VIBRATION OF THIN-WALLED LIQUID-FILLED PIPES

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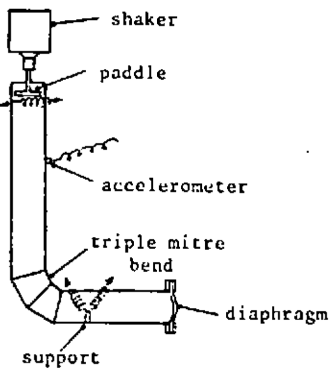
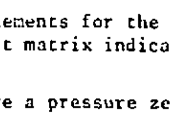




INTRODUCTION An earlier paper, Wilkinson (1978) gave a transfer matrix theory for predicting vibrations of thin-walled liquid-filled pipe systems and some comparisons with experiment on acoustic mode shapes for an L-pipe. This paper describes some further experimental and theoretical work on the L-pipe model.

EXPERIMENT The L-pipe is sketched on the right. It was about 1 m long, 70 mm diameter, 0.9 mm wall thickness stainless steel. It was driven acoustically by a paddle and shaker dipping in the water. A rubber diaphragm at the bottom retained the water and provided a boundary condition.

To get consistent repeatable results several features had to be developed. To support the pipe while permitting it to vibrate various arrangements of polystyrene blocks, foam rubber etc. were tried but stiffness and damping were not small and could vary with time, water spillage, etc. A spring support system was developed having equal stiffness in all 3 directions and 3 rotations low enough to give a vertical resonance of 4Hz and adequate stability. Calculation of the transfer matrix elements for the support as a constraint gave less than 1% deviation from a unit matrix indicating negligible support effects.

The rubber diaphragm was initially thought to give a pressure zero but it was found that its stiffness was not small enough to neglect and a measurement was attempted. Statically a dial gauge was used to give deflection as a function of pressure, varying water height in the vertical pipe. However, comparisons with theory indicated a dynamic value of about 4½ times as much. A dynamic measurement was attempted with an air filled pipe and a shaker powered piston pushing on the centre of the diaphragm. For a known piston mass resonant frequency and peakwidth imply diaphragm stiffness and damping. However, these are also functions of loading and displacement which is basically conical in the piston tests as against spherical under water loading. Analysis of the results and conversion to test values is still in progress. Stiffness problems were removed at one stage by using a rigid tufnol diaphragm but acoustic damping was then so low that peak amplitude could not be measured reliably. The rubber diaphragm was re-fitted but it was also noted that its damping varied with temperature. In order to get reliable peak amplitudes it was necessary to keep the temperature fairly constant.

Another reason for temperature control was to stop air bubbles forming in the water. For saturated water a 1°C temperature rise causes 1.6 mℓ of air to come out of solution on a 3.8 ℓ model. This represents 3000 1 mm diameter bubbles with a total non-dimensional compliance $\Delta u/p = -i6.6$ which has a major effect on results. It is difficult to fill the rig without introducing bubbles and temperature cycling will cause them to appear. In a small rig tilting and shaking can remove most of them. In a larger rig, particularly with horizontal pipes periodic vacuuming down is probably the answer and we are developing this.

Some tests were run without water in the pipe to avoid fluid-structure effects and give the bend flexibility factor from the first structural mode frequency. Excitation by the same paddle proved entirely adequate.

Measurements included hydrophone and accelerometer traverses. Acoustic mode shapes proved difficult to measure because of the need to keep the hydrophone at the centre of the tube as it traversed along the length on a wire. If off-centre it would pick up not only the desired $n = 0$, plane wave, component but also various amounts of the unknown $n = 1, 2, 3$ etc. internal sound field components. Many attempts were made to hold the hydrophone at the centre by means of spiders but all, however delicate they seemed, changed the frequency and amplitude of the signal relative to a free hydrophone, so no valid acoustic mode shapes were obtained.

