ESTIMATION OF THE DYNAMIC PLATE MODULUS OF VISCOELASTIC MATERIALS BY INVERSION OF IMPEDANCE TUBE MEASUREMENTS

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1. INTRODUCTION

Impedance tubes have been used to measure the acoustic performance of polymer materials in underwater applications for many years. It is now important to re-evaluate the importance of this apparatus for use as a tool for the development and evaluation of polymer materials in the light of the recent advances in standards of contemporary digital data acquisition equipment and signal processing.

This paper presents a technique for estimating the complex acoustic parameters of polymer materials over the bandwidth 2-10kHz using acoustic pulses in a 7cm internal diameter impedance tube. Values of the complex plate modulus are obtained from measurements of transmission magnitude and phase by using an optimisation routine to minimise the difference between these values and the predictions of a plane wave model. Measured reflection magnitude is used to evaluate the success of this approach. Results are presented for a series of polymer (polyurethane) materials. The technique is shown to be most successful, although the ring of water that surrounds the samples in the impedance tube is believed to influence the ultimate accuracy of measurements made with impedance tubes.

2. BACKGROUND

The problem facing a manufacturer who wishes to design polymer systems for the marine environment with specific acoustic properties is one of relating chemistry and conditions of manufacture to the dynamic properties of the finished product. The performance of acoustic materials can be modelled once the values of the physical parameters which control the materials' elastic behaviour are known, but knowing these parameters is often difficult, especially in the case of lossy elastomeric materials. There are few satisfactory techniques available for easily testing the dynamic properties of materials at acoustic frequencies over a range of environmental conditions and Lane et al.[1] have outlined the need to standardise reliable acoustic or dynamic test procedures which are easy to use and independent of operator skill, in order to improve quality control. In 1985 they described the situation as "unsatisfactory".

A popular technique that is widely accepted in this field of materials testing is ***dynamic mechanical thermal analysis*** (DMTA) which uses the Williams, Landel and Ferry equation [2] to relate low frequency mechanical measurements made over a range of temperatures to the behaviour at higher frequencies. However successful measurements rely on the skill and experience of the operator. There is also a significant problem associated with the degree of sub-sampling involved in this technique which uses only small strips of material.

Accurate measurements can be achieved using panel specimens in acoustic tanks and this approach is most useful for characterising samples for oblique incidence. However, these measurements have the disadvantage of requiring both a large sample size and the expenditure of considerable amounts of time and money if they are to be made under pressure.

The impedance tube is an alternative acoustic apparatus which allows quick measurement capability over a range of environmental conditions and on samples of reasonable size. This apparatus is a derivative of Kundt's tube adapted for underwater measurements. The tube is built with thick walls so that the it is rigid enough to act as a water filled acoustic waveguide. When operated below the cut-off frequency of the first mode of the pipe, analysis is simplified to the consideration of plane waves. With these apparatus, measurements of acoustic impedance or echo reduction and transmission loss can be achieved over a range of pressures and temperatures.

Sabin [3] refers to a variety of approaches for calculating the acoustic impedance of viscoelastic materials from reflection measurements using a tone-burst source on samples with sound-hard backings towards the end of the tube. More recently, Townend[4] has reported on improvements made to increase the accuracy of this technique. In place of mounting samples on absorbing cones at the end of the impedance tube, samples are now suspended down the tube to permit time separation of signals from the sample surfaces and the ends of the tube, and also to remove the need for reference measurements to be performed. Broad-band, pulsed signals are used in place of tone-bursts at each frequency and an improved analysis uses Fourier transform spectroscopy.

A similar technique is reported here, however the aim is to use measurements of transmission and reflection to deduce the ***complex plate modulus*** **P** of an impedance tube sample which defines the propagation of longitudinal plane waves through it. This can be written as $\mathbf{P}=\mathbf{K}+4/3\mathbf{G}$ where **K** is the complex bulk modulus, **G** the complex shear modulus, or as $\mathbf{P}=\rho\mathbf{c}_p^2$, where $\mathbf{c}_p$ is the complex longitudinal velocity and ρ the material density. Although the moduli of viscoelastic materials have a strong frequency dependence they can be considered at each frequency of interest in the same manner as an elastic material by inclusion of an imaginary component of the velocity of propagation which also changes with frequency to account for viscous losses (see Shaw and Bugl[5]).

