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## NUMERICAL CALCULATION OF THE PROPAGATION OF LASER LIGHT THROUGH WATER

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### INTRODUCTION

In any analysis of underwater laser systems it is imperative to model accurately the propagation of a pulse of laser light from the source through the sea to the receiver. This is so because of the exponential attenuation law where small uncertainties in the attenuation coefficient lead to large errors in the predicted laser power requirements for a given range. Only when single scattering occurs can analytic techniques be used to accurately predict the beam spread and attenuation of the pulse. When multiple scattering of the laser pulse occurs analytic methods are no longer adequate and numerical techniques must be used. The most useful numerical technique that may be used is the Monte Carlo method [2].

The 'Monte Carlo technique' is a method of computer simulation of a physical system using random numbers. There are two main forms of Monte Carlo techniques available, namely the probabilistic and deterministic methods. The probabilistic method generates random numbers such that they simulate the physical random processes of the system under discussion, a prime example being the simulation of the propagation of a laser pulse through a random medium. The deterministic approach is applied to the case when there is considerable theoretical understanding of the system under investigation and will not be discussed any further here.

The general optical propagation properties of water have been reviewed by Cooke and Dickinson [1] in an accompanying paper where particular attention was paid to the absorption and scattering effects. This paper will first discuss the usefulness of Monte Carlo methods with particular reference to a specific code developed at NOAA [2]. In this code the approach is to model the transport of photons through the water as a series of individual absorption and scattering events. The mean free path and the scattering angle are chosen randomly but weighted to the various inherent properties of the water such as the volume scattering function and the absorption coefficient. The code then predicts the spatial and temporal spreading of an initially narrow laser pulse, and has been applied to the problem of propagating a laser pulse over large distances in 'clean' sea water [1].

We also discuss the propagation of laser pulses through water containing air bubbles. The scattering properties of a single bubble are well understood, allowing algorithms to be constructed to model by, Monte Carlo methods, the

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propagation of a pulse through a medium containing a random distribution of bubbles of varying size.

### MODELLING OF THE PROPAGATION OF PULSED LASER LIGHT THROUGH WATER USING MONTE CARLO TECHNIQUES

When we wish to design an underwater laser system it is essential to carry out accurate systems analysis to characterise the performance of the system. To this end a Monte Carlo code has been obtained from NOAA that models the propagation of laser light through a given type of water. The code was designed to model the propagation of individual photons through water as a series of random absorption and scattering events. This allows both the spatial and temporal forms of a given laser pulse to be predicted at a given range.

#### The Computer Code

In the code the scattering properties of sea water are defined by the volume scattering function  $\beta(\theta)$ , which characterises the scattering properties for a given type of water and the 'albedo for single scattering'  $\omega_0$ , which is the fraction of the incident energy at each scattering event that is not absorbed [2].

The Monte Carlo code is a probabilistic code where individual photons are followed through separate scattering events by generation of a random number between 0 and 1. This random number is then used to calculate the position of the next scattering event and the scattered angle by weighting with respect to  $\beta(\theta)$ . Absorption of photons is simulated by retaining a weighting factor, related to  $\omega_0$ , at each scattering event.

The code obtained from NOAA was constructed to model the experimental readings from an airborne lidar system that measures the depth of coastal waters surrounding the American shore-line. It follows the downward path of individual photons from the surface of the water to the sea bed for different angles of incidence. The photons that are reflected from the sea bed are then followed up to the surface using downward trajectories randomly chosen from an archive file. This allows the paths of the upwelling photons to be followed without having to generate them again. On reaching the surface, the path of the photon is followed back to the laser source. This allows the form of an initial pulse of laser light to be followed through the entire path.

The code has been used to study the one-way propagation of pulses of laser light to large depths for a given water type, namely 'clean' water [1]. In our example the albedo was chosen to be  $\omega_0=0.75$ . From Cooke and Dickinson [1] we find that for this type of sea water the scattering coefficient,  $b$ , equals  $0.27\text{m}^{-1}$ , and the diffuse attenuation coefficient  $K_d$  is  $0.115\text{m}^{-1}$ . The code is used to calculate the spatial and temporal distributions of the remaining

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photons at given optical depths, in this case at 4, 8, 12 and 16 respectively, giving physical depths of 11, 22, 33 and 44 metres. The simulations presented are for  $10^4$  initial photons.

