THE EFFECT OF DEPTH PRESSURE ON A FLEXTENSIONAL TRANSDUCER

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1. INTRODUCTION

There is great interest in the design and development of flextensional-type transducers for underwater acoustic purposes. Much emphasis is being placed on obtaining compact, high power sources, to operate at low frequencies for a variety of applications. The flextensional design of projector is a promising choice to meet these requirements. Here a simple shell is excited at resonance electromechanically, usually by means of a piezoelectric ceramic stack. The surface of the shell flexes and so produces compression and rarefaction in the surrounding fluid, which in turn radiates to the far-field. Several different types of flextensional transducer exist [1].

The use of a compliant shell has a number of advantages. The shell may be excited at selected regions to exploit any mechanical advantage provided by its shape. Composite materials may be readily used, providing greater design flexibility. More importantly, a large volume displacement in the fluid is possible, provided by the large active surface area of the shell. This allows high acoustic power outputs to be achieved.

The relatively large surface area exposed to the surrounding fluid makes flextensional transducer designs sensitive to large external static pressure. This may adversely affect the operation of the device to a lesser or greater extent as a function of depth. In extreme cases the outer shell may buckle catastrophically. More likely, the electromechanical drive may become uncoupled from the shell or the bias pre-stress of the ceramic stack is reduced below a critical value. These phenomena result in the projector failing to produce sound of sufficient intensity below a certain depth. A more subtle effect is the change in the structural stiffness of the transducer due to the hydrostatic load. This stress-stiffening changes the resonance frequency as the transducer changes depth. This results in a loss of efficiency as the driving frequency is usually fixed.

To counteract the hydrostatic loading requires some mechanism to compensate for the resultant forces without recourse to elaborate and cumbersome, reinforcing of the shell. An obvious scheme would provide a corresponding internal shell pressure which would balance the outside hydrostatic load. This internal pressure must be allowed to vary appropriately as depth.

The Finite Element (F.E.) method is a useful tool in general structural analysis [2]. The method has been employed with success in transducer design studies [3]. In this paper, we employ the method to predict the effect of hydrostatic loading on a flextensional transducer. In particular, the stress-stiffening effect. The F.E. method is also used to assess the effectiveness of a possible depth compensation scheme. This amounts to a complicated non-linear problem, in general. Not only must stress-stiffening terms be included, but material non-linearites may become significant. Large structural displacements can occur and surfaces may come into contact providing an abrupt change in boundary conditions. The ABAQUS structural analysis code [4] is used here.

The effect of dynamic fluid loading must be incorporated into the analyses as this greatly affects the resonance frequencies and mode shapes of any submerged structure. This is done by surrounding the structure by a volume of fluid with suitable absorbing boundary conditions. We ignore the far-field radiation pattern as only near-field effects matter here.

We begin with a brief description of the finite element model of an idealized 350 Hz. flextensional transducer [5]. A simple discussion of the non-linear structural and dynamic fluid loading within the context of finite element theory follows.The results from the calculations on specific cases of interest are given with particular emphasis on illustrating the design of a possible depth compensation scheme. Some brief conclusions end this paper.

2. THE MODEL

We consider a type IV flextensional transducer as a typical design. Here a cylindrical shell with elliptical cross section is excited harmonically, at the resonant frequency of the first flexural mode, by small motion along the major axis (fig. 1a). This produces large surface motion around the minor axis during flexure of the shell. A large volume of fluid is excited. The acoustic wavelength in water is much greater than the typical dimensions of the transducer which radiates omnidirectionally.

The transducer is taken to consist of a ceramic stack, an aluminium insert and a GRP shell. All the cited materials are assumed to be linear and isotropic. Piezoelectric coupling is ignored as we are interested solely in the frequency response and not in predicting the device's acoustic power levels. Since the motions and forces are essentially constant over the axial length of a working device a two-dimensional finite element mesh was adopted for simplicity. The essential physics remain as we are not concerned with far-field effects. The surrounding fluid has an absorbing boundary condition to simulate the out-going radiation condition (fig. 1b).

