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DIFFUSION-LIKE GROWTH OF DECOUPLING AND ENHANCEMENT DEPTHS IN SHALLOW-WATER ACOUSTIC PROPAGATION

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ABSTRACT

The simple transmission loss formulae for acoustic propagation in constant-depth isovelocity water are recapitulated; for transducers well away from the boundaries; and covering regions with A spherical spreading, B cylindrical spreading, C stripping and D the single-mode condition. Modified formulae allowing for decoupling at the boundaries are presented, applicable for any source and receiver depths and important near both boundaries. These turn out to describe in addition the enhanced propagation when the source and receiver depths are the same. In the C stripping region the depth extents having decoupling or enhancement grow as $(\text{range})^{1/2}$ in a process that mimics diffusion.

1. INTRODUCTION

In 1971 the writer presented a theoretical description of shallow-water sound propagation (Ref 1) which had two main features. One was the attempt to provide a very simple and compact account, admittedly only approximate and only for isovelocity water, and with a very simple derivation. This was done by using where possible a flux approach, as opposed to one based on rays or modes. The other feature was the coverage of all ranges and most frequencies in a series of four connecting main regions.

Most of the relevant literature has assumed both transducers to be well away from the boundaries, but there are exceptions in Refs 1 - 3. And Denham (Ref 4) gives a full treatment for the case with one transducer near the surface, and suffering a dipole or Lloyd's Mirror loss. Here we extend the main guiding regions of Ref 1 for the case where one or both transducers may be placed in the critical depth interval near either the surface or bottom. The results should be suitable for incorporation in simple computer models, an excellent example of which is INSIGHT (Ref 5).

2. RECAPITULATION FOR TRANSDUCERS DISTANT FROM BOUNDARIES

Table 1 collects together the original formulae from Ref 1, for constant-depth isovelocity water and transducers at some undefined but fairly central depth. The level F is quoted relative to that at unit range in a free field, and the formulae allow for geometrical spreading and for boundary losses but not for bulk absorption. Figure 1 shows the boundary loss assumptions, ϕ being a general grazing angle and θ the critical angle at the bottom. The parameter H may be taken simply as the water depth, nominally between two free boundaries. But it is better to interpret H as an effective depth, enhanced by the order of one acoustic wavelength in the water because of the penetration of the sound field into the bottom. The improvement proposed in Table 1 helps to smooth the transition from region C to region D.

The different propagation regions are shown in the range-frequency plane in Figure 2, which is based on Ref 1. The close-range region A simply assumes spherical spreading. In region B the energy trapped below the critical angle spreads out cylindrically. In region C the energy at relatively steep angles is preferentially stripped away. By region D there is virtually just one attenuating mode left, for which the amplitude is already known to have a simple sinusoidal dependence on both source and receiver depths. Regions E and F need not concern us here.

3. CYLINDRICAL REGION B FOR ANY TRANSDUCER DEPTHS

The approach here has much in common with that in Ref 6 and some results are borrowed. For a unidirectional plane wave incident on a free surface the level at a distance Z_a from the surface has a correction factor $2 \sin^2 (k Z_a \sin \phi)$, where k is wavenumber $2\pi/\lambda$. The coefficient 2 is chosen so that the mean factor is unity. Since grazing angles are low $\sin \phi$ will be approximated as ϕ. If the so-called plane wave starts at a distance Z_b from another parallel free surface, at a sufficiently great total distance that the wave may be treated as plane, the combined correction factor may be written

$$G = 4 \sin^2 \phi k Z_a \sin^2 \phi k Z_b. \qquad (1)$$

When this is averaged over the angle range 0 to θ, with a flat weighting, we get

$$G_B = 1 - \frac{\sin 2\theta k Z_a}{2\theta k Z_a} - \frac{\sin 2\theta k Z_b}{2\theta k Z_b} + \frac{\sin 2\theta k (Z_b - Z_a)}{4\theta k (Z_b - Z_a)} + \frac{\sin 2\theta k (Z_b + Z_a)}{4\theta k (Z_b + Z_a)}. \qquad (2)$$

This is the necessary correction to F_B, as included in Table 2. The only special point to watch in its derivation is that one must, of course, combine the depth effects for the two transducers before averaging. Z_a and Z_b refer to source and receiver, and it does not matter which is which. Z itself refers to the depth below the sea surface <u>or</u> to the height above the displaced bottom:use whichever is the lesser.