The spatial results for this case are illustrated in figure 1. The effects of absorption mean that the intensity of the beam falls as the depth increases. The effects of scattering tend to broaden the beam, in this case by about 14 degrees. Figure 2 shows the temporal results for an initial triangular pulse of 17ns width. The broadening of the pulse with increasing range is clearly evident. In figure 2 each of the curves are normalised separately.

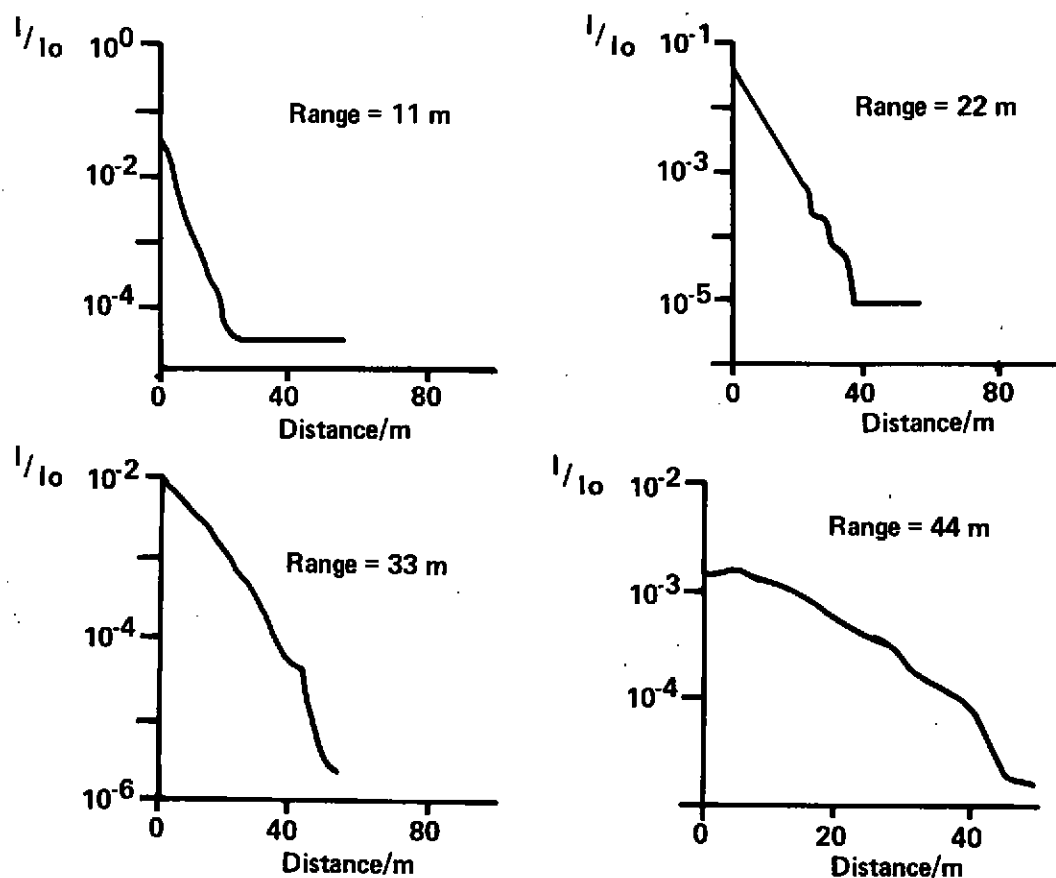


Fig.1 Spatial results at increasing optical depths for 'clean' sea water with  $\omega_0=0.75$ .

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More information may be obtained from the code with minimal modification, e.g. it may be modified to include either the effects of a receiver's field of view (FOV) on the observed intensity at a given depth or be used to consider the effects of inhomogenous water columns. In principle we can simulate the propagation of laser light to larger depths, but accurate results may only be obtained by using many more photons than we have used in the simulation.

