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LIGHT DIFFRACTION BY ULTRASOUND AS EVIDENCE OF FINITE AMPLITUDE DISTORTION

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INTRODUCTION

The availability of plastic membrane electret hydrophones has greatly simplified accurate observation of ultrasound pressure waveforms and most of the evidence for finite amplitude distortion derives from hydrophone measurements. The sceptic may however postulate that waveform distortion could be due to other causes, including the presence of harmonics, hydrophone defects such as delamination, finite amplitude distortion in the hydrophone itself and the discontinuity caused by the hydrophone membrane. This paper presents some supporting evidence from a completely different source, optical diffraction. By its nature this involves negligible disturbance of the sound field and is an integral effect over a considerable volume in contrast to the normal hydrophone measurement.

A continuous wave ultrasound beam can be regarded as a phase contrast diffraction grating for a coherent light beam perpendicular to it and a schlieren beam visualisation system happens to provide the ideal optical set-up for observing the diffraction patterns under Fraunhofer (far field) conditions. It was observed that these diffraction patterns became progressively more asymmetrical as sound intensity was increased and that the degree of asymmetry directly related to the waveform distortion shown by a PVDF hydrophone. It is suggested that the diffraction asymmetry is due to differences in positive and negative axial pressure gradients in the ultrasound beam resulting from finite amplitude distortion.

EQUIPMENT AND MATERIALS

Optical System

The schlieren system has been described elsewhere [1]. It comprises the classic zig-zag system using concave mirrors, corrected for astigmatism and with a test field diameter about 140 mm. The converging mirror focal length is 1230 mm, the diffraction patterns being produced at the focal plane and observed and photographed through a transmitting diffusing screen. A one mm diameter zero-order stop was used. The light source was a 50W quartz-iodine projector lamp used with a red gelatine filter having maximum transmission about 650 nm.

Electronics and Ultrasound Systems

The transducer used for all observations was a 15 mm diameter unmounted lead zirconate titanate disc (vernitron type BMFI5F-5A) driven continuously at its resonant frequency of 2.026 MHz. A

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Marconi bi-laminar hydrophone type Y-33-7611 was used without a hydrophone amplifier but with a 50 ohm load, for which the calibration factor was 0.28×10^{-6} VPa^{-1}.

All hydrophone measurements were made nominally on the beam axis, with the hydrophone being moved slightly to maximise output at a given depth. As the hydrophone would not go into the schlieren test tank, hydrophone and schlieren measurements had to be made separately. The medium was ordinary tap water left standing at least overnight, but a check against distilled water showed no significant differences. Hydrophone output and transducer drive voltage were monitored with a Hewlett-Packard 1715A 200 MHz dual channel oscilloscope.

OBSERVATIONS

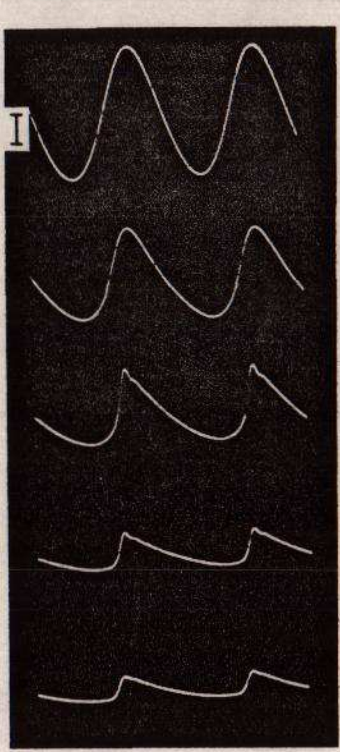

Fig 1. The marker indicates 70 KPa approx.

Figure 1 is included to illustrate the type of distortion involved in this experiment. In this instance the transducer drive was constant and the waveforms are the hydrophone output at increasing distances from the transducer; from the top down at 4, 12, 24, 48 and 65 cms respectively. This progressive distortion could be explained on the basis of the bulk modulus increasing with pressure, so that high pressure regions travel faster and try to overtake lower pressure regions. (a small hydrophone resonance at about 25 mHz is apparent in the lower traces of fig 1).