There are several special cases concerning equation (2). In region B we normally assume many modes to be present, so that for some typical depth values all arguments will be large, ie

$$Z_a, Z_b, (Z_b - Z_a), (Z_b + Z_a) \gg (2\theta k)^{-1}, \qquad (3)$$

and all bar the first term of equation (2) can be neglected. $G_B \approx 1$ and we are back to Table 1.

If Z_b is large but Z_a is in the surface dipole region we have, as in Refs 6 and 4,

$$G_B \approx 1 - (\sin 2\theta k Z_a)/(2\theta k Z_a), \qquad (4)$$

$$\approx (2/3)(\theta k Z_a)^2 \text{ for } Z_a \text{ very small}. \qquad (5)$$

If Z_a and Z_b are both very small we have to take three terms in the expansion of the sine functions to reach

$$G_B \approx (4/5)(\theta k Z_a)^2 (\theta k Z_b)^2. \qquad (6)$$

If all arguments are large except that involving $(Z_b - Z_a)$ we have

$$G_B \approx 1 + \{\sin 2\theta k (Z_b - Z_a)\} / \{4\theta k (Z_b - Z_a)\}. \quad (7)$$

$$\approx 3/2 \text{ for } Z_b = Z_a. \quad (8)$$

We have without trying recovered the wave-theory peaking effect of Ref 6, where for source and receiver at the same depth or at complementary depths the range-averaged level is multiplied by 3/2 or 1.8 dB. Ref 6 goes on to show that for source and receiver at the special depth of midwater the multiplier becomes 2. This is not modelled here, because we have assumed a continuous distribution in angle, but the error is small.

4. STRIPPING REGION C FOR ANY TRANSDUCERS DEPTHS

In the stripping region we have to average the G correction factor from equation (1) using a Gaussian weighting, where the Gaussian has value $\exp(-\pi/4)$ at the ϕ_e effective angle of Table 1. The result is

$$G_C = 1 - \exp\left\{-\frac{Hk^2 Z_a^2}{RQ}\right\} - \exp\left\{-\frac{H k^2 Z_b^2}{RQ}\right\} + \frac{1}{2}\exp\left\{-\frac{H k^2 (Z_b - Z_a)^2}{RQ}\right\}$$

$$+ \frac{1}{2}\exp\left\{-\frac{H k^2 (Z_b + Z_a)^2}{RQ}\right\}. \quad (9)$$

In Table 2 this has been applied to F_C, together with the first-mode attenuation factor already discussed.

There are special cases as for region B.. Thus for general transducer depths with all exponentials numerically large we find again $G_C \approx 1$.

For Z_b large and Z_a in the dipole region we have, with related formulae in Refs 6 and 4,

$$G_C \approx 1 - \exp(-H k^2 Z_a^2 / RQ). \quad (10)$$

$$\approx H k^2 Z_a^2 / RQ \text{ for } Z_a \text{ very small}. \quad (11)$$

If we treat Z_b as the source depth the first version, equation (10), shows us that the dipole or decoupling region spreads inwards from either boundary as range increases, this depth extent varying as $R^{1/2}$, rather like a diffusion process (see Figure 3a). Its second version, equation (11), shows there is an extra R^{-1} dependence of level on range and overall dependence is $R^{-5/2}$, as previously predicted in Refs 1, 3 and 4.

When Z_a and Z_b are both very small we have to make a three-term expansion of the exponentials to find

$$G_C \approx 3 H^2 k^4 Z_a^2 Z_b^2/R^2 Q^2. \qquad (12)$$

The depth extent of decoupling associated with the receiver must still spread inwards as $R^{1/2}$. But now G_C provides an extra R^{-2} range dependence, with $R^{-7/2}$ for the overall dependence of level on range (Refs 1, 3 and 4).