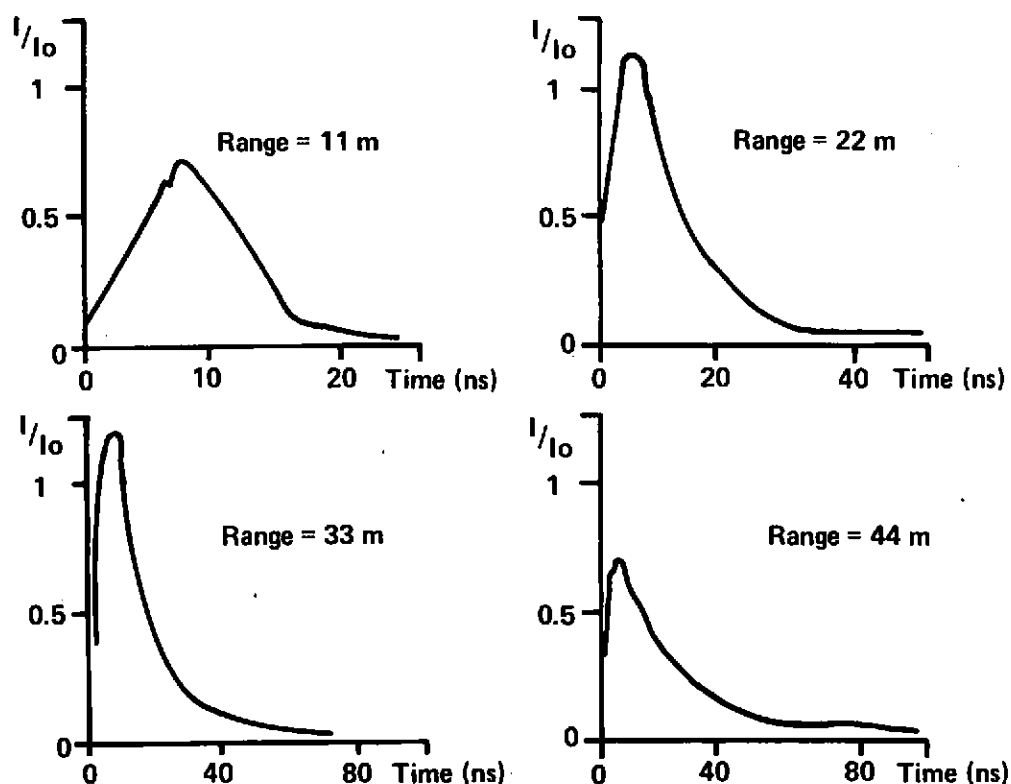


Fig.2 Temporal results for an initial beam width of 17ns at increasing optical depths for 'clean' sea water with  $\omega_o=0.75$ .

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### MODELLING OF THE PROPAGATION OF PULSED LASER LIGHT THROUGH BUBBLY WATER

In discussing bubbly water we will assume that the bubble size is much greater than the wavelength of the incident light allowing the effects of diffraction to be ignored.

Both theoretical and experimental studies have been carried out by a number of authors [3] to [6] into the scattering properties of a single bubble of air in water. In these papers the theoretical results have been compared favourably with experimental data. However, very little work has previously been carried out on the propagation of pulsed laser light through bubbly water, i.e. water containing many bubbles. It is the intention in this section to discuss this problem using a simple two-dimensional model.

We will first review the scattering properties of individual bubbles in water. We then introduce a two-dimensional Monte Carlo algorithm that models the propagation of single photons through bubbly water and presents some results for a randomly generated distribution of 15 by 15 bubbles. It should be noted that the algorithm neglects the absorption and scattering properties of the water, although this could easily be included in a more complete model.

#### Scattering of light by a bubble in water

As stated earlier we will assume that the bubble size is much greater than the wavelength of the incident light, allowing the effects of diffraction and interference to be ignored. It is assumed that parallel rays fall upon a spherical bubble. In general, bubbles will not necessarily be spherical, but a generalisation to more realistic shape is straightforward to consider.