A single small B&K accelerometer was moved from point to point to find the mechanical mode shapes. At each point an HP real time analyser, using the 'periodic' signal to drive the shaker, was tuned to each main peak in turn to find amplitude and phase. At each axial station 8 or 16 equally spaced circumferential points were measured and the results Fourier analysed to find the $n = 0, 1, 2 \ldots$ component amplitudes and phases. A plot of these components against axial distance then gave the mode shape for each component at that frequency. Although the modes could be mainly said to be of $n = 1$ or 2 or 3 types there were always components of the others present as well.

THEORY This was substantially as described in the earlier paper. Only 8 x 8 T.M.s were needed for this in-plane case. This would nominally require 16 equations to be solved but these can be reduced to 8 as follows. The T.M. relating vibration state vectors [y] at the top, 1, and bottom, M, of the pipe represents 8 equations.

$$\underset{8\times1}{[y]_M} = \underset{8\times8}{[A]_{1M}}\underset{8\times1}{[y]_1} \tag{1}$$

The four boundary conditions at each end give another 8 equations.

$$\underset{4\times8}{[BC]_1}\underset{8\times1}{[y]_1} = \underset{4\times1}{[RHS]_1} \tag{2}$$

$$\underset{4\times8}{[BC]_M}\underset{8\times1}{[y]_M} = \underset{4\times1}{[RHS]_M} \tag{3}$$

Eliminating $[y]_M$ between equations (1) and (3) gives

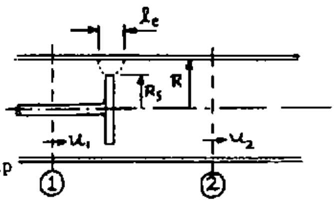
$$\underset{4\times8}{[BC]_M}\underset{8\times8}{[A]_{1M}}\underset{8\times1}{[y]_1} = \underset{4\times1}{[RHS]_M} \tag{4}$$

Then equations (2) and (4) can be combined to give 8 equations for $[y]_1$ which may be solved instead of the original 16 from equations (1), (2) and (3). This reduction can always be used to halve the number of equations to be solved since any two points on a pipe without a junction between them will always have a relation of the form (1) between them so one of the vectors can be eliminated. While the saving is trivial in this case it becomes more significant for larger numbers of equations in more complex systems. It was also more convenient to incorporate the RHS into the T.M. making it 9x9 or 15x15 in order to keep down to 8 or 14 equations for a pipe with an internal source.

All measured accelerations were expressed as a Transfer Function using shaker acceleration as a reference. In the theory a corresponding acceleration source can be defined which has a source, or RHS, term and an imaginary source impedance which affects resonant frequencies.

$$p_2 = p_1 - \frac{i\omega\ell e}{c}\,\frac{u_1}{1-R_s^2/R^2} + \frac{\ell e}{c}\,\frac{as}{R^2/R_s^2-1}$$

$$u_2 = u_1$$



where as = shaker acceleration xρc
ℓe = effective length of fluid in the gap
u = ρcu' and u' = particle velocity

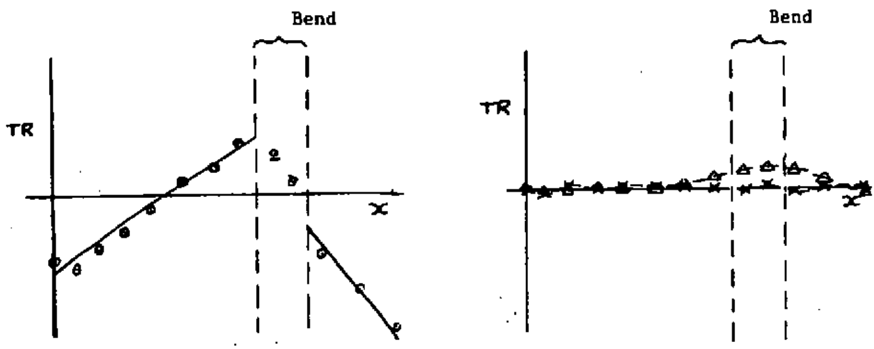
The T.M. theory accounts only for n = 0 and 1 modes. It uses static bend flexibility factors and may be expected to be inaccurate above the n = 2, 3 etc. resonant frequencies which may produce an unknown dynamic bend flexibility. For this pipe these frequencies from Bentley and Firth (1974) for a water-filled pipe are

u	2	3	4	5
f_{res}(HZ)	296	915	1879	3199

RESULTS Experimental frequencies with the main mode type were

f(Hz)	55	275	310	368	460	542	615	724	774	914	938	992
n	1	2	2	1	2	2	1&Acous.	2	2	2	2	3