When source and receiver depths (or depth and complementary depth) are close we have the peaking phenomenon as before (Ref 6),

$$G_C \approx 1 + \frac{1}{2} \exp\left\{-\frac{H k^2 (Z_b - Z_a)^2}{RQ}\right\}, \qquad (13)$$

$$\approx 3/2 \text{ for } Z_b = Z_a. \qquad (14)$$

The widths of the high-level regions also vary as $R^{1/2}$ (Figure 3b), producing parabolic sections in the range-depth plane. The level in both the normal and high regions varies as $R^{-3/2}$, but if we take out the azimuthal or cylindrical spreading we are left with $R^{-1/2}$, again like a diffusion process.

Figure 3c illustrates the special case of a source at half depth, when $G_C \approx 2$ for $Z_b = Z_a$. We also have the correction factor $G_C \approx 2$ for the longer ranges when source and receiver are only approximately at half depth. Thus the high region at the source depth and the ghost region at the complementary depth spread and eventually merge, as happens in Figure 3b.

It is worth commenting that, although the present range dependence resembles the time dependence in diffusion, there is no close analogy in the physical mechanisms. However one does encounter such analogies elsewhere in underwater acoustics, eg the spreading in ray angle encountered when the propagation is in a randomly inhomogeneous medium.

5. CONCLUSIONS

The main results of this paper appear in Table 2. Region C shows a curious behaviour in which the depth extent to the boundary decoupling and the depth extent of the enhancement near the source and complementary depth all grow as $(\text{range})^{1/2}$, in a process that mimics diffusion.

6. ACKNOWLEDGEMENTS

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Table 1. Levels and controlling angles in the main propagation regions, with parameters at changeovers. For both source and receiver well away from boundaries. For environment independent of range, based on Ref 1. A proposed improvement is to multiply F_C by $\exp(-R\lambda^2 Q/4H^3)$.

	Range R	**Level F**	**Controlling angle ϕ**
Region A Spherical	-	$F_A = 1/R^2$	-
Changeover AB	$R_{AB} = H/2\theta$	$F_{AB} = (2\theta/H)^2$	θ
Region B Cylindrical	-	$F_B = 2\theta/RH$	θ
Changeover BC	$R_{BC} = \pi H/4Q\theta^2$	$F_{BC} = 8Q\theta^3/\pi H^2$	θ
Region C Strip	-	$F_C = (\pi/R^3HQ)^{1/2}$	$\phi_e = (\pi H/4RQ)^{1/2}$
Changeover CD	$R_{CD} = \frac{\pi H^3}{\lambda^2 Q}$	$F_{CD} = \frac{\lambda^3 Q}{\pi H^5}\exp\left(-\frac{\pi}{4}\right)$	$\phi_1 = \frac{\lambda}{2H}$
Region D Single-mode	-	$F_D = \frac{\lambda}{RH^2}\exp -\left(\frac{R\lambda^2 Q}{4H^3}\right)$	$\phi_1 = \frac{\lambda}{2H}$

Table 2. Amended levels in the guiding regions B C and D for any source and receiver depths, environment <u>independent</u> of range. The improvement with an F_C attenuation is incorporated.

$$F_B = \frac{2\theta}{RH}\left\{1 - \frac{\sin 2\theta k Z_a}{2\theta k Z_a} - \frac{\sin 2\theta k Z_b}{2\theta k Z_b} + \frac{\sin 2\theta k (Z_b - Z_a)}{4\theta k (Z_b - Z_a)} + \frac{\sin 2\theta k (Z_b + Z_a)}{4\theta k (Z_b + Z_a)}\right\}$$

$$F_C = \left(\frac{\pi}{R^3 H Q}\right)^{1/2} \exp\left(-\frac{R\lambda^2 Q}{4H^3}\right) \left[1 - \exp\left\{-\frac{H k^2 Z_a^2}{RQ}\right\} - \exp\left\{-\frac{H k^2 Z_b^2}{RQ}\right\} + \frac{1}{2}\exp\left\{-\frac{H k^2 (Z_b - Z_a)^2}{RQ}\right\} + \frac{1}{2}\exp\left\{-\frac{H k^2 (Z_b + Z_a)^2}{RQ}\right\}\right]$$

$$F_D = \frac{\lambda}{RH^2} \exp\left(-\frac{R\lambda^2 Q}{4H^3}\right) 4\sin^2\left(\frac{\pi Z_a}{H}\right) \sin^2\left(\frac{\pi Z_b}{H}\right)^2$$