Davis [3] produced the first comprehensive model for the scattering of light by a bubble of air in water. He showed that the scattered intensity in three dimensions, a distance  $D$  from the centre of the bubble, could be calculated by a two-dimensional model and multiplying the results by a factor  $0.25 \times (a/D)^2$ . The model that he introduced is shown in figure 3 where the bubble radius is  $a$ . The diagram shows the paths of the reflected and refracted components of a ray with an angle of incidence  $i$  and angle of refraction  $r$ . The scattering angle  $\theta$  gives the direction of rays leaving the bubble. He then used the Fresnel conditions to calculate the intensity of the scattered light at infinity. A significant feature of results was that there is an abrupt decrease in the intensity for scattering angles greater than  $82^\circ$ , the critical scattering angle in water.

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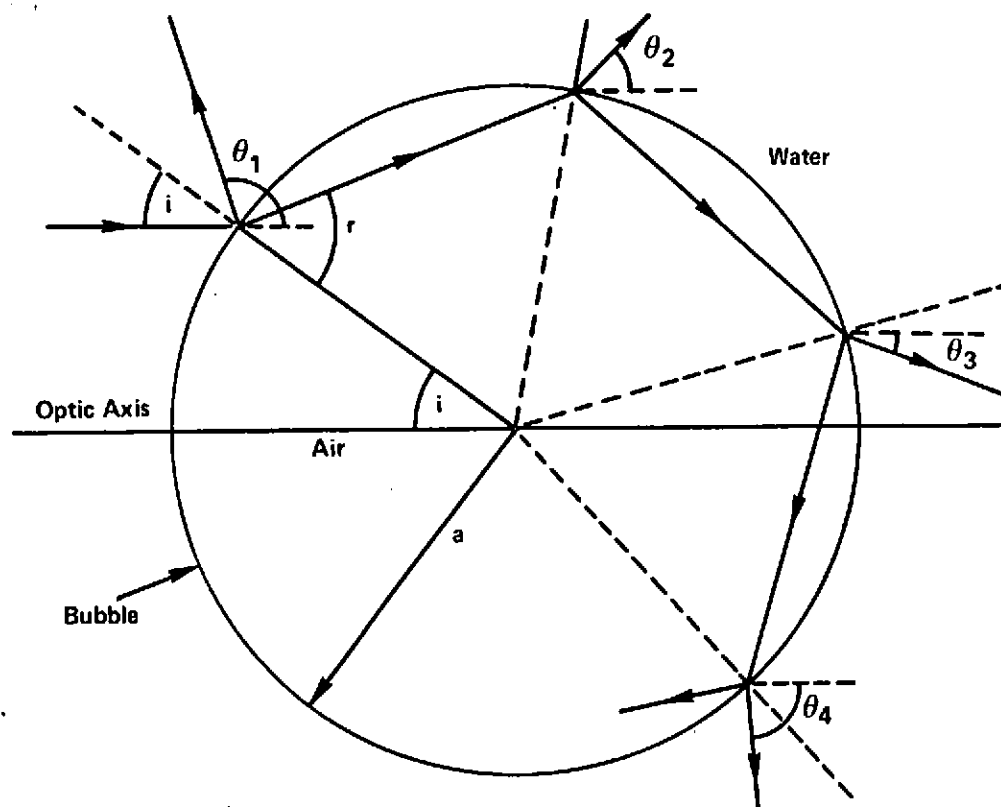


Fig.3 Reflection and refraction of an incident ray on an air bubble in water.

It should be noted that the cut-off in intensity as predicted by the geometric optics model implies that the gradient of the intensity is discontinuous at this point contradicting diffraction theory. In a series of papers Marston et al [4] to [6] have studied the diffraction and interference effects around a bubble using both physical optics and Mie scattering algorithms [7].

On comparison of these results with Davis' earlier model it was found that the gross features of the scattering were unchanged for the sizes of bubbles under discussion.

#### Two-dimensional Monte Carlo Code

Before proceeding with the description of the two-dimensional Monte Carlo code we define the governing parameters of the bubbly water. As we are dealing with a pulsed source of laser light it may be assumed that the time evolution of the distribution of bubbles need not be considered. The basic parameters are as follows:  $a$  is the average radius of the bubbles,  $R$  is the average separation of the bubbles with respect to their centres and finally  $L$  is the distance over which the pulse is required to propagate. To proceed we will assume that the

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bubbles occur in a square of side  $L$ . In this simple model the scattering properties are governed by two non-dimensional parameters of the form  $a/R$  and  $(L/R)^2$ . These parameters are the mean separation of the bubbles and the number density respectively.

