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ATTENUATION OF SPIRAL MODES IN A CIRCULAR AND ANNULAR LINED DUCT

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1. INTRODUCTION

Theoretical understanding of duct attenuation has advanced rapidly in recent years. The work of Rice (1), Mungar and Gladwell (2) and others, has built on the earlier analysis by Morse (3) and shown how theoretical models may be constructed which take account of most of the influential parameters, namely duct geometry, source modal distribution, mean and sheared flow and of course the duct wall impedance. Most often the wall boundary condition is taken as that given by a locally reacting material. This is for the practical reason that most of these studies have been financed by the aircraft industry for whom structural integrity generally dictates that these partitioned or honeycomb backed structures be used. The alternative boundary condition of a non-locally reacting type has been investigated by Scott(4) and Bokor(5) and is amenable to the same kind of analysis, but results for non-locally reacting attenuators generally indicate a reduction in effectiveness

Experimental work has generally lagged behind that of the theoreticians although sufficient results are available (see, e.g., references (6) and (7)) to conclude that the above models are realistic ones. However, the majority of this experimental work does not establish the modal distribution and any theoretical comparisons must necessarily be based on assumptions of, or guesses at, this parameter. Thus the objective of the present experimental work has been to isolate and examine the attenuation of individual modes. Because of the earlier work of Tyler and Sofrin(8) it was known that the noise radiation of rotating machinery such as compressors and turbines is dominated by high order circumferential (spiral) modes. Consequently the behaviour of these modes is of special interest to the aero-engine designer, and a knowledge of their properties is vital in specifying acoustic liners for a given situation.

2. THEORY

2.1 <u>The cylindrical duct.</u> Morse's equation for the propagation of sound in a cylindrical duct lined on the outer wall with a locally reacting material of impedance Z = R + iX (see reference (9))

$$ihe^{i\phi} = -\frac{m}{w} - iw\frac{J_{m-1}(-i\pi w)}{J_m(-i\pi w)} \qquad (1)$$

where $h = 2b/\lambda|Z|$ and $\phi = \tan^{-1}(X/R)$. The radial wavenumber $k_r^m = -i\pi w$ can be found by Newton's iterative method, the application of which is well documented and has previously been used for this type of problem (10, 11). The attenuation of a given mode is then determined by the imaginary part of the axial wavenumber k_x which is related to k_r by the equation

$$k_x^2 = k^2 - k_r^{m2} \qquad (2)$$

thence attenuation/unit length = 8.68 Im(k_x). (3)

By using measured values of the wall impedance and the developed computer programme it is then possible to compare directly experimental and theoretical decay values for particular modal solutions.

2.2 The annular duct. In addition to the cylindrical duct work some experiments were carried out on an annular duct. This was created simply by inserting various diameter steel annuli within the lined outer cylinder so that the duct was lined on the outer wall only. The exact annular duct solution in principle presents little further difficulty than is involved with the cylindrical duct calculations. The radial functions become somewhat more complex as the Neumann functions, Y_m, now provide an additional solution. However the detail work involved in obtaining the exact modal solutions for the annular duct was not attempted here. For a very narrow annulus the annular solution will tend to the equivalent width rectangular duct solution also lined on one side (see reference (12)) so that these calculations later provide an interesting basis of comparison with the experimental annular results. The available rectangular duct computer programme (available by courtesy of Rolls-Royce Ltd.) was easily adapted to handle three-dimensional modes by including the third (i.e. circumferential) wavenumber. Spiral modes in a narrow annulus can thus be modelled.

For the infinitely narrow annulus the circumferential wavenumbers are given by

$$k_z = m/b \tag{4}$$

As we wish though to imitate a finite annulus it is more reasonable to use the correct wavenumbers which are readily available from tables such as those of reference (8). By this artifice the circumferential wavenumber and hard wall mode cut-off frequency are given their correct values and unnecessary distortion of the frequency scale is thus avoided. The "radial" wavenumber is calculated by ignoring the duct curvature and solving the rectangular coordinate boundary condition, which, in the notation of Morse, is given by (9)

$$g \tanh(\pi g) = ihe^{i\phi} \tag{5}$$

where k_y, the "radial" wavenumber, is equal to πg.

The solutions k_y and k_z are then combined to give the axial wavenumber, k_x, which of course determines the axial decay rate as before.