SHALLOW-WATER DECOUPLING AND ENHANCEMENT DEPTHS

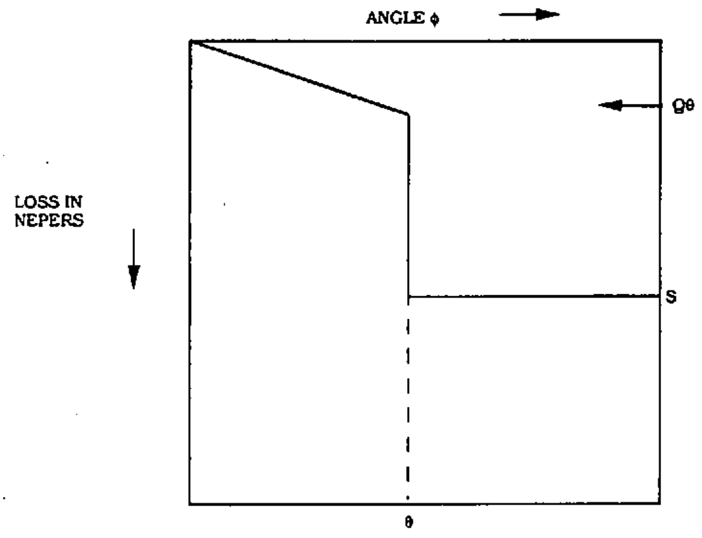

FIGURE 1. SCHEMATIC OF BOUNDARY REFLECTION LOSS ASSUMPTIONS. THE DASHED LINE PERTAINS TO REF 1

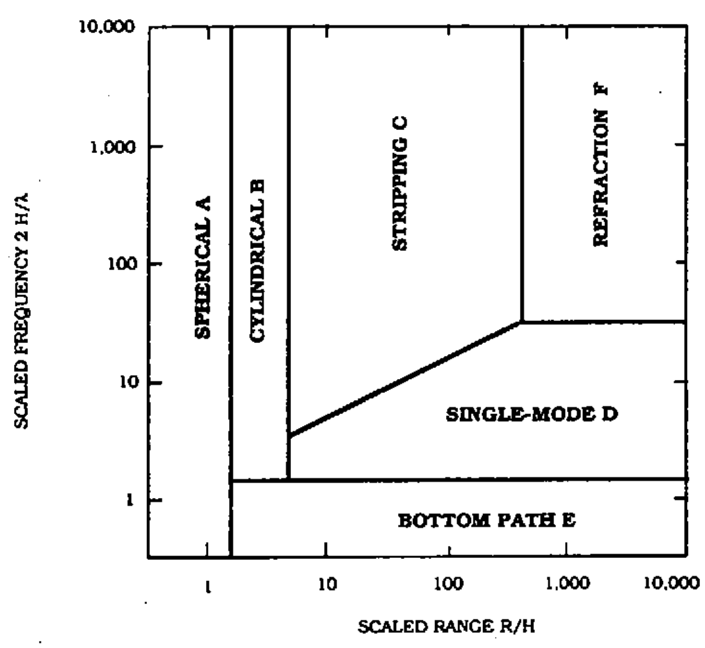

FIGURE 2. DIAGRAM SHOWING PROPAGATION REGIONS FOR