The problems that we consider are:

1. The effect of the external pressure on the transducer natural frequencies *in vacuo*.
2. The effect of the external pressure on the transducer resonances *in water*.
3. The efficacy of an internal pressure as a counter to the depth effect (see fig. 2).

3. THEORY

The primary effect of an external static load on the dynamics of a shell or plate-like structure is to change its stiffness [2]. This phenomenon is readily observed in the buckling of columns under axial compression. A dynamic example is the vibration of turbine blades where the centripetal forces produce a change in natural frequency as a function of rotation speed. The change in stiffness is brought about by the slight deviation from the initial shape by the load. Even though the strains may still be small there is no longer linear compatibility relating strain to the spatial derivatives of displacement. Extra terms may come into play producing membrane forces that affect the resistance of a plate to bending, say. In the case of submerged structures the hydrostatic loading tends to reduce the overall stiffness.

An elastic structure's response to a loading is determined by its stiffness. Within the approximation of linear elasticity, the response is proportional to the loading and the stiffness is a constant. To account for the change in structural stiffness due to the action of applied forces requires an additional stiffness term, dependent on the response. This extra stiffness is known as a geometric stiffness or initial-stress stiffness. At lowest order, the geometric stiffness is proportional to the internal forces opposing the applied load.

A key aspect of the F.E. method is the approximation to the structural stiffness by an assembly of small, discrete element stiffnesses forming a matrix. In similar vein, the geometric stiffness, dynamic mass and damping terms are reduced to matrices [2]. In this way the problem takes algebraic form, which lends itself to solution by computer algorithms.

The solution of these matrix equations is straight forward, at least in principle, for linear problems. Since the F.E. method is essentially a linear approximation, an iterative procedure is required for the more complicated non-linear case. The non-linear terms are dependent on the response and are collected together as a forcing term to an essentially linearized problem. Successive approximations are made to solve the linearized equations by assuming small increments in the loading and hence changes in response. The principle of equilibrium is ensured throughout the solution phase. This requires that the residual forces generated by the next iteration of the approximate solution should be small. Solving these equations, we obtain the difference between the approximate and the exact discrete equilibrium solution. This procedure is repeated until convergence is satisfied for a single load increment after which the next small load increment is added. This process continues until the required loading is reached.

The convergence criteria are specified by the user or automatically by the solution algorithm. Obviously the more stringent the convergence, the more expensive the solution. Typically, residual forces should be at least an order of magnitude less than any applied load. A modern proprietary F.E. code like ABAQUS has several solution algorithms which are suited to different forms of non-linear structural behaviour. It requires careful consideration on behalf of the user to ensure that his problem is well posed.

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The fluid requires the provision of special acoustic finite elements. A single pressure degree of freedom is assigned to each node. The classical Helmholtz equation is assumed to be valid throughout the acoustic medium which is characterized by its density and bulk modulus. To account for the fluid-structure interaction, ABAQUS provides coupling elements. The fluid may exhibit viscous damping through the introduction of a volumetric drag coefficient.

4. PROCEDURE

Case 1. is simply solved using the combined stiffness matrix, in the matrix eigen-equation for the natural frequencies and mode shapes. The geometric stiffness matrix, is first constructed from a non-linear statics solution algorithm, in this case a form of Newton's method.

The solution of case 2. requires a static derivation of the geometric stiffness matrix, as before. The acoustic fluid elements having been effectively removed, temporarily. This is followed by a direct solution of the coupled steady-state time-harmonic equations including the acoustic fluid, over a selected frequency range. The excitation is provided by a harmonic force of unit magnitude along the axis of the ceramic stack. The damping is dominated by the acoustic radiation damping contribution and by the absorption of the out-going waves at the boundary of the fluid.

The resonances are determined by identifying the maxima of the receptance (defined to be the ratio of the normal surface velocity to the applied force) of the shell at the minor axis as a function of frequency. The matrices derived from the coupled acoustic-structural interaction equations are non-symmetric. This is a consequence of the inclusion of the absorbing boundary condition imposed by the introduction of a volumetric drag coefficient matrix for the outer layer of acoustic fluid.