3. MEASUREMENT TECHNIQUE

The impedance tube system installed by Avon Technical Products is 4.5m long and machined from a single length of stainless steel, mounted vertically through two building floors above an access pit. Nicholls[6] reports that tubes should ideally be mounted horizontally to prevent thermal stratification. However, mounting the tube vertically is more practical for the insertion and removal of samples. A piezoelectric piston transducer, mounted to the base of the tube is used to make measurements over the frequency bands 1-10kHz. The waterborne signals are received on hydrophones mounted through the wall at three positions along the tube. The bore of the tube is $\approx 71mm$ which corresponds to a cut-off frequency of 12kHz (the cut-off frequency for a

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circular opening is $\nu_{cut-off} = 0.585c_w/D$ where D is the bore). Water circulating through a jacket around the tube, provides temperature conditioning between 0°C and 20°C through use of thermostatically controlled cooling or heating units. The system can be pressurised up to 900*psi* (or 6*MPa*, corresponding to a water depth of 600*m*).

For this investigation, signals were captured using a digital storage oscilloscope with IEEE interface controlled using a PC. This allowed recording of 1k, 8 bit traces at up to 1MHz. A broad-band pulse waveform was produced by gating a 5kHz sinusoid over a single cycle and this was amplified and fed into the transducer. Figure 1 shows a typical signal train showing a pulse reflected from the open water surface at the top of the tube as recorded on a side hydrophone. The Fourier transform of the outgoing pulse is shown, illustrating the convolved response of the transducer and hydrophone system. This shows that the useful bandwidth of measurement with this pulse lies in the region of 2-9kHz. From the recorded traces evidence can also be seen of other signal contributions arriving within the measurement windows of the waterborne signals, most likely to have travelled through paths in the wall of the tube. This is the cause of small ripples in the spectrum.

Typically, one hundred signals would be averaged to improve the signal to noise ratio, though no anti-alias filtering was performed because it was known that the acoustic signals were band limited by the response of the transducer in the tube. The sampling period was typically 10μs making the Nyquist frequency 50kHz.

Measurements were made on solid cylindrical samples of material suspended down the tube on monofilament line and attached to the rear of the sample by nylon screws threaded into the material. Care was taken to ensure that no air was trapped in this thread as this has a pronounced scattering effect in soft materials. The samples were machined to a diameter that allowed the samples to be inserted to the correct depth in the tube. Kuhl and Oberst have reported on the influence of the ring of water that exists between a sample and the tube wall. This can, at certain frequencies, act as an acoustic short circuit. Indeed Giangreco and Audoly[8] have reported on this effect which they have observed with steel samples. Simmonds[9] has accurately modelled this effect in high modulus materials but suggests that the effect is less significant for materials with acoustic impedances closely matched to water. For this reason he reports that measurements made with an impedance tube on well characterised but acoustically hard materials cannot be used to evaluate the accuracy with which measurements can be made on softer materials.

The transmission coefficient of a sample was estimated by recording the signals received by hydrophones above and below the sample position with and without the sample in place. Signals were windowed using an 1/8th cosine taper before using FFT to obtain the complex spectrum of the selected segment. Complex division neatly accomplished the unwrapping of the relative phase displacement information to give the transmission magnitude and phase change caused by insertion of the sample. Because of the difficulty in knowing the exact position of the sample down the tube, reflection magnitude only was estimated using the record from the lower hydrophone.

This analysis calculated the relative *changes* in magnitude and phase between signals.

