

## DIFFUSION AND ABSORPTION OF QUADRATIC RESIDUE DIFFUSERS

D. E. Commins, N. Auletta, B. Suner

commins-bbm, consultants in acoustics  
9-11 Kensington High Street - London W8 5NP

### I - INTRODUCTION

The need for homogeneous diffusion of sound in large concert halls has led to the design of quadratic residue diffusers using Schroeder's theory. Such a solution seems most appropriate when "contemporary music" is to be performed with non-classical distributions of musicians throughout the hall or in the case of multipurpose concert halls, where musicality should not be obtained at the expense of clarity or intelligibility.

The diffusion of sound by quadratic residue diffusers is due to the interference of sound waves reflected by the bottom of each cavity and of incident waves. The key parameter is the variable depth of the successive cavities: it is computed using the elementary number theory of Legendre and Gauss.

The diffusing characteristics of quadratic residue diffusers can be defined according to

- the number of directions of diffusion within a frequency range,
- the energy efficiency.

From the first characteristic and the selected design frequency, one can define the following parameters:

- the prime number  $N$ , which is equal to the number of cavities for one period,
- the cavity width

and compute the cavities depth.

### II - EXPERIMENTAL STUDIES OF A QUADRATIC RESIDUE DIFFUSER

Because of the important role that such objects would play if used in large amounts in a concert hall, preliminary laboratory studies have been conducted on prototypes to determine :

- their diffusing properties,
- their absorption coefficient

The prototype quadratic residue diffuser (fig. 1) was designed as follows :

- normal design frequency of 400Hz,
- effective frequency range 400 to 2000Hz,
- prime number  $N = 23$ ,
- cavity width : 8,5cm,
- thickness of separating walls : 0,8cm.

From these values, the depth of the cavities were obtained and the quadratic residue diffusers were built in plywood.

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### II.1 TEST OF DIFFUSING PROPERTIES

Two series of tests have been performed on 1/4 scale diffusers. The measured sample was made of four complete periods (4N). The first series of measurements has been performed in a normally reverberant industrial hall; this means that between the source, a loudspeaker, and the receiver, an omnidirectional microphone, many different sound paths are possible. Time delay spectrometry was used and the following conditions were carefully checked :

- separate the reflected sound from the direct sound,
- make sure that all the contribution of the diffuser is contained within the analysis window,
- make certain that none of the energy reflected by the room is contained within the analysis window.

The loudspeaker signal was a sine-wave with linear frequency modulation. The third-octave bands measured were those between 1600 and 8000Hz, corresponding at a 1/1 scale to 400 to 2000Hz.

The measurements were performed in the far-field to test the plane-wave assumption and compare experimental and theoretical results.

The distances between source and quadratic residue diffuser (D) and between quadratic residue diffuser and receiver (R) were respectively set at  $D = 5\text{m}$  and  $R = 2,5\text{m}$  (which corresponds at a 1/1 scale to 20 and 10m).

To verify the properties of quadratic residue diffusers, similar measurements were performed on a flat panel of identical dimensions : thus, the ability of the method to identify a specular reflection can be demonstrated.

The tests were performed for three angles of incidence :  $0^\circ$ ,  $30^\circ$  and  $60^\circ$  (larger angles are of course not accessible in a reverberant room). Only the 30 diagrams are shown here (fig. 2) since they are quite representative; they show striking agreement with the theoretical results :

- number of diffraction lobes,
- effective frequency range,
- negligible effect of angle of incidence (fig. 3).

### II.2 MEASUREMENT OF THE ABSORPTION COEFFICIENT OF QUADRATIC RESIDUE DIFFUSERS

At the design stage of a concert hall, control room or studio, it is essential to have a grasp of the absorption characteristics of quadratic residue diffusers. For this purpose a  $10\text{ m}^2$ , 1/1 scale, quadratic residue diffuser was built and tested in a  $200\text{ m}^3$  reverberation chamber, according to standard NFS 31003.

#### II.2.1 Description of prototypes and results

The first type of quadratic residue diffuser (fig. 1) is made of a wooden frame and of a 3cm thick bottom with the following dimensions :  $L = 2 \times 2,282\text{m}$   $l = 2,191\text{m}$   $h = 0,365\text{m}$ . The vertical partition walls are made of 8mm thick plywood with a 8,5cm interval. The bottom of each cavity is made of a wood brace 70mm thick.