In the latter case, we investigate the use of a free-flood rubber bladder as a simple means of providing an internal counter pressure. This in principle requires the solution of a highly non-linear static problem followed by a no less complicated dynamic one. The rubber material is *hyperelastic* giving rise to large elastic strains and its action on the shell is an involved contact problem. To render the procedure tractable requires some simplification.

Firstly, we calculate the reactive forces on the inside of the shell due to the action of the pressurised rubber bladder counteracting the external pressure. The hyperelasticity of the rubber requires an iterative solution scheme based on a modified Riks method [5]. The rubber is assumed to be incompressible and well described by a Mooney-Rivlin material model. The contact procedure requires the use of gap-type elements and correct evaluation of the surface normals at each increment. The pressurized bladder works against the shell. The reactive force on the shell due to the compensation system is then included in the calculation of the geometric stiffness matrix. This two-stage process is valid provided the shell does not move significantly compared to the bladder during the incremental contact phase. Having determined the overall structural stiffness, the determination of the resonances follows cases 1 (*in vacuo*) and 2 (in water).

5. RESULTS AND DISCUSSION

The natural frequency for the operating mode *in vacuo* is predicted to be 730 Hz. This compares with measured values of between 660 Hz. and 690Hz [5]. The disparity may be reconciled by the fact that the F.E. method generally is derived via a variational principle and tends to predict upper bounds to natural frequencies of linear structures [3]. The adoption of a [2D] mesh imposes further constraints and again we expect a high calculated value. Also in the actual transducer, the ceramic stack does not entirely extend throughout the axial length of the shell [5]. The stack comprises four regularly spaced columns of ceramic material.This makes the real transducer less stiff than the F.E. idealization. In addition the measured values are derived from admittance loop measurements (maximum conductance) and are not strictly natural frequencies. As the real structure has a relatively small Q factor (Q<15) these experimental values may very slightly underestimate the true natural frequencies.

The calculated resonance frequencies in water show much better agreement with experiment. Experimental values of between 330 Hz. and 360 Hz for the operating resonant frequency, straddle our value of 348 Hz. inferred from the receptance calculations The limitations of our *in vacuo* calculations are not as apparent with the inclusion of fluid loading. This is due to the dominant influence of the surrounding water. In principle fluid of infinite extent is required to ensure correct hydro-acoustic loading. However, to account for the correct amount of water the mean radius of the fluid volume was chosen to be large enough so that the receptance curves as functions of frequency were insensitive to first order changes in the radius. Typically the radius was chosen to be greater than a quarter of the lowest acoustic wavelength considered. Here the fluid is 20 times more massive than the structure. This allowed the following calculations to be performed with a good degree of confidence.

The predicted shift in frequency due to the influence of the external pressure, for the lowest operating mode *in vacuo* is seen to be 0.4 Hz./metre of water for pressures equivalent to depths less than 200 m. of water (1 MPa. ≅ 100 m. depth). Unfortunately there are no experimental data for this case to compare with this work.The change in the natural frequencies is proportional to the applied external pressure indicating that the load produces a small change in stiffness. Beyond 200 m. equivalent depth the change in resonant frequency becomes more severe, indicating that the structure is becoming rapidly less stiff with increasing hydrostatic pressure (fig. 5). At effective depths greater than 800 m. convergence problems arise. This is born out by the fact that the linear buckling depth for the uncompensated structure is predicted to be around 1080 m.

When account of the dynamic fluid loading is made, the effect of the hydrostatic pressure on the resonances is less marked than *in vacuo*. As the shell does not deform appreciably while under the hydrostatic loading, no re-zoning of the fluid mesh was necessary. Our receptance calculations indicate that the predicted shift in resonance frequency is 0.21 Hz./metre of depth (fig. 3). This is in excellent agreement with experiment [6]. The observed frequency shift is seen to be between 0.2 and 0.3 Hz./metre of depth for depths less than160 m (neglecting the point at 60 m. depth). Again the derived frequency measurements are obtained from admittance loop tests. Below 120 m. depth, the transducer's efficiency was impaired by the external pressure tending to reduce the bias prestress inside the stack.