With the hydrophone position fixed and the transducer drive increased from a low level, distortion appeared in a similar sequence to fig 1 as expected, but of course with accompanying amplitude increase. At less than about 8 cm distance the more extreme distortions of fig 1 could not be reached, due to lack of sufficient drive power and also transducer overheating.

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Figure 2 shows a selection of photographs of the diffraction pattern in the schlieren system as transducer drive is increased, with the whole volume of the test tank illuminated. (The peak pressures indicated are only estimates of the values at about the mid point of the 14 cm ultrasound beam length).

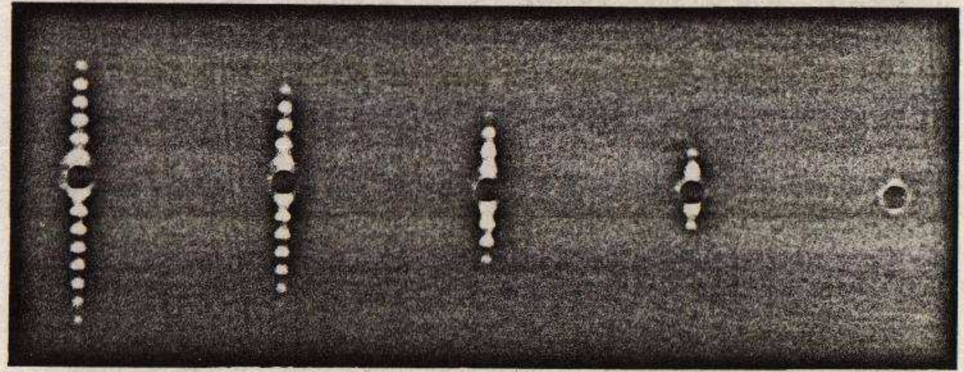

Fig 2. Schlieren system diffraction patterns for peak pressures of 300, 200, 125, 42 and 0 Pa approximately. The spot spacing of the patterns is actually 1.07 mm.

The spacing of the spots of the patterns agrees with ordinary diffraction grating theory, but the envelope function does not since the sound beam modulates only the phase of the light, due to refractive index variations, but not the intensity. It will be noticed also that the patterns become distinctly asymmetrical as sound intensity increases.

To investigate this further and to try to tie it up with the hydrophone measurements, the diffraction patterns were observed with the light beam restricted in area so that only 4 cm of the sound beam length was sampled, the centre of the sampled region being placed in turn at 4 and 12 cm from the transducer. Figure 3 shows typical results at 24 V RMs drive represented by the larger amplitude waveform, the maximum drive available being 28 V RMs.

The coma-like flare in the left hand pattern appeared to be due to an optical defect in this particular area.