The results show (fig. 4, curve A) that quadratic residue diffusers show excess absorption around the design frequency, namely 0,45 at 400Hz. In lower octave bands, where the diffuser acts as a quasi reflective plane, the absorption coefficient goes as high as 0,5 (at 250 and 315Hz). In higher octave bands, the absorption coefficient is greater than or equal to 0,2.

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Absorption measurements were also performed on another prototype made of a damped aluminium sandwich. The results show a similar excess absorption (fig. 5).

### 11.22 Reduction of excess absorption

An investigation of this absorption peak, following Schroeder's initial work, concludes that the phenomenon is linked to the depth of the deepest cavities. Therefore, as a pragmatic solution, the quadratic residue diffusers were modified so that these cavities (fig. 6) have a variable depth thus hoping to spread the absorption peak without deteriorating the diffusing characteristics. This test was performed on 1/1 and 1/4 scale quadratic residue diffusers. Of course, both diffusion and absorption had to be measured.

#### - 1. Diffusion of modified quadratic residue diffusers

This test was performed on modified 1/4 scale diffusers and took place in a semi-anechoic chamber to eliminate some of the problems encountered in the reverberant chamber. The TDS method was similar and comparative measurements were also performed on a reflecting plane.

Both the quadratic residue diffuser and reflecting panel were placed at 2,28m from the reflecting floor of the semi-anechoic chamber. The distances between source and sample and between sample and microphone were identical to those described above ( $D = 5m$ ,  $R = 2,5m$ ).

The experimental results are shown for a  $30^\circ$  incidence angle in figure 7. They clearly show that the diffusing properties of the quadratic residue diffusers have not been significantly altered by the modification.

#### - 2. Absorption of modified quadratic residue diffusers

The absorption coefficient has been measured on a 1/1 scale modified quadratic residue diffuser under the conditions described in section 11.2. The result is shown in figure 4, curve B and a significant decrease of the absorption coefficient is obtained :

400Hz = 0.25 or a 44% decrease  
2000Hz = 0.15 or a 25% decrease

The absorption peak is still present around 250Hz but is somewhat "damped" :

250Hz = 0.30 or a 40% reduction.

## III - CONCLUSIONS

Quadratic residue diffusers are of interest to acoustical consultants involved in the design of concert halls, theatres, studios, etc, since they provide quasi- omnidirectional diffusion and since their diffusing properties can be computed accurately, unlike those of, for instance, the "Cariatides" of the Musikverein. Therefore they have been introduced in several building projects such as the new Brest multi-purpose hall which requires good intelligibility or clarity as well as acoustic "equilibrium". It has also been planned in the new Paris concert hall which will be the one of the Ensemble Intercontemporain and Pierre Boulez since multidirectional diffusion would "depolarize" the hall and make it possible to distribute the musicians in various ways.

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Nevertheless, in the design, the absorption characteristics of quadratic residue diffusers should not be forgotten ; they should actually be measured, not only on prototypes, but also on samples of the very diffusers that will be installed in the hall. The method to be used for these measurements deserves to be improved to take into account the specific features of quadratic residue diffusers such as their thickness. Standardized absorption coefficient measurement methods were used blindly here; even though their accuracy is questionable, one may conclude that they provide reasonably reliable relative values.

The authors hope that other designers and users of quadratic residue diffusers will investigate thoroughly their diffusing and absorbing properties and confirm or disprove the existence of intrinsic excess absorption.