Our receptance calculations exhibit somewhat narrow band peaks (fig. 4). Mechanical losses in the F.E. model are neglected to achieve unrealistically high Q values. This allows easy identification of the frequency at which occurs the receptance maximum for a given hydrostatic pressure. For convenience we normalise the receptance with respect to the maximum amplitude (fig. 4).

The effect of applying an internal pressure to counteract the hydrostatic load is seen to be successful (fig. 5). A simple internal pressure (90% of the external pressure) applied to the inside of the shell reduces the shift in resonant frequency due to stress-stiffening (*in vacuo*) by over an order of magnitude. The inclusion of a rubber bladder as a scheme for providing the internal counter pressure is shown (fig. 2). The bladder experiences an internal pressure of the same magnitude as the external hydrostatic pressure. This scheme produced almost indistinguishable results compared with the simple application of the internal pressure. Only the active part of the bladder was included in the analysis. At pressures exceeding 5 MPa. the bladder suffers extreme distortion and convergence was difficult to achieve. Thickening the rubber did little to improve matters. The depth compensation mechanism is seen to be equally efficacious for the fully fluid loaded projector (fig. 5).

6. CONCLUSION

The effect of the hydrostatic pressure on the operational dynamics of a flextensional transducer is predicted using the non-linear finite element method. A simple [2D] mesh is employed. Account of the fluid loading is made and the calculated results show good accord with experiment. A scheme for countering the external pressure effects is investigated and found to be effective for depths over half the catastrophic buckling depth.

7. REFERENCES

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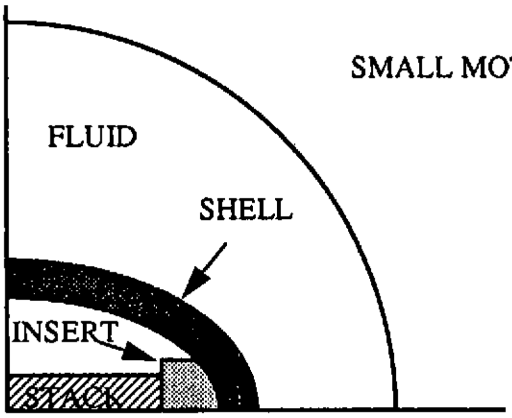
Figures 1a. & 1b.



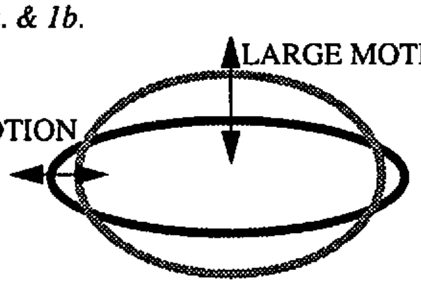


Figure 2.