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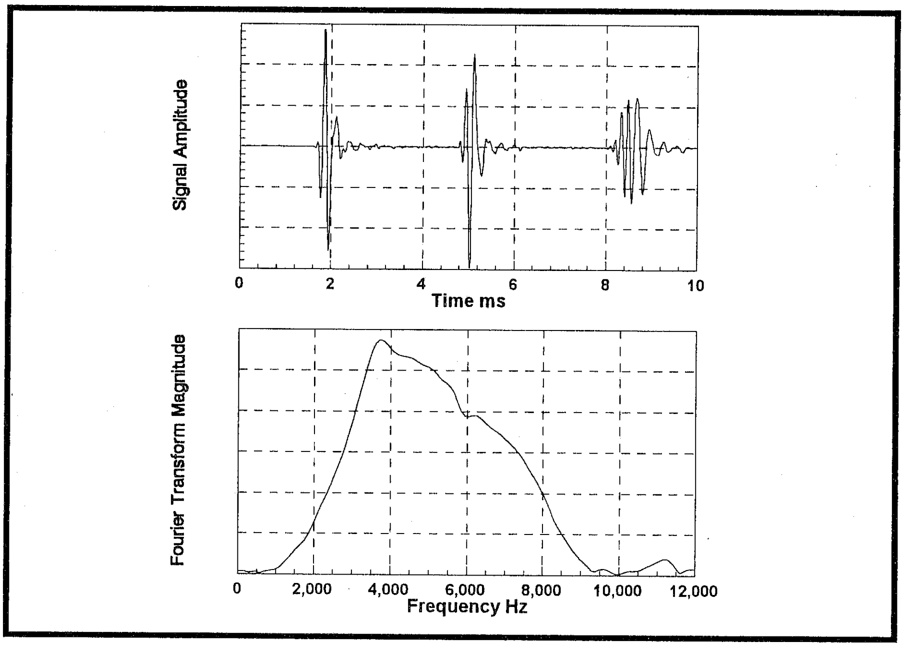

FIG 1.5kHz pulse train recorded on a side hydrophone. Also shown is the magnitude of the Fourier spectrum of the outgoing pulse. The second and third echoes are reflections from the open water surface and the transducer face.

4. INVERSION

An optimisation routine was coded in the MATLAB macro language based upon Brekhovskikh's[10] single layer plane wave analysis for normal incidence. MATLAB is an interactive modelling environment designed for handling matrices and is now widely used by scientists and engineers for the speed with which problems can be formulated. Although the central calculations are performed using complex velocities, the input acoustic parameters for the layer are specified in terms of the dynamic complex plate modulus magnitude and loss tangent. The density and the thickness of each layer are also required and these are easily measured.

The optimisation routine used is provided within the PCMatlab environment which uses a *Nelder-Meade* simplex algorithm (see [11]). This is held to be one of the most efficient general search routines available. Optimisation was carried out with respect to the real part of the plate modulus P' and the appropriate loss tangent $\tan\delta$. The objective function was chosen to be the squared vector difference between experimental values and values calculated using the single layer model. For example, when optimising against measurement of complex transmission the function $(T-T_g)^2$ was chosen, where T is the measured transmission coefficient (relative to an equivalent path through water) and T_g is calculated by the model from initial (and subsequent) estimates of the two material modulus parameters.

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The question then remains as to how it is possible to verify the results obtained by optimisation given that these materials have not been accurately characterised by other means in the frequency régime of interest. The approach used was to assess the self-consistency of the data. That is, the acoustic parameters deduced from the transmission characteristics of sample materials were used to predict the measured reflections.
A series of computer investigations was carried out to discover the loci of guesses which would converge onto optimal values of the complex modulus. For instance, these revealed that for a hypothetical polyurethane material at 5kHz the estimated tolerance ranges were 1-10GPa on the plate modulus, P'=2.5GPa, and 0.03-0.3 on the loss tangent of $\tan\delta = 0.1$, although these limits are both pessimistic. It was also found that, in this case, optimisation is more likely to succeed if the modulus is an overestimate and the loss tangent an underestimate of the true values.

5. EXPERIMENTAL RESULTS

The procedure for optimisation starts with measurement of the density and length of the samples. Three polymer samples were fabricated based on a polyurethane mix with lengths between 150cm and 180cm.

- a.) RB1 Polyurethane base polymer with no fillers.
- b.) RB2 Polyurethane with nominal 5% expancel air inclusions by volume (although air content estimated from the measured density is nearer 2%).
- c.) RB3 Polyurethane with barytes filler to increase density to 1500kg/m^3.