3. EXPERIMENTAL PROCEDURE

3.1 General description of rig. On one end of an acoustically lined duct a noise source is provided. The modes produced by the source travel along the duct, being gradually attenuated, until they either reach negligibly low levels or are absorbed by the duct termination. The acoustic pressure in the duct is measured by a probe microphone and it is this rate of decay per unit length which is the main factor of interest. The attenuating portion of the duct is sufficiently long (36") so that over a considerable length only the least damped mode will normally be present. Two different noise sources were used and either of these could be connected to various lengths of six inch diameter ducting with rigid walls, as well as the fixed length of absorbent duct. Inside this duct a series of inner tubes could be inserted so that an annular channel of hub-tip ratio equal to 0.5 or 0.75 was created. To measure pressure distribution within the duct a probe microphone or 1/8" diameter microphone cartridge was used. This microphone could be traversed along the duct axis at any angular location and it was also possible to traverse circumferentially at any axial position. The attenuating duct was constructed of a "rigimesh" material (PMS 1526)

The original flow resistance of this material was approximately 400 Rayls (S.I.). The rigimesh was mounted over a honeycomb backing of ½" cell size which divides and compartments the combined structure with the aim of forming a locally reacting boundary. The bonding of the material resulted in a blockage of approximately 50%. A more precise impedance measurment of the material was made by using a standing wave tube.

3.2 Noise sources. In the first instance a siren was used as the generator of desired duct modes. By varying the number of holes in the rotor and stator any required spiral mode can be forced into excitation and by choosing the numbers with care the presence of other unwanted modes can be minimised (see reference (8)).

An alternative laboratory rig was built for the plane and first-order modes by using a twin loudspeaker assembly. When the two drivers are driven in anti-phase they will tend to excite the first-order circumferential mode whose cross-sectional pressure distribution given by

$$f(r,\theta) = J_1(k_{mn} r)\cos(\theta)\cos(\omega t) \tag{6}$$

and this function, particularly the angular distribution, when measured experimentally, provides a useful check on modal purity. The siren rig, or indeed a fan or compressor, will, because of the special generating mechanism, produce modes of the form

$$f(r,\theta) = J_1(k_{mn} r)\cos(m\theta - \omega t) \tag{7}$$

so that the pressure amplitude is now invariant with for normal measurements. Thus a valuable piece of information is lost unless correlation equipment is available. It was found that the setting up of a symmetrical pressure distribution necessitated an accurate alignment of the ducting so that it was normal to the source plane. The care needed in setting up increased as higher frequencies were approached. Close to the cut-off frequency it was difficult not to excite almost a pure (1, 0) mode.

3.3 Measurement of modal decay rates. When the required source modal distribution was set up as far as possible the axial pressure trace within the absorbent section was mapped. Where this decay curve was linear with distance it could reasonably be concluded that only a single mode was present. In the case of the loudspeaker rig the circumferential order of this mode could be ascertained by an angular traverse at selected points, whereas when using the siren rig the mode order was assumed from the rotor and stator hole numbers. In many cases the axial pressure trace is not linear. Several different causes for this behaviour are possible. Vibratory motion may be transmitted along the walls of the duct and on radiating be interpreted as air carried signals. This type of interference must be eliminated and a major restriction to this structure-borne noise is provided by the honeycomb partitioning of the lining material. As a further means of damping the duct itself was contained within a sand filled box.

If the attenuation rate is small and the duct termination is other than perfectly absorbent standing waves will be formed along the duct length. Their character will depend upon the attenuation rate and the absorption coefficient of the termination. Another type of interference may be caused by the presence of more than one mode. Because any two modes must have different phase velocities it follows thatthey will pass continually in and out of phase as they travel along the tube and so form an interference pattern. Experimentally it is necessary to recognise these patterns and so some theoretical patterns were calculated.

4. RESULTS

For the circular duct the theoretical results have been calculated exactly based on a locally reacting model. Exceptionally good agreement between theory and experiment was found over a wide frequency range and for all modal orders investigated for this circular case. The local reaction model for the lining was therefore verified.

Theoretical calculations for the annular duct cases were made by using an approximate theory based on a thin annular assumption. Reasonable agreement between theory and experiment was found for these cases, with experimental attenuation values being generally somewhat higher than theory.

The experimental results show that attenuation of the higher order modes characteristic of jet engine noise radiation can be much larger than in the equivalent plane wave case especially for an open duct, so that care in design and testing is necessary. The results demonstrate that theory can be used with reasonable confidence in designing silencing devices for aircraft compressors although it should be noted that the effects of flow were not considered in this study, nor were possible effects of non-linearity investigated.

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