We describe a numerical code that has been constructed to model the propagation of many individual photons through bubbly water. For specific values of  $a/R$  and  $L/R$  we create a distribution of bubbles using an appropriate random number generator to calculate specific positions and radii of bubbles that are constrained in a square of side  $L$ .

The aim of the code is to follow single photons through a given distribution of bubbles. The photons enter the bubbly medium in a beam centred about the mid point of one of the boundaries and is followed until a bubble is reached. When this occurs, the angle of incidence is calculated together with the values of the reflectivity coefficient and transmittance. The geometry described in figure 3 yields a number of exit points and the probability of exiting at a given point related to the value of the reflectivity coefficient. A random number is then applied to choose the actual exit point, allowing the new direction of the photon to be defined. This continues until the boundary of the bubbly region is reached, allowing the scattered angle to be counted into angular bins. The direction of the scattered photon is defined in terms of the initial laser direction.

We will illustrate the form of the results we expect from the algorithm by considering a  $15 \times 15$  distribution of bubbles produced for  $a/R=1/6$  and  $L/R=15$ , corresponding to bubbly water with a large concentration of bubbles, and count the exciting photons in 5 degree bins. The total number of photons input to the square is  $10^4$  in steps of 500. The photons are released into the square centred about the mid-point of one of its sides, with beam size given by the average diameter of the bubbles, and individual positions within the beam chosen at random.

Figure 4 shows a typical output from the code. It can be seen that the scattering is severe compared to normal sea water.

### CONCLUSION

This paper described the application of numerical techniques to the propagation of pulsed laser light through water containing particulate matter and also water containing air bubbles. The code developed to study propagation through 'bubbly' water was based purely on geometrical optics and is currently being extended to include the effects of diffraction and non-sphericity of bubbles.

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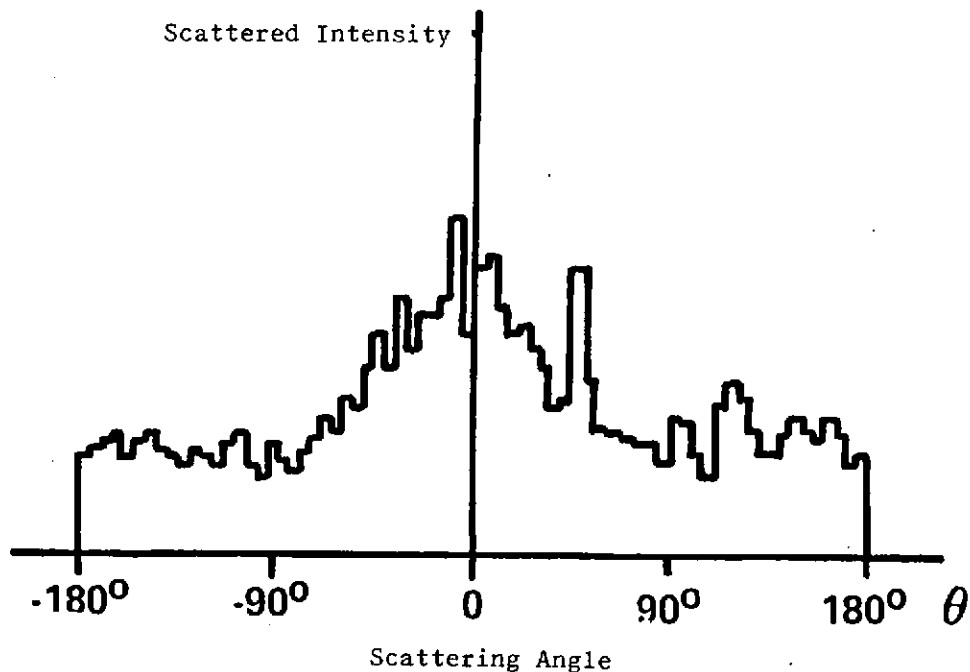


Fig.4 Typical output from 2-D Monte Carlo code for air bubble in water.

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