Reflection magnitude, transmission magnitude and phase were measured relative to the displaced element of water. An initial estimate of the acoustic velocities in the materials was made by estimating the path difference in the sample using the recorded signals from hydrophones above and below the sample position in the tube. The velocity of sound in water for the tube was re-calculated using Wood's Equation[1] and allowing for the flex in the tube (see [9]). For instance at 15°C this is $\approx 1440 m/s$ (free field value is 1464m/s). This agreed well with that estimated by measuring the time of flight between the hydrophones.
Values of the real part of the plate modulus were calculated from these and were used as the initial estimates for the optimisation. The estimated values are given in table 1.
Initially, optimisation of the measured complex transmission coefficients for the three samples was attempted. The optimisation routine was applied point by point to the experimental data using the same initial estimates of modulus and loss tangent across the frequency band. The initial estimates of plate modulus and loss tangent supplied for each sample are given in table 2. The loss tangent values were chosen as typical of a lossy polyurethane operating at room temperature.

[1] Velocity of sound in water v_w=1410 +4.21T -0.037^{T}2 where T is the temperature in centigrade.

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	WATER	RB1	RB2	RB3
Velocity m/s	1,440	1,565	943	1,440
Density kg/m^3	1,000	1,133	1,112	1,475
P' GPa	2.07	2.78	1	3.06

Table 1. Plate modulus values estimated from sample path difference.

	RB1	RB2	RB3
P' GPa	4	1.5	4
tanδ	0.4	0.4	0.4

Table 2. Estimates used for optimisation of transmission data.

The optimised values of real plate modulus and loss tangent are presented in Figures 2-4, to the left of the experimental data. Dotted lines drawn on the experimental data show values of magnitude and phase which the model calculates from the optimised values (these lines are only just visible because the optimisation routine has managed to match the values exactly in each case).

The optimised values of modulus appear to have adopted reasonable values, typical of urethanes at this temperature and pressure. The deduced modulus of the base material, RB1, of 3GPa,

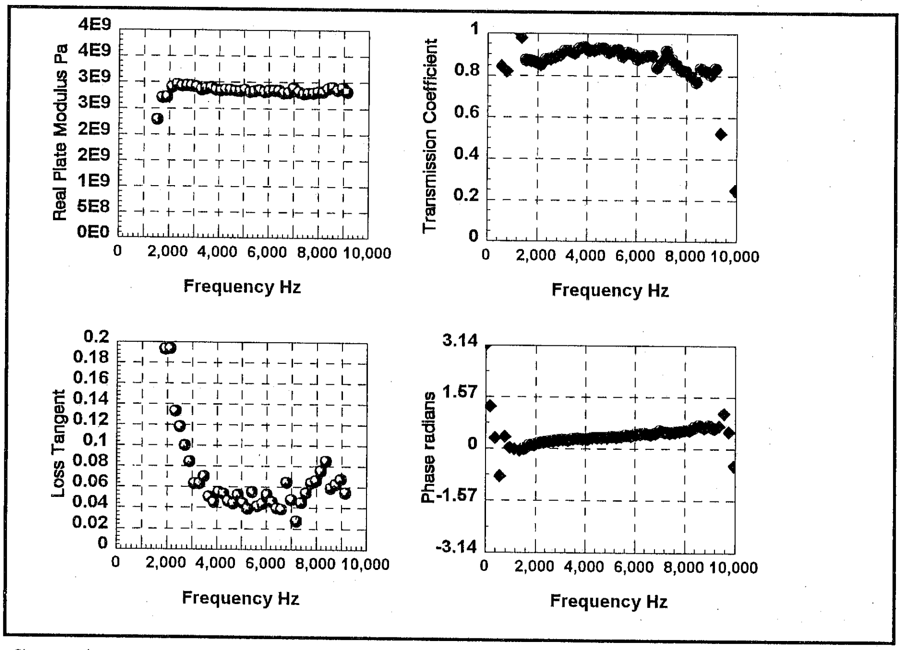

Figure 2. Sample RB1. Results of optimisation against measured complex transmission coefficient of real plate modulus P' and loss tangent tan δ , also showing fitted data