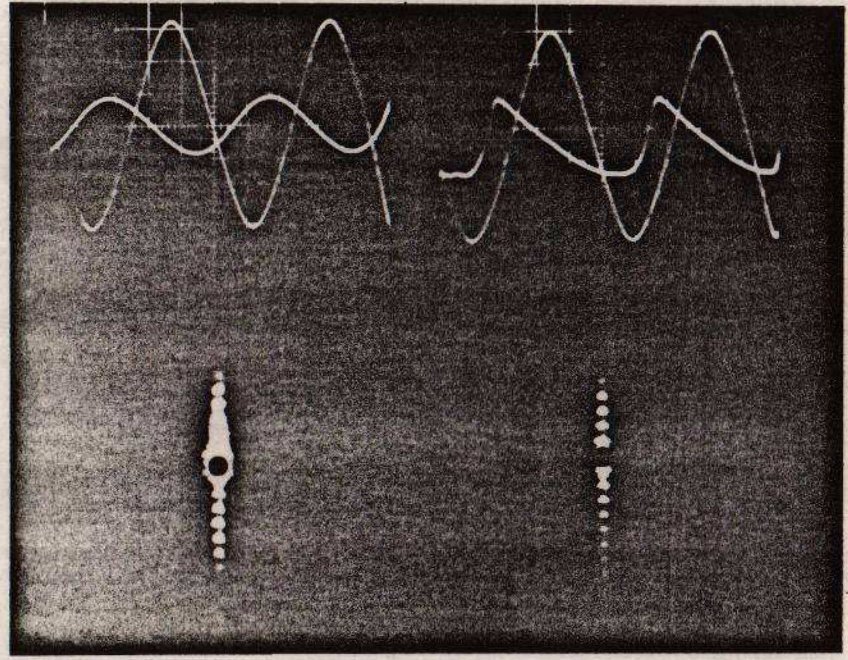

Fig 3. Comparison of hydrophone waveforms and diffraction patterns. Hydrophone output is smaller waveform.
Left At 4 cms. Hydrophone calibration 360 K Pa per division.
Right At 12 cms. Hydrophone calibration 180 K Pa per division.

RESULTS AND THEORY

Results

There was always good visual agreement between the degree of diffraction asymmetry and hydrophone waveform distortion. To quantify this the extent of a diffraction pattern each side of the central stop was estimated by counting the number of spots clearly visible and multiplying by the known spot spacing in mm. The ratio of these two numbers was taken. The positive and negative slopes of each corresponding waveform were also measured for the best visual straight line fit over 50% of the peak to peak amplitude and the ratio again taken. The results are plotted in Fig 4a and follow approximately a square law.

Plots were also made to relate the extent of the diffraction pattern to sound pressure or gradient. For sinusoidal waveforms the two are of course related by a constant factor, but including the distorted waveforms it was more fruitful to use pressure gradient (for example in fig 3 the peak to peak pressure of the right hand waveform is 0.7 of that of the left whereas the diffraction patterns are about equal overall). Fig 4b shows this plot with the diffraction amplitude converted to radians. The knee in the curve roughly corresponds with moderate waveform distortion being visible. Below this the slopes were averaged but above plotted as two points separately.

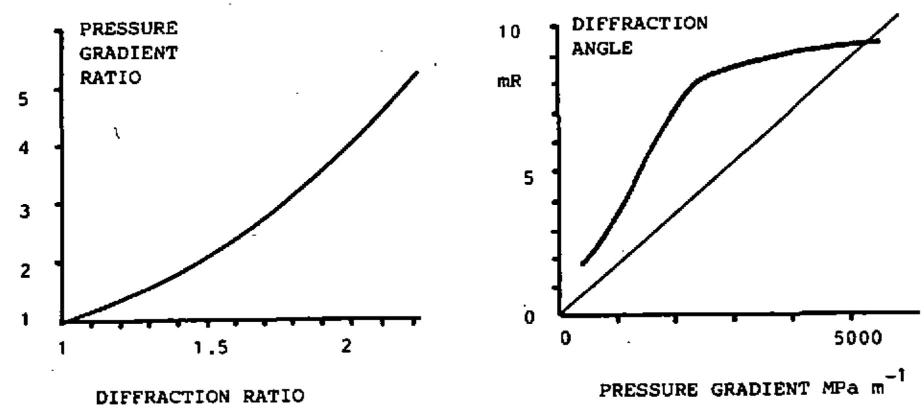

Fig 4a (Left) Hydrophone pressure gradient ratio versus diffraction amplitude ratio

b (Right) Diffraction amplitude versus pressure gradient.

Theory

A full analysis of the light-sound interaction must be complex. For a first attempt an expression was derived relating angle of refraction to pressure gradient using simple geometrical principles, on the basis that this would indicate the maximum angle of light contributing to the diffraction pattern.

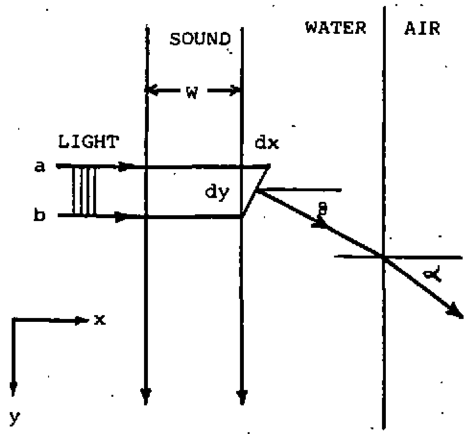

Figure 5 represents a section through a uniform parallel sound beam, width w and wavelength λ_{acc} travelling in the y direction. A light pencil width dy ($<< \lambda_{acc}$) crosses the sound beam and is refracted by a refractive index gradient $\frac{dn}{dy}$ within it.

