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TRANSMISSION OF VIBRATION IN BUILDINGS

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1. Summary

Vibration and noise criteria for human tolerance, for structural integrity and for the satisfactory performance of internal equipment must all be considered in the design of buildings.

Such "vibration sensitivity criteria" are discussed as these reflect the ultimate aim of the designer to achieve a satisfactory building for the purpose for which it is intended.

The sources of input vibration are discussed and categorised as either external or internal. The former includes the effects of wind loadings, earthquakes, explosions, impacts, passage of moving vehicles, pile-driving, etc. The latter specifically includes building systems (e.g. fans, boilers, etc.) and other internal equipment which are possible sources of vibration.

The transmission of vibration from source to the various points of interest in the building structure is considered fundamentally and various simple mathematical analyses derived to explain the relevant factors which affect transmissibility.

For a mass-spring-damper system a single degree of freedom analysis enables the question of resonance, the role of damping and the use of a dynamic transfer function to be introduced. A two-degree of freedom analysis leads to the concept of the dynamic absorber and vibration isolation. A multi-degree of freedom analysis is shown to follow as a logical extension of the single degree of freedom analysis and this can represent a simplified mathematical model for a complex building structure.

In any general problem involving many degrees of freedom (i.e. many modes) it is shown that the output vibration at a point depends upon the product of the input force times the modal displacement at the input point, times the modal displacement at the particular output point. This product is summed over all modes.

From this approach the relevance of nodal points is discussed and the utilisation of these for vibration isolation is considered.

The pattern of the vibration response of complex building structures is considered and various alternative assumptions discussed for the idealisation of such structures; e.g. equivalent cantilever, shear building with and without rigid joints, completely flexible construction. Some recent literature will be reviewed which comments on the accuracy involved in the various degrees of structural refinement.

2. Basic Dynamic Analyses

Fig. 1 shows a single spring-mass-damper system for which eqn. (1) is the governing differential equation of motion

$$m\ddot{x} + c\dot{x} + kx = F_{(t)} \tag{1}$$

By taking the Laplace transform the corresponding transfer function is

$$\frac{\bar{X}}{\bar{F}} = \frac{1}{m(s^2 + 2zw_o s + w_o{}^2)} \tag{2}$$

where $w_o{}^2 = k/m$; $z = c/C_o$; $C_o = 2\ \text{km}$.

For a multi-degree of freedom system a similar transfer function can be written for each mode. Therefore, for the three degree of freedom system of Fig. 2, eqn. (3) is the corresponding expression to (1) written in matrix form for three forcing functions F_1, F_2, F_3 (damping has been omitted for clarity only).

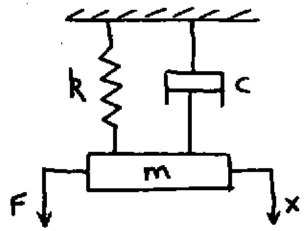

FIG. 1

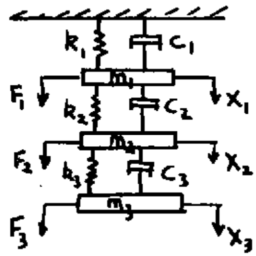

FIG. 2

$$\begin{bmatrix} m_1 s^2 + k_1 + k_2, & -k_2, & 0 \\ -k_2, & m_2 s^2 + k_2 + k_3, & -k_3 \\ 0, & -k_3, & m_3 s^2 + k_3 \end{bmatrix} \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \bar{X}_3 \end{bmatrix} = \begin{bmatrix} \bar{F}_1 \\ \bar{F}_2 \\ \bar{F}_3 \end{bmatrix} \tag{3}$$

and in a simpler form

$$\left[M\, s^2 + K\right]\left[\bar{X}\right] = \left[\bar{F}\right] \tag{4}$$

Hence, expressions can be found for each $\bar{X}$ in terms of the various F's and if