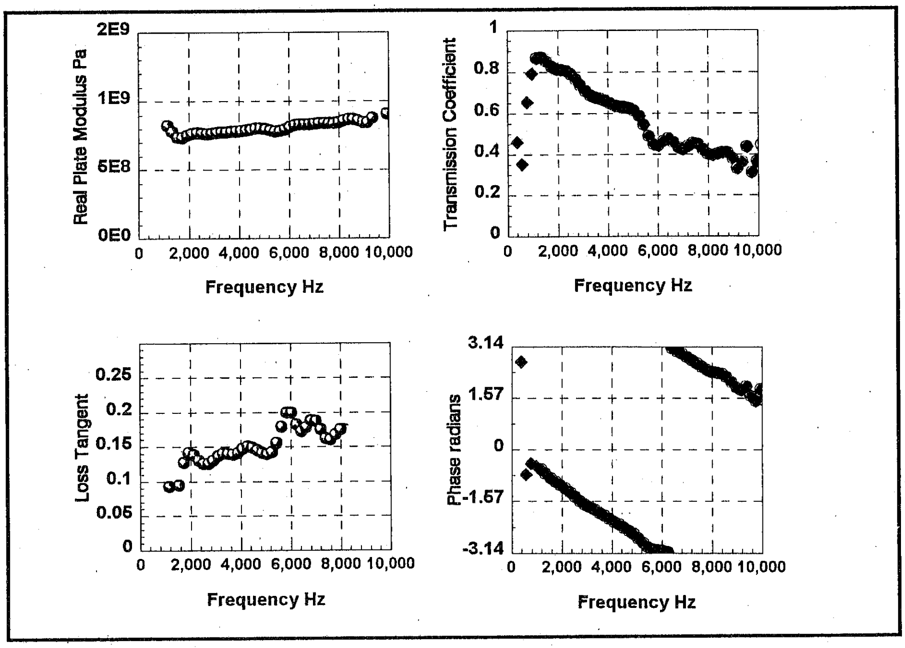

Figure 3. Sample RB2 Results of optimisation against measured complex transmission coefficient of real plate modulus P' and loss tangent tan δ, also showing fitted data

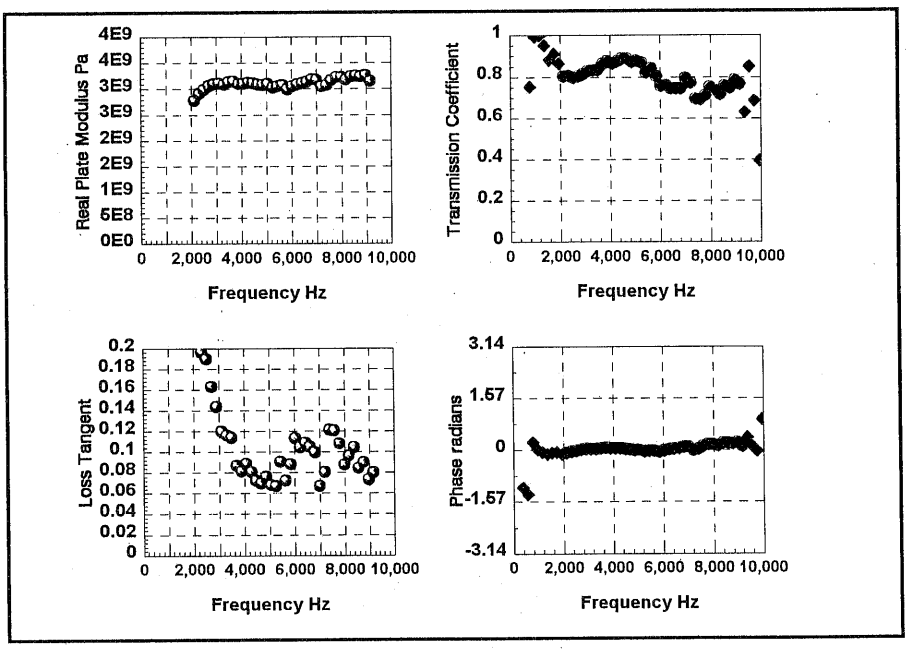

Figure 4. Sample RB3. Results of optimisation against measured complex transmission coefficient of real plate modulus P' and loss tangent tan δ, also showing fitted data.

