Proceedings of The Institute of Acoustics

THE NON-LINEAR ACOUSTIC PROPERTIES OF FLAMES

D J Adams,

Scientific Services Department
South Eastern Region,
Central Electricity Generating Board.

INTRODUCTION

A satisfactory description of the problem of combustion driven oscillations in industrial furnaces can, in many cases be obtained by treating the flame as a linear amplifier of acoustic pressure. Systems having a well defined, low loss acoustic resonance appear to respond well to this type of treatment. There may be cases, however, where such a treatment cannot wholly account for observed properties and where the problem proves resistant to solutions based upon such a theory. Plant which is characterised by having exceptionally high throughput burners, overcapacity draught fans and high values of heat flux within the furnace may fall into this category. Such conditions might generally be expected to lead to significant departures from a linear regime and observations made on one such plant have led to an interest in the potential non-linear behaviour of flames to the passage of acoustic waves.

This paper outlines the development of a simple non-linear model and describes the results of some initial experiments to confirm the existence of such effects.

The possible importance to combustion driven oscillations in industrial plant is discussed briefly.

DEVELOPMENT OF THE MODEL

When a periodic acoustic wave passes through a flame it causes a fluctuation in the rate of heat release at points down the length of the flame. This perturbation in the flame can be thought of in terms of two distinct processes which affect the passage of the pressure and heat waves through the flame.

Firstly the change in rate of heat release due to the incident pressure wave must also be accompanied by a change in temperature of the medium through which the wave passes. The wave number K for the pressure wave is then no longer constant in time, but undergoes a periodic variation. The phase of the pressure wave arriving at a point X along the flame is then also no longer fixed in time with respect to the inlet conditions, but is subject to modulation due to the change in temperature.

Secondly the rate of heat release from the surface of the flame at some point X and time t is dependent on the mixture quality arriving at that point in space and time. This is largely governed by the flow conditions existing at the inlet to the burner flame and on the transport lag down the flame. Since the effective transport velocity may be small, the phase lag down the flame for the heat release wave may be significant and will be a function of both the steady state flow velocity into the flame and the acoustic velocity. Thus the phase of the heat release wave at some point X down the flame is no longer constant in time but is also subject to modulation due to the acoustic flow.

Both processes therefore introduce phase modulation and effectively lead to a highly non-linear system response. A mathematical outline of these processes is given in Appendix A. Results of this analysis may be summarised as follows:-

The acoustic pressure wave at point X is determined by:-

$$P(x,t) = P_o(w) \sum_q \sum_n J_q(m_m) \sin (Y-gZ_n) + (-1)^q \sin(Y+qZ_n)$$

where Y = wt + phase constant

Z_n = $w_n t$ + phase constant

where the depth of modulation is determined by the temperature perturbation ϕ_n at frequency w_n

$$m_n \alpha \phi_n$$

The heat release wave at point X is determined by:-

$$h(x,t) = \eta_{(x)} \sum_q \sum_m g_m J_q(m_m) \sin (Y'-qZ'_m) + (-1)^q \sin(Y'+qZ'_m)$$

where Y' = wt + phase constant

Z' = $w_m t$ + phase constant

where the depth of modulation is determined by the velocity perturbation g_m at frequency w_m

$$m_m \alpha g_m$$

In practice of course both processes occur and interact simultaneously. Thus if we considered a system with 2 acoustic sources at frequencies w_1 and w_2 then we would expect to produce heat (and pressure) perturbations at frequencies w_1; w_2; $w_1 - w_2$; $w_1 + w_2$; $w_1 - 2w_2$ etc.

EXPERIMENTAL RESULTS

Some experimental work has been carried out on two small gas rigs to investigate this model.

The initial work was carried out at the Polytechnic of the South Bank on a small pre-mixed gas rig. The limited objectives of these tests were:-

a) To establish whether or not heat release perturbations could be observed at the first difference frequency $w_1 - w_2$ using 2 acoustic sources w_1 and w_2.

b) To establish whether or not heat perturbations at the first difference frequency could excite an acoustic response from a furnace.

THE NON-LINEAR ACOUSTIC PROPERTIES OF FLAMES

In these tests a range of source frequencies w_1 and w_2 was used and in all cases as w_1 approached w_2 variations in flame brightness were clearly visible at the difference frequency, the effect being most pronounced towards the tip of the flame. A tube approximately 1m long was then placed over the flame to act as a resonant furnace. The source frequencies w_1 and w_2 were then adjusted such that the difference frequency corresponded to the natural resonance of the tube (~ 150 Hz). A strong acoustic response was then obtained, which decreased as either w_1 or w_2 was varied or as the signal level of either source was reduced. This test was repeated with similar results for a range of values of w_1 and w_2.