### IV - ACKNOWLEDGEMENTS

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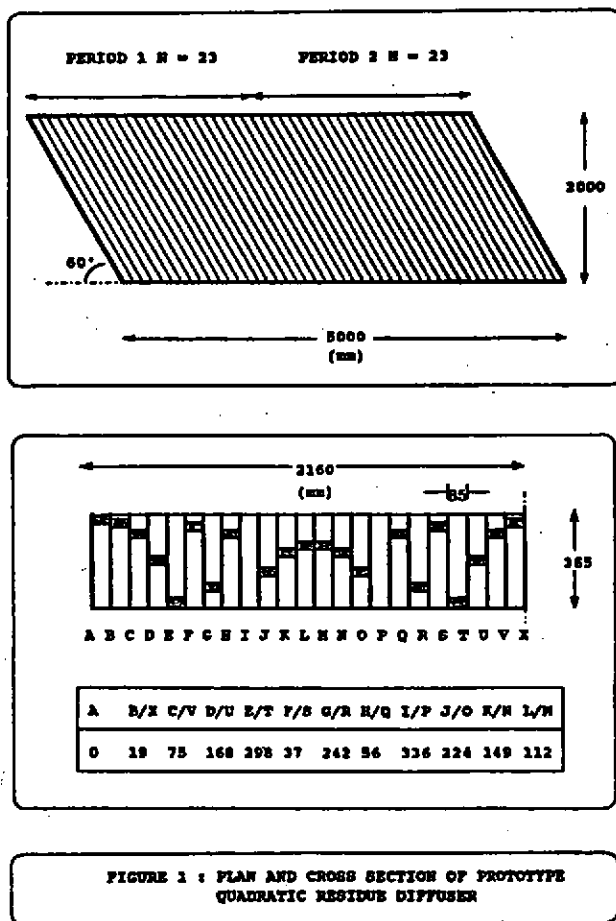
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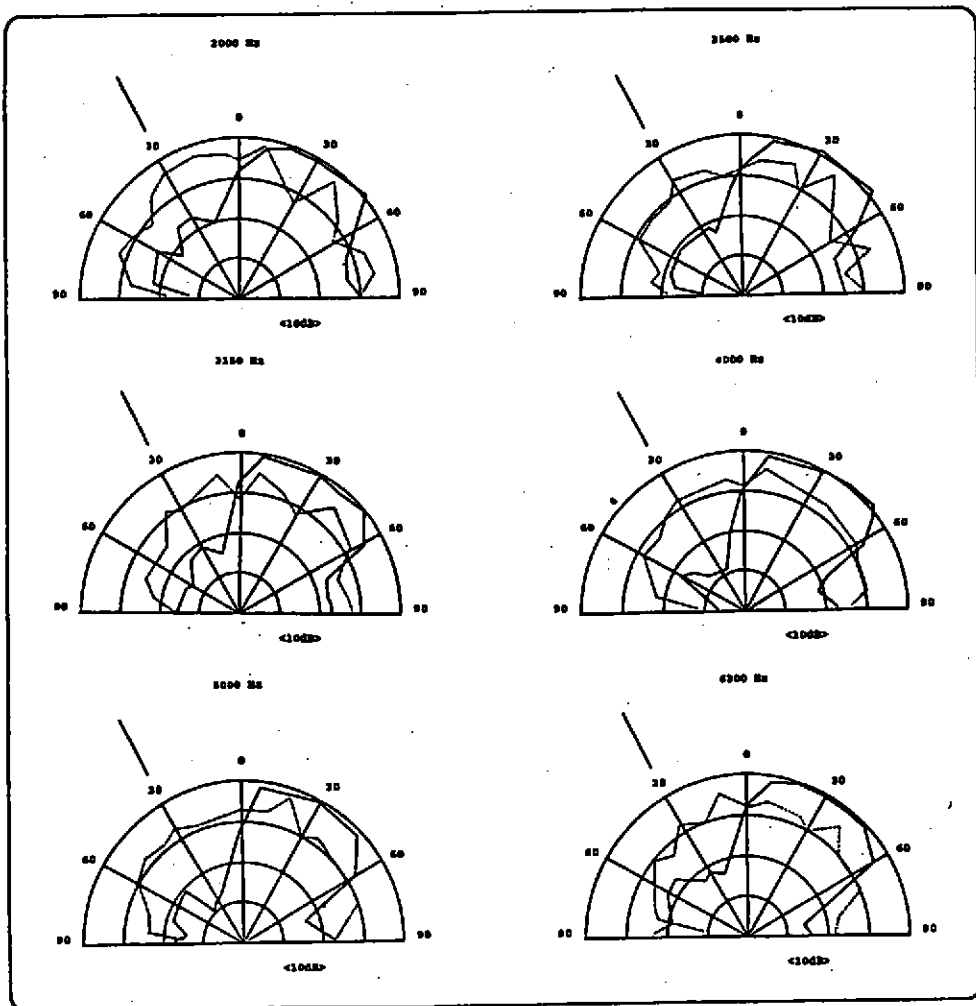