compared with that of the 2% volume air foam, RB2, of 0.8GPa, would imply that the shear modulus, G, of this polyurethane is in the region of 20MPa (see Kerner's formula, in Lane et al. [1]), which is a reasonable estimate. The optimised modulus of the denser polymer, RB3, is little different to that of the base material. This is to be expected since dense fillers do not tend to alter the base material's acoustic properties significantly for the quantities specified for this material. All three moduli increase smoothly towards higher frequency in line with the normal behaviour of viscoelastic materials. A large increase in modulus is not to be expected over such a limited band of frequency.

The predicted loss tangent values are less regular, although the values and trends do not contradict viscoelastic theory except below 3kHz. The loss tangent of the foam is larger than those of the other two samples, as predicted, and has the smoothest variation of all three across the useful measurement bandwidth. The other two materials have low losses and the observed variation can is attributed to experimental error. The objective function used by the optimisation routine was shown to be less sensitive to the loss tangent than to the modulus, so, conversely, any experimental error will have a greater effect on the optimised loss tangent.

The next step taken to assess the self-consistency of the optimised data was to calculate the predicted magnitude of the reflection coefficient from these values and then to compare the results with the experimentally measured values. These results are presented in Figures 5-7.

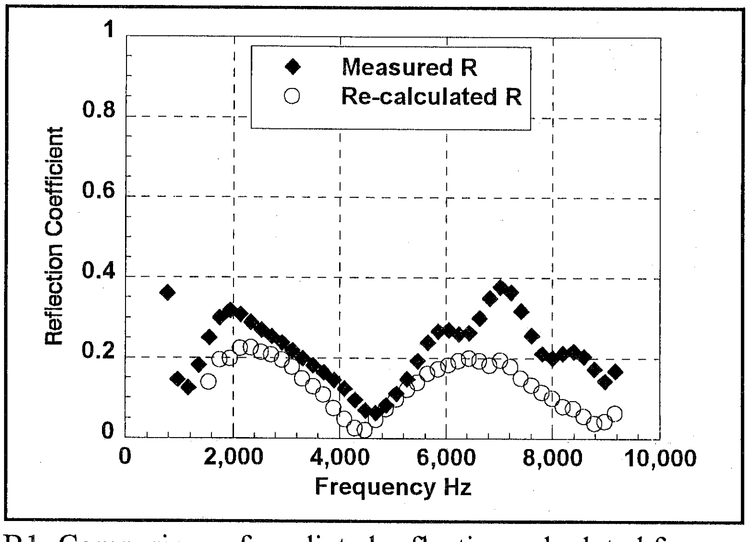

Figure 5. Sample RB1. Comparison of predicted reflection calculated from optimised value of P' and tan δ with the measured reflection.

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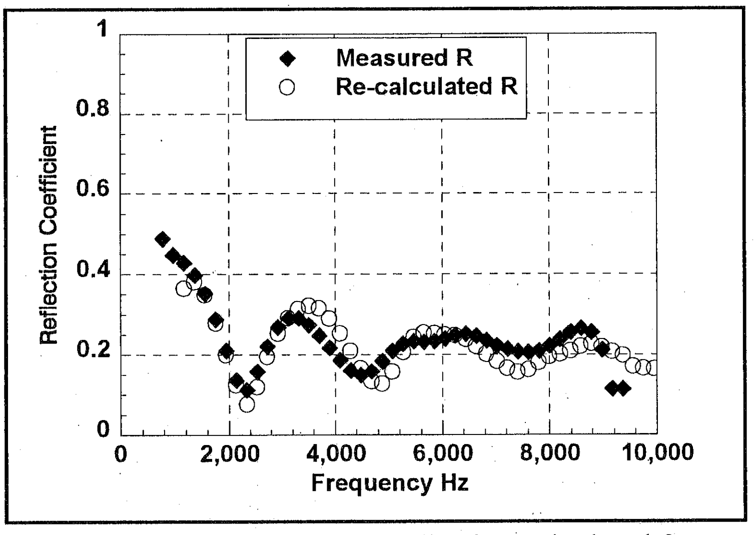

Figure 6. Sample RB2. Comparison of predicted reflection calculated from optimised value of P' and tan δ with the measured reflection.