Two points are also worthy of note. In all cases an intense acoustic output was obtained when both sources were present in the correct frequency relation, but the individual sources were themselves inaudible. Secondly the bandwidth of this process appears to be surprisingly wide; in one test source frequencies of greater than 300 Hz and 450 Hz were used to produce a strong response [1].

A second series of tests have been carried out on a small CEGB gas rig, aimed at determining the approximate strength of the heat wave modulation index and to check that the observed modulation effect was due to the flame and not due to non-linearities in the acoustic sources. An outline of the system is given in Fig. 1.

Measurements of the coherent transfer function between flame brightness and inlet acoustic pressure were obtained using a single acoustic source over a range of frequencies (Fig. 2). This was then repeated for the same flame conditions using two acoustic sources and obtaining the transfer function for the first difference frequency (Fig. 2). The transfer functions for the two tests would be similar if the observed modulation in flame brightness was due to non-linearities in the sources. They in fact differ by approximately an order of magnitude, indicating a strong non-linear effect within the flame itself and not in the acoustic sources.
The strength of the modulation process may be appreciated from a typical frequency spectrum plot of flame brightness (Fig. 3), which has a classical form.

DISCUSSION

It is clear the even small flames exhibit a substantial non-linear characteristic over a wide range of frequencies. The above results indicate that a flame has the ability to act as a (phase) modulator to the passage of acoustic waves and thereby to transfer pressure energy from external sources at frequencies w_1 and w_2 into heat and pressure energy at the modulation products $w_1 \pm n\, w_2$. Under certain circumstances this preferential infeed of energy may result in a build up of pressure energy in a system which would otherwise remain stable. The possible importance of these findings to problems in large scale plant is therefore of some interest.
Heat energy will be transferred into modulation products if two (or more) acoustic sources are present. A build up of acoustic energy inside a furnace system may occur if [2]:-

$$\int_{flame} \dot{\underline{H}}\ (jw,\ x)^{*}\ \underline{P}_f\ (jw)\ dx > \frac{w}{Q}$$

where the nett heat release over the flame, in-phase with furnace pressure P_f, is itself a strong function of such factors as mixture velocity at the base of the flame, flame heat release profile etc.

Combustion driven oscillations may therefore occur in systems which are intrinsically highly tuned (high Q), being triggered by noise within the system under the appropriate burner operating conditions and leading to high intensity standing waves in the system. The above analysis and results indicate, however, that oscillations may also occur in much more lossy systems (low Q) given the presence of two (or more) acoustic sources at frequencies outside the response band of the furnace, but having a modulation frequency within that band. In these circumstances attempts to "de-tune" the burner air supply side, for example, may have little effect on the ability of the plant to generate high levels of acoustic pressure and vibration of its structure since the oscillation is primarily sustained, not by a standing wave through the furnace/windbox system but, by travelling waves coming from the external sources. Elimination of the acoustic sources might therefore be a more appropriate solution in these cases.

ACKNOWLEDGEMENTS

This paper is published with the permission of the Central Electricity Generating Board.

REFERENCES

[1] Roberts J.P, Amplification of an Acoustic Signal by a Laminar, premixed, gaseous flame. Combustion and Flame 33, 79-83 (1978).

[2] Rayleigh J. The Theory of Sound, Vol.2, Dover, New York 45.

[3] Morse P M, Ingard K V, Theoretical Acoustics, McGraw Hill, 68.

[4] Woods, Advanced Calculus.

APPENDIX A

Consider a notional cylindrical volume enclosing the flame surface, its internal oil/air mixture and the surrounding air in close proximity to the flame surface. At each plane x down the axis of the cylinder we can ascribe a mean temperature of the medium under steady state conditions:-

$$T^{o}(X) = \frac{1}{2\pi R} \int_{0}^{R} \int_{0}^{2\pi} T^{o}(r,\Theta)\ dr\ d\Theta \Big|_{X} \qquad (A1)$$