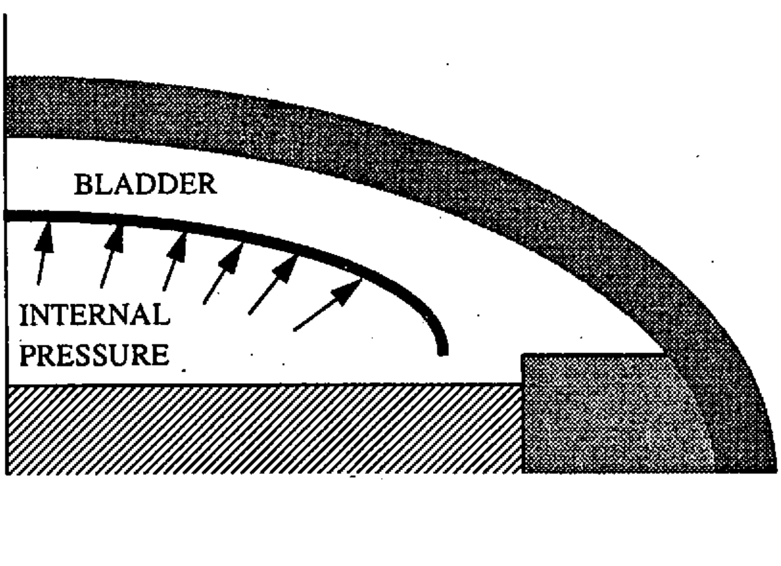
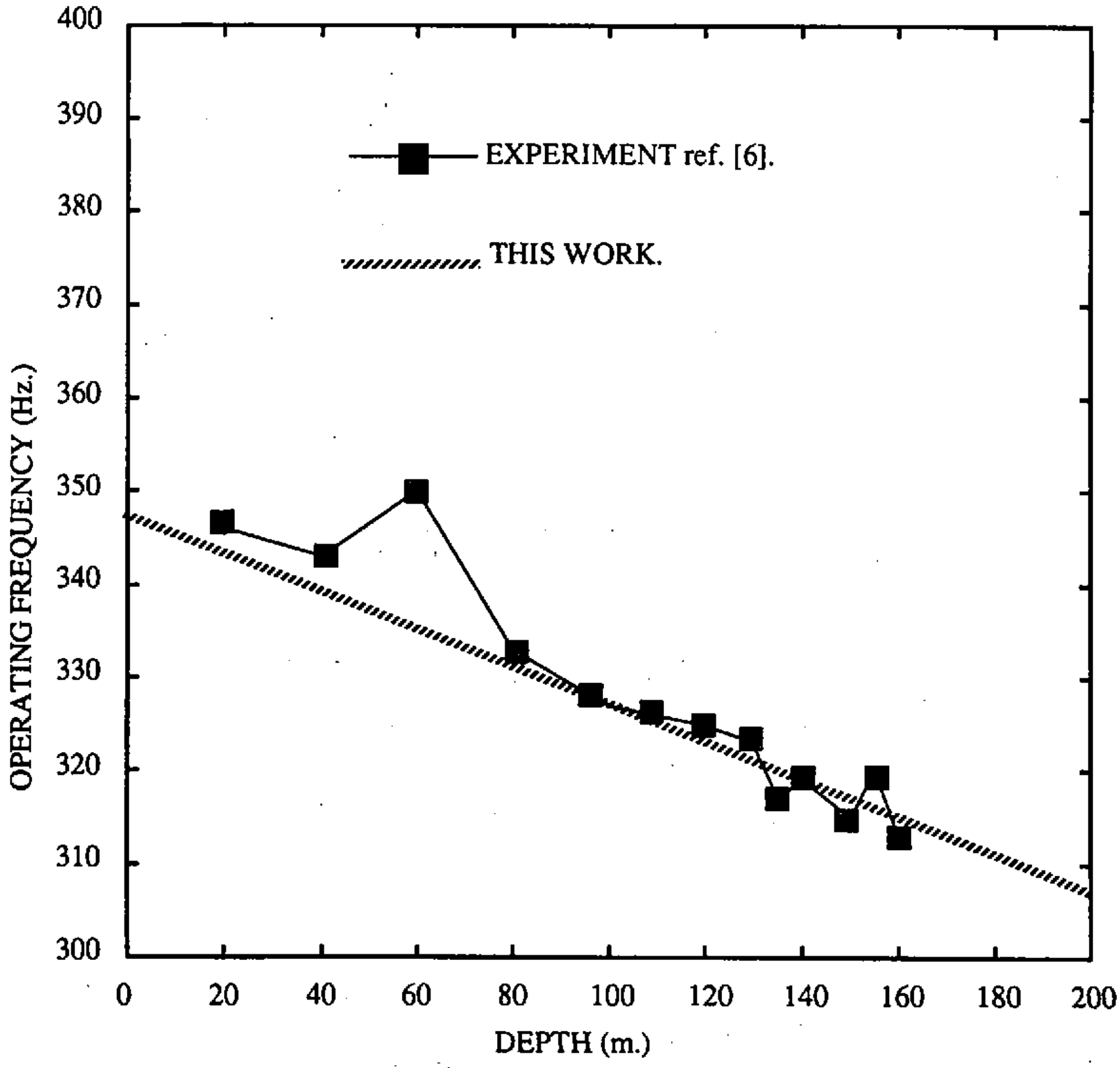


Figure 3.



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OPERATING FREQUENCY (Hz.)
DEPTH (m.)
EXPERIMENT ref. [6].
THIS WORK.

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Figures 4. & 5.