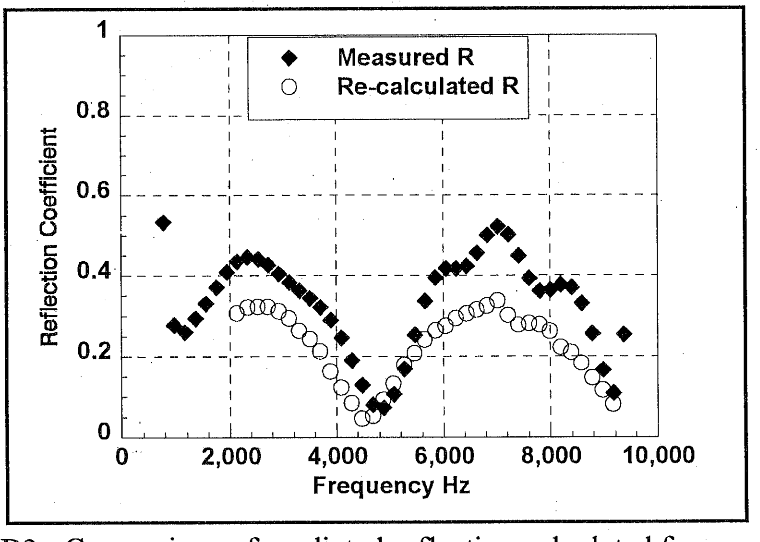

Figure 7. Sample RB3. Comparison of predicted reflection calculated from optimised value of P' and tan δ with the measured reflection.

As can be seen in Figure 6 the agreement between the measured reflection coefficient and that predicted using the optimised values of the modulus and loss tangent is remarkably good. This is a great success for the optimisation technique outlined in this thesis.

For the other two samples the predicted reflection coefficient is sympathetic with the form of the measured values but differs by up to 50% in magnitude. Believing that these discrepancies might simply be due simply to a systematic experimental error, it was decided to proceed with the optimisation using both reflection magnitude and the transmission magnitude and phase data to

consider if improvements could be made. However the results produced by this method gave only a negligible change in these results.
The optimised specific acoustic impedance values and velocities are given below in table 3.

	RB1	RB2	RB3
P' GPa	2.9	0.8	3.1
tan δ	0.05	0.16	0.09
Velocity m/s	1,620	850	1,450
Characteristic Impedance MPa.s/m	1.8	0.94	2.14

Table 3. Approximate mean values of the optimised acoustic parameters

6. CONCLUSIONS

The optimisation process has been demonstrated to work most successfully for a lossy polyurethane foam, typical of materials used in anechoic design. This has been achieved using transmission data alone. No significant change occurred by including the reflection data in the optimisation.
The results of optimisation of the other materials are also encouraging, producing expected values for the modulus, although values for the loss tangent display a greater variability, especially below 3kHz. The loss tangent appears to be most susceptible to experimental error, although deviations are almost imperceptible in the original data.
However, these results appear slightly inconsistent when the measured reflection coefficient is compared with that calculated with the optimised parameters. The form of the optimised reflection coefficient is still encouraging but the amplitudes are up to 50% adrift. The observation that the thickness effects can be seen in the reflection characteristic immediately implies that the slot impedance is not low enough to short-circuit the sample and hence that the slot velocity has not been significantly reduced. The measurements on these higher modulus materials seem to imply that the reflection coefficient is greater than would be predicted by the plane wave model. The same is probably true of the transmission coefficient. It is suggested that these inconsistencies are caused by the ring of water around the sample which appears to still have an influence on the apparent acoustic impedance of the material, though further investigation is required.
As a final note, I am grateful to David Townend of the DRA who has suggested that the measurement technique can be extended to deducing the acoustic behaviour of polymer materials over a wider frequency range by the application of the WLF equation.

ACKNOWLEDGEMENT

The studentship for this work was funded by the Scientific and Engineering Research Council and by Avon Technical Products, to whom I am most grateful.

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