A pressure wave entering the system at the burner quarl can be described by:-

$$p(X,t) = p_o(w)\ \sin\ (\bar{k}_{(X)}X - wt + \varnothing_w) \qquad (A2)$$

where:-

$$\bar{k}_{(X)} = w/\bar{C}_{(X)}$$

and

$$\bar{C}_{(X)} = \frac{1}{X} \int_o^X C(x,\ t-\tau)\ dx \qquad \text{(A3)}$$

with $\tau = \frac{X-x}{\bar{C}^o}$

Now the effect of a change in the temperature of the medium on the phase velocity [3] is given by:

$$C = C^o \left(1 + \frac{\theta}{2T^o}\right) \qquad \text{(A4)}$$

thus:-

$$\bar{k}_{(X)} = \frac{w}{\bar{C}^o_{(X)}} \left[1 - \frac{1}{2\ \bar{C}^o_{(X)}\ X} \int_o^X \frac{C^o(x)}{T^o_{(x)}}\ \theta(x,t-\tau)\ dx \right] \qquad \text{(A5)}$$

Consider the integral term:-

$$I = \frac{1}{X} \int_o^X \frac{C^o(x)}{T^o_{(x)}}\ \theta(x,t-\tau)\ dx] \qquad \text{(A6)}$$

Now since:

$$\left(\frac{C^o(x)}{C_a}\right) = \left(\frac{T^o(x)}{T_a}\right)^{\frac{1}{2}}$$

and:-

$$T^o(x) = T_a \left[1 + \alpha x\right]$$

Then it follows that:-

$$\frac{C^o(x)}{T^o_{(x)}} = \frac{C_a}{T_a} \frac{1}{(1+\alpha x)^{\frac{1}{2}}} \qquad \text{(A7)}$$

A cyclic change in heat release will produce a temperature perturbation in the medium:

$$\theta_{(x,t)} = \alpha X \sum_n \phi_n \sin w_n t \qquad \text{(A8)}$$

Thus we have:

$$I = \alpha \frac{C_a}{T_a} \sum_n \phi_n \int_o^X \frac{1}{(1+\alpha x)^{\frac{1}{2}}} \sin w_n\ (t-\tau)\ dx \qquad \text{(A9)}$$

This can be reduced to the standard form of Fresnel Integrals by a series of substitutions to give:-

$$I = \sqrt{2\pi}\,\frac{C_a\ \alpha X\ \Sigma\psi_n}{T_a\ \bar{C}^o\ \tau_1\beta^{\frac{1}{2}}}\left[\sin A\left\{C\{\beta_1^{\frac{1}{2}}\} - C\{\beta^{\frac{1}{2}}\}\right\} + \cos A\left\{S\{\beta_1^{\frac{1}{2}}\} - S\{\beta^{\frac{1}{2}}\}\right\}\right] \quad (A10)$$

where $\beta_1 = \beta\,(1+\alpha\bar{C}^o\,\tau_1)$

$$\beta = \frac{w_n}{\alpha\bar{C}^o}$$

$$\tau_1 = \frac{X}{\bar{C}^o}$$

and the Fresnel Integrals are:-

$$S\{x\} = \frac{1}{\sqrt{2\pi}}\int_0^{x^2}\frac{\sin Z}{Z^{\frac{1}{2}}}\,dZ$$

$$C\{x\} = \frac{1}{\sqrt{2\pi}}\int_0^{x^2}\frac{\cos Z}{Z^{\frac{1}{2}}}\,dZ$$

Thus the propagation coefficient is given by:

$$\bar{k}_{(x)} = \bar{k}^o_{(x)}\left\{1 - \Sigma_n\, m^1\,\phi_n\,\sin\,(w_n t + \zeta_n)\right\} \quad (A11)$$

where:

$$m^1 = \frac{\sqrt{2\pi}\ \alpha^{\frac{1}{2}}\ C_a}{2\bar{C}^{o\frac{1}{2}}\ T_a\ w_n^{\frac{1}{2}}}\quad\frac{1}{\left\{[C\{\beta_1^{\frac{1}{2}}\} - C\{\beta^{\frac{1}{2}}\}]^2 + [S\{\beta_1^{\frac{1}{2}}\} - S\{\beta^{\frac{1}{2}}\}]^2\right\}}$$

$$\zeta_n = \tan^{-1}\left|\frac{S\{\beta_1^{\frac{1}{2}}\} - S\{\beta^{\frac{1}{2}}\}}{C\{\beta_1^{\frac{1}{2}}\} - C\{\beta^{\frac{1}{2}}\}} - w\tau - \beta\right|$$