For the three 'n = 1' modes the mode shapes were measured and compared with theory.



55 Hz

	n
Experiment ⊖	1
Δ	2
x	3

Theory ———

EXPERIMENTAL AND THEORETICAL MODE SHAPES

Good agreement was found at 55 and 368 Hz for n = 1 with higher order, n = 2, 3 components limited to the bend region. Poor agreement was found at 615 Hz for the n = 1 mode with n = 2, 3 components large away from the bend. An air filled pipe test gave 93 Hz for the first mechanical mode confirming a static bend flexibility about 65% above the reference book value for a multi-mitre bend. Theory predicted the acoustic resonance at 615 Hz even though the mechanical mode shape was wrong.

It was concluded that the T.M. method worked well at lower frequencies but was unreliable much above the n = 2 ovalling frequency for mechanical resonances with some suggestion that acoustic predictions might hold to higher frequencies.

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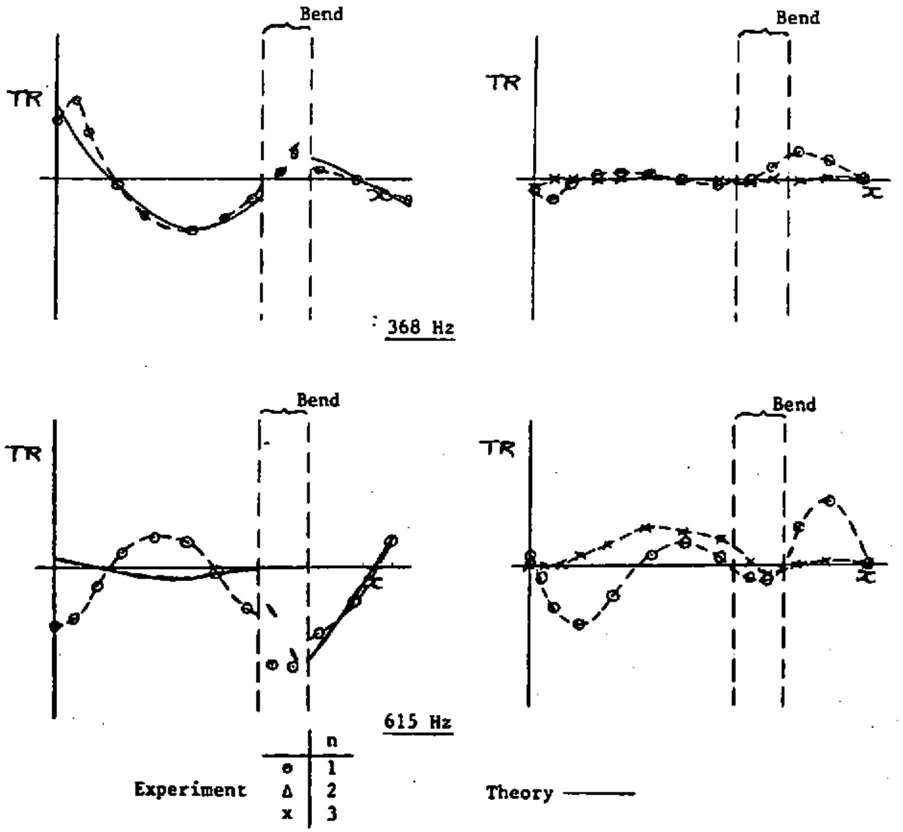

EXPERIMENTAL AND THEORETICAL MODE SHAPES