CONSTANT-DEPTH WATER WITH SLIGHT LAYERING

SHALLOW-WATER DECOUPLING AND ENHANCEMENT DEPTHS

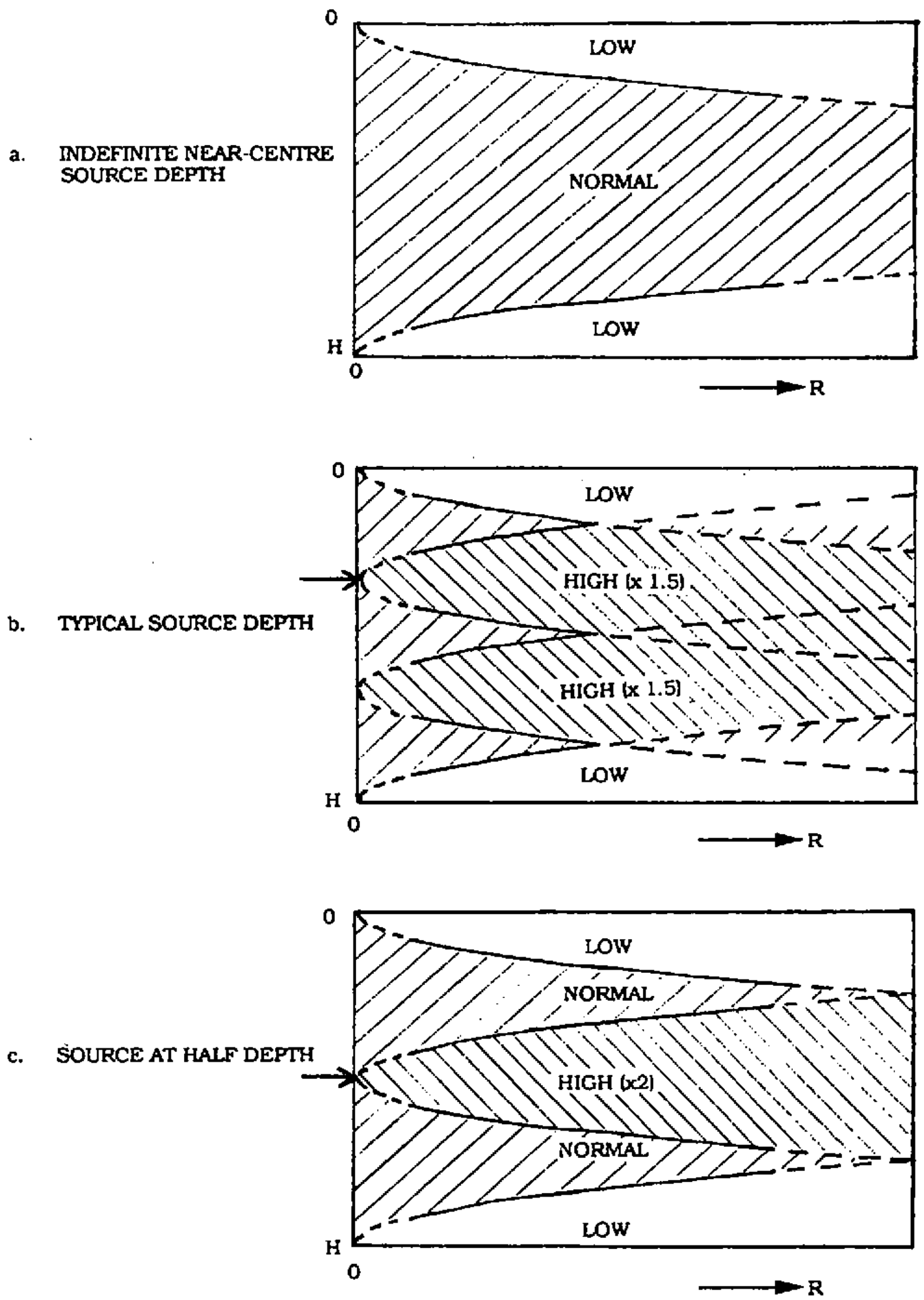

FIGURE 3. SCHEMATIC SHOWING GROWTH OF DECOUPLING (LOW) AND ENHANCEMENT (HIGH) REGIONS IN THE RANGE-DEPTH PLANE AS HIGHER MODES ARE STRIPPED AWAY.