The acoustic pressure in the early part of the flame is then given by:

$$P(X,t) = P_o(w)\,\sin\left\{\bar{k}^o\,X - wt + \emptyset_w + \bar{k}^o\,\Sigma\,m^1\,X\,\psi_n\,\sin\,(w_n t + \zeta_n)\right\} \quad (A12)$$

which by a simple change of variables is then seen to have the general form of a phase modulation process:

$$P(X,t) = P_o(w)\,\sin\left[Y - \Sigma\,m_n\,\sin\,Z_n\right] \quad (A13)$$

THE NON-LINEAR ACOUSTIC PROPERTIES OF FLAMES

where:

$$Y = \bar{k}^o X + \phi_w - wt$$

$$m_n = m^1 \phi_n \bar{k}^o X$$

$$Z_n = w_n t + \zeta_n$$

The solution to equation A.13 takes the form of a series of Bessel functions [4]:-

$$P(X,t) = p_{o(w)} \sum_q \sum_n J_q(m_n) \left[\sin (Y-qZ_n) + (-1)^q \sin (Y+qZ_n)\right] \quad (A14)$$

The passage of the acoustic wave also modifies the rate of heat release from the flame. The normal assumption is that this is dependent on the acoustic velocity, and for the sake of simplicity we will assume that the conditions at the flame inlet are the governing factors. Thus we have:-

$$\dot{h}(x,t) = \eta_{(x)} u (O, t - \tau_3) \quad (A15)$$

where τ_3 is the phase propagation delay given by:-

$$\tau_3 = \left(\frac{X}{u^o_b + u_c^o + u (o,t)}\right) + \tau_k \quad (A16)$$

Now the acoustic velocity can generally be represented by:-

$$u(o,t) = \sum_m g_m \sin (w_m t + \phi_m) \quad (A17)$$

and hence the heat wave is given by:-

$$\dot{h}(X,t) = \eta_{(X)} \sum_m g_m \sin (\bar{k}_h^o X + \phi_m - w_m t - \sum_m m_m \sin w_m t) \quad (A18)$$

where $m_m = \dfrac{g_m}{(u_b^o + u_c^o)} \bar{k}_h^o X$

$$\bar{k}_h^o = \frac{w_m}{(u_b^o + u_c^o)}$$

Again this has the form of a modulation process

$$\dot{h}(X,t) = \eta_{(X)} \sum_m g_m \sin (Y' - \sum_m m_m \sin Z'_m) \quad (A19)$$

with a solution similar to that for equation (A13).

THE NON-LINEAR ACOUSTIC PROPERTIES OF FLAMES

NOTATION

C	-	wave (phase) velocity
g	-	component of fluid velocity field (at frequency w_m)
h	-	variation in heat release
k	-	wave number
m	-	modulation index
p	-	acoustic wave pressure
u	-	fluid velocity field
$H(j\omega)$	-	Fourier transform of h(t)
J_q	-	Bessel function of order q
$P(j\omega)$	-	Fourier transform of acoustic pressure p
R	-	radius of burner quarl
T	-	steady state temperature
$U(j\omega)$	-	Fourier transform of u
w	-	angular frequency
α	-	normalised temperature gradient $\frac{1}{T_a}(\partial T/\partial x)$
ρ	-	gas density
θ	-	variation in temperature
ϕ	-	phase angle
τ	-	time delay
η	-	constant relating fluid velocity and heat release
ϕ_h	-	contribution to temperature perturbation due to change in heat release at frequency w_n

Co-ordinate directions:

x, r, θ

Subscripts:

a	-	air
b	-	burning
c	-	mixture
f	-	flame
h	-	heat wave
k	-	chemical reaction (rate)
n	-	component at (angular) frequency w_n
m	-	" " " " w_m
o	-	inlet to burner.

Superscripts:

o	-	steady state
‾	-	space average from origin to point x

THE NON-LINEAR ACOUSTIC PROPERTIES OF FLAMES

FIGURE 1.

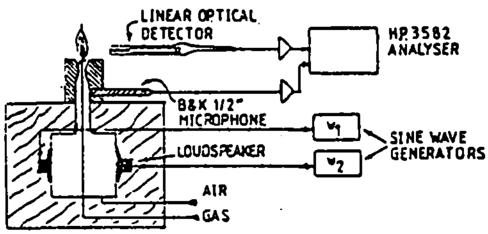

FIGURE 2

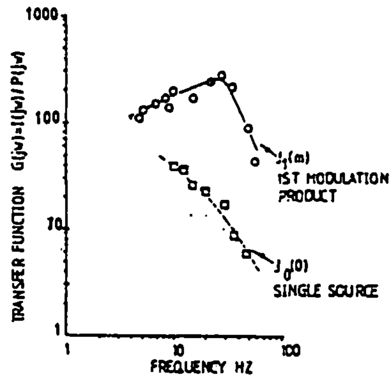

FIGURE 3