FIGURE 2 : COMPARED DIFFUSING CHARACTERISTICS OF DIFFUSERS AND OF A FLAT PANEL FOR AN INCIDENT ANGLE OF 30°

DIFFUSER

FLAT PANEL

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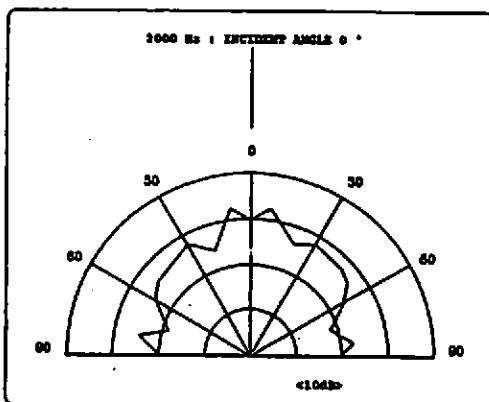
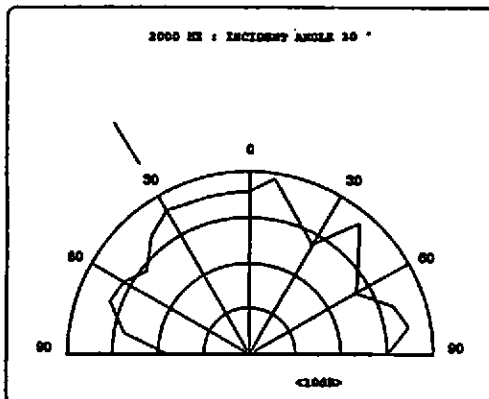
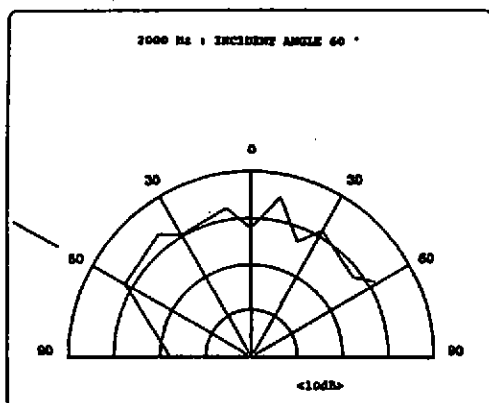


FIGURE 3 : EXPERIMENTAL DIFFUSION AT 2000 Hz FOR INCIDENT ANGLES OF 0°, 30° AND 60°

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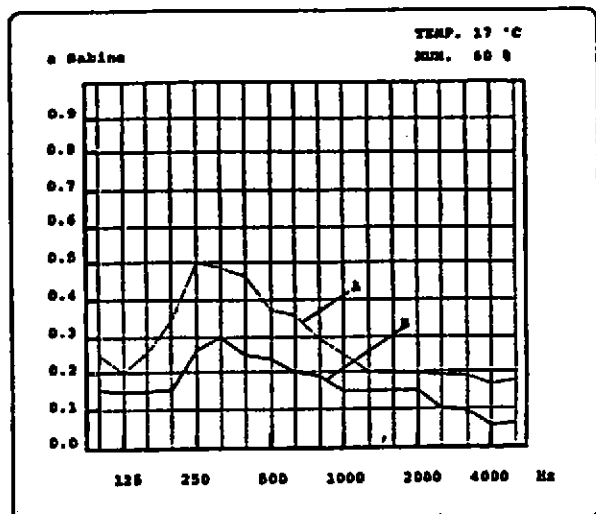


FIGURE 4 : ABSORPTION COEFFICIENT PER 1/3 OCTAVE BANDS FOR WOODEN QUADRATIC RESIDUE DIFFUSER  
A : ORIGINAL RECTANGULAR CAVITIES  
B : CAVITIES WITH VARIABLE DEPTH