$$\left[\bar{X}\right] = \left[T\right]\left[A\right] \tag{5}$$

where T is the matrix of eigenvectors $\emptyset_{ij}$ and A_i the modal generalised co-ordinates, subsequent matrix manipulations lead to the result that

$$A_1 = \frac{\emptyset_{11}\bar{F}_1 + \emptyset_{21}\bar{F}_2 + \emptyset_{31}\bar{F}_3}{\bar{\bar{M}}_1\ (s^2 + w_1{}^2)} \tag{6}$$

where w_1 is the uncoupled frequency of the m_1 mass-spring system and similar expressions are found for A_2, A_3. Therefore from eqns. (5) and (6) the X at a point is given by the eigenvector Ø in T times A which is itself the product of F times an eigenvector divided by a single mass-spring transfer function. The correspondence of this to eqn. (2) is now clear.

Obviously if a dynamic input is concentrated at a nodal point in a particular mode there will ideally be no dynamic output. Similarly there can be no output at a nodal point.

For the corresponding two degree of freedom analysis eqn. (3) yields the following result for the amplitude of mass m_1 when only $F_{1(t)}$ is acting. Therefore if

$$F_{1(t)} = F_1 \sin wt \tag{7}$$

$$X_1 = \frac{F_1(k_2 - m_2w^2)}{(k_1 + k_2 - m_1w^2)(k_2 - m_2w^2) - k^2} \tag{8}$$

and if $k_2 = m_2w^2$ $X_1 = 0$, but if the denominator becomes zero (at two other values of w) two other resonance peaks occur. This well-known principle of the dynamic vibration absorber can be developed further with the inclusion of damping and it can be shown that X_1 is not reduced to zero at the frequency $w = (k_2/m_2)^{\frac{1}{2}}$. The characteristics of the damper also determine the relative heights of the resonance peaks at the adjacent values of w.

3. Vibration Sensitivity

From a subjective point of view vibration may be defined as any fluctuating mechanical force which may be perceived by senses other than hearing. High levels of vibration can cause discomfort and may interfere with the performance of manual or mental tasks. The body is an extremely complex dynamic structure which has equally complex resonance modes and at low frequencies (1 - 30Hz) these involve whole body resonances. The most important of these are at 4 - 6Hz (abdominal mass motion); 10 - 12Hz (longitudinal oscillations of the spinal column); 20 - 30Hz (head oscillates relative to the torso). Below about 4Hz the body moves as a single mass without resonance but motion sickness and vision blurring are then more likely. Cases are on record where wind induced motions have induced these last effects, and similar motions due to internal equipment should be avoided.

Human sensitivity to vibration may be a function of the vibration amplitude, velocity or acceleration depending on the frequency (Ref. 1). The "degree of strain" K is given in Table 1 in terms of the amplitude and frequency of vibration and a value of K greater than 1 - 3 is undesirable. Acceleration is thought (Ref. 1) to be the dominant criterion for frequencies below 2 - 5Hz. This and much other useful data is summarised in Ref. 2a.

Table 1 K Values (Ref. 1)

Vertical Vibrations	Horizontal Vibrations
Up to 5Hz $K=25Af^2$	Up to 2Hz $K=50Af^2$
5–40Hz $K=125Af$	2–25Hz $K=100Af$
Above 40Hz $K=5000A$	Above 25Hz $K=2500A$

(A = amplitude in inches; f = frequency in Hz)

Structural sensitivity to vibration is discussed in Ref. 2a and 2b and possible damage boundaries presented in terms of vibration amplitude and acceleration for various vibration frequencies.

Similar boundaries for equipment sensitivity cannot be generalised because of the wide diversity in types of equipment. Ref. 2b presents some data.

From the information available (Ref. 2a) the likely range of frequencies for very tall steel framed buildings may begin as low as 0.1Hz, whereas 3 - 6 storey buildings have typical frequencies of 3 - 5Hz. An acceptable conservative empirical formula for the vibration period of the fundamental mode is $T = 0.05h/b^{\frac{1}{2}}$. (Tsec; h, b in feet).

4. References

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