *Linear algebra and its applications*, Academic Press Inc (London) LTD, 1976

[10] Cadzow J.A., "A high resolution direction-of-arrival algorithm for narrowband coherent and incoherent sources", *IEEE Trans* Vol.ASSP-36, 1988 pp.965-979.

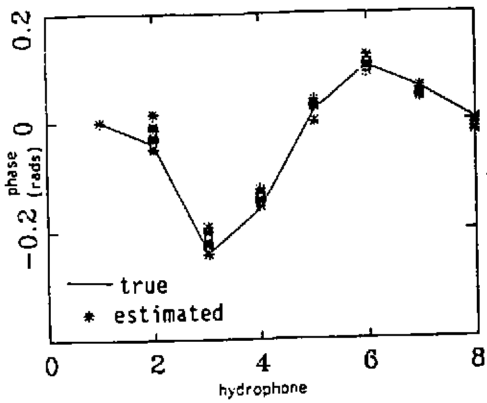

Figure 1
phase deviations on each sensor
100 snapshots

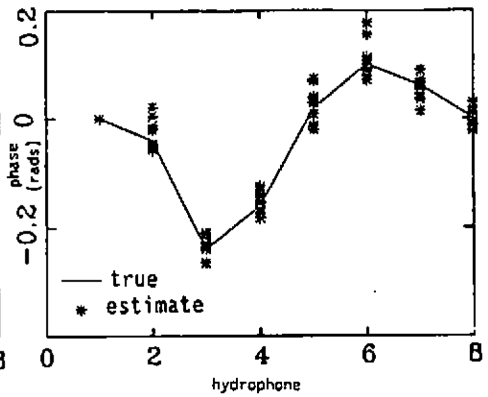

Figure 2
phase deviations on each sensor
20 snapshots

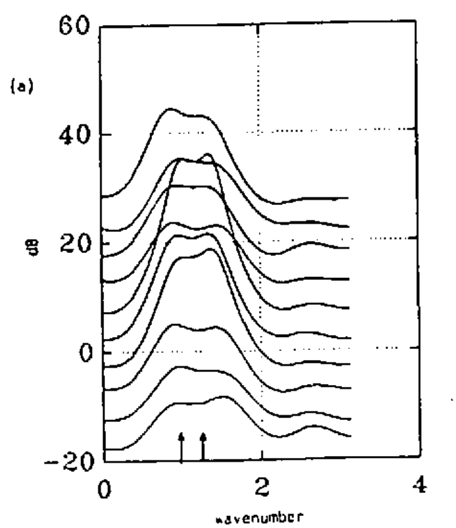

Figure 3(a) MUSIC spectrum

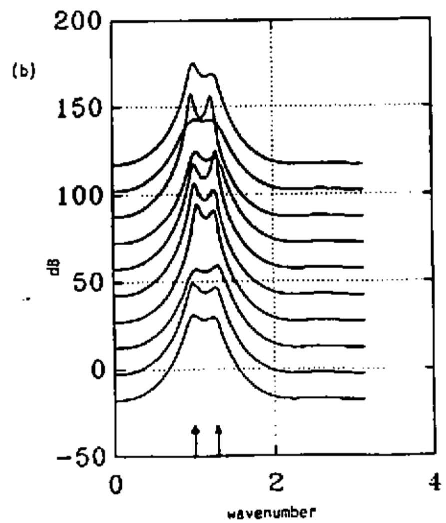

Figure 3(b) enhanced MUSIC spectrum