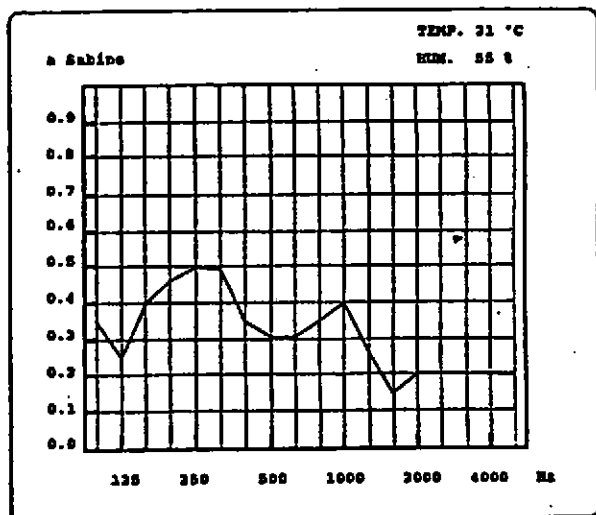


FIGURE 5 : ABSORPTION COEFFICIENT FOR ALUMINIUM SANDWICH QUADRATIC RESIDUE DIFFUSER



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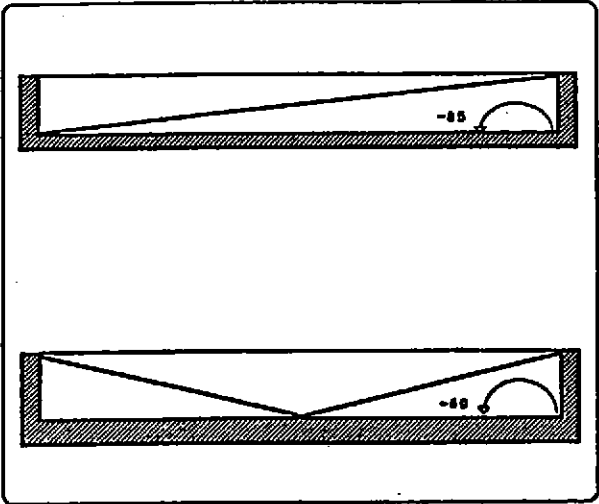


FIGURE 6 : CROSS SECTIONS OF MODIFIED CAVITIES

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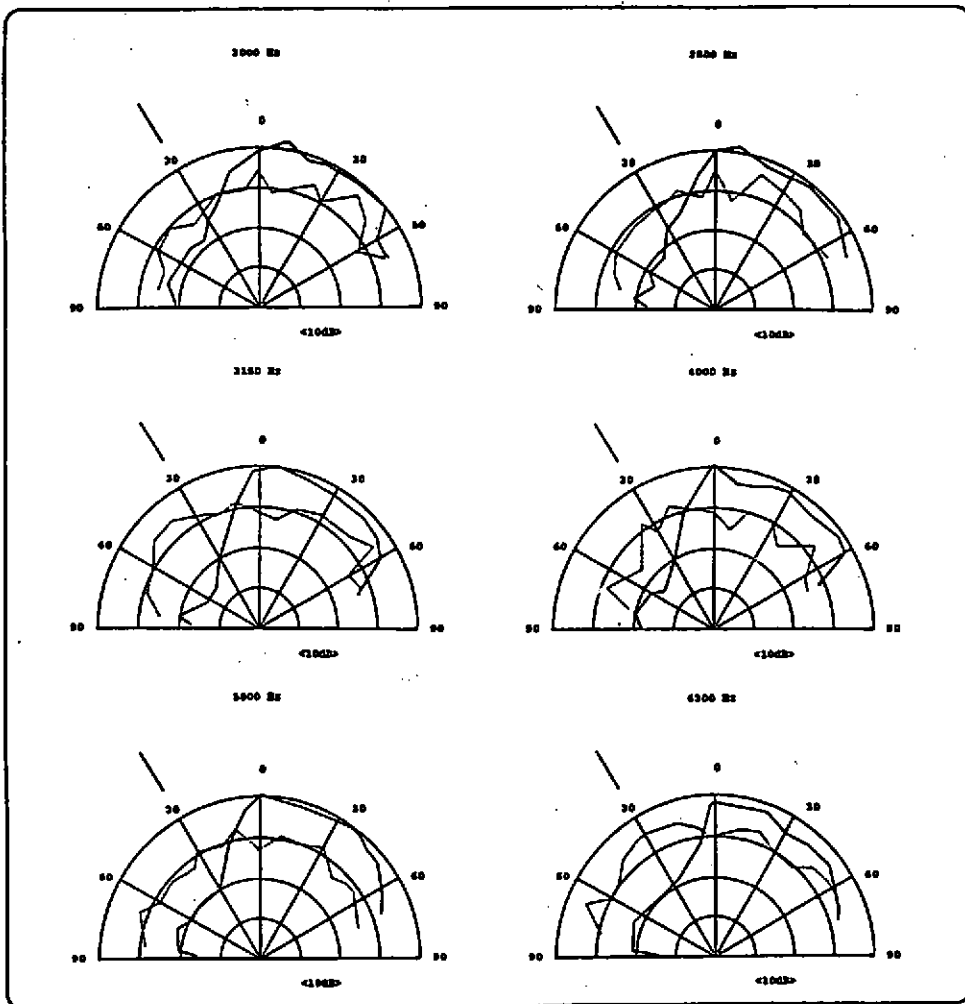


FIGURE 7 : COMPARED DIFFUSING CHARACTERISTICS OF MODIFIED DIFFUSERS AND OF A FLAT PANEL FOR AN INCIDENT ANGLE OF 30°

DIFFUSER ——— FLAT PANEL - - - - -