To simplify calculation the rays a, b are considered to be undeflected within the sound beam but b suffers a retardation dx relative to a. Applying Huygens construction, for small angles we obtain:

$$\theta = \frac{dx}{dy} = \frac{w}{n}\frac{dn}{dy} \tag{1}$$

Refractive index n and density ρ are related by the Lorenz-Lorentz law

$$(n^2 - 1) = (n^2 + 2)\,\rho k$$

where k is a constant [2]. Differentiation and elimination of k yields:-

$$dn = \frac{(n^2 - 1)\,(n^2 + 2)\,d\rho}{6\,n\rho} \tag{2}$$

Density changes are related to pressure changes by the bulk modulus k:-

$$d\rho = \frac{\rho}{k}\,dP \tag{3}$$

Substituting from (3) and (2) into (1) gives

$$\theta = \frac{w\,(n^2 - 1)\,(n^2 + 2)\,\frac{dP}{dy}}{6\,n^2\,k} \tag{4}$$

Finally, multiplying by n to allow for refraction at the tank-air interface,

$$\alpha = \frac{w\,(n^2 - 1)\,(n^2 + 2)}{6\,n\,k}\,\frac{dP}{dy} \tag{5}$$

The -6dB beam width was about 11 mm and putting this value for w and $k = 2.2 \times 10^9$ Pa m^{-1} gave the result shown by the additional thin line in fig 4b. Theory at first underestimates the refraction and does not predict the limiting effect. In an attempt to improve this, diffraction grating theory as expounded in classical optics textbooks was applied to the concepts of fig 5. Some rather unpleasant integrals resulted but it was possible to extract the maximum deviation from the envelope function: this was almost the same as eq 4 with a different numerical factor making the error worse.

DISCUSSION

There is no doubt that the spread of the diffraction pattern depends on some function of sound pressure or intensity, and from the evidence presented here pressure gradient is the likely significant physical quantity. Can we then be sure that the asymmetries seen in the patterns really represent distortions in the sound beam? Two other possibilities were considered, the first being an optical defect. Firing the transducer upwards with an absorber at the water surface confirmed that the asymmetry reversed, exonerating the system. Secondly, perhaps curvature of the sound wave fronts was responsible, a possible factor because the nominal near point ($D^2/4\lambda$) of the transducer was at 7.5 cm and so most of our observations were in the far field in a divergent beam. However, a check in the convergent beam of a focussed transducer did not reverse the asymmetry.

As regards theory, a limiting effect might be expected if light in its traverse of the sound beam is displaced by an amount comparable with an acoustic quarter wave length; the deviations measured might just be significant in this respect. Obviously a more rigorous treatment is desirable.

CONCLUSIONS

(1) The overall spread of the diffraction pattern observed at the stop or knife edge of a schlieren beam visualising system increased with increasing pressure gradient in the sound beam, initially fairly linearly but then levelling off.

(2) Asymmetries observed in the diffraction pattern were the result of ultrasonic pressure waveforms with unequal positive and negative pressure gradients.

(3) The pattern asymmetry increased as the sampled volume of sound beam was moved away from the transducer, correlating with distortion measured with a hydrophone.

(4) The diffraction asymmetry offers very good independent evidence for finite amplitude distortion building up in a 2

MHz continuous wave ultrasound beam with increasing distance from the transducer, for initial pressure gradients above about 1500-2000 MPa m^{-1}.

(5) There could be the possibility of the diffraction pattern spread providing a measure of ultrasound intensity.

REFERENCES

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[2] R S Longhurst. "Geometrical and Physical Optics" 3rd Edn 1973, Longmans, p 506.