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Sound Generation by Open Supersonic Rotors.

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Introduction

The development of the ducted transonic fan as a fundamental component of modern aeroengines has led to a need for a better understanding of the sound generation mechanisms of very high speed rotors. However, a complete theoretical description of the sound generation by such fans appears some way off, so it is necessary to explore first some simplified models. One such model which has proved valuable for fully subsonic fans is the open (unducted) rotor, so in this paper we analyse the noise characteristics of open rotors when they operate at supersonic tip speeds.

Theory

The crucial simplification inherent in the open rotor model is that the sound waves radiate into free space. Consequently the acoustic radiation problem can conveniently be formulated in terms of the Lighthill aerodynamic noise theory, without any complications arising from the duct walls. This implies that the acoustic generation by the rotor blades can be simulated by a distribution of monopole sources representing the displaced volume, combined with a dipole distribution representing the forces exerted on the air. As usual, the quadrupole stress sources are not included. This model implies the following general expression for the radiated acoustic pressure;

$$4\pi p(t) = -\frac{\partial}{\partial t}\int\left[\frac{\rho_0 \underline{U}.\nabla h}{r|1-M_r|}\right]dS - \frac{\partial}{\partial x_i}\int\left[\frac{f_i}{r|1-M_r|}\right]dS \qquad (1)$$

The notation in (1) is standard, except that h is the local thickness of the blades, and $\underline{U}$ the source convection velocity. S is the planform area of the blades. This result can also be derived from the general theory of noise generation by moving surfaces (Ref. 1.)

To progress with the analysis, it is now assumed that only the steady acoustic sources are important, so all the unsteady sources are neglected. (Here steady sources are those whose strengths are constant in blade fixed coordinates, not laboratory ones). This assumption is made because when steady sources move supersonically, they can radiate very efficiently into the far field, and so the steady sources on the supersonic sections of the blade will dominate the acoustic radiation . This is the primary difference between subsonic and supersonic rotors; for fully subsonic rotors it is the unsteady sources which are most important.

The steady sources generate an acoustic field which is periodic with a fundamental frequency equal to the blade passing frequency $B\omega$. Consequently it can be expanded as a harmonic series, and by using the techniques of Ref. 2 it can be shown that in the far field the (complex) magnitude of the mth harmonic of blade passing frequency is

$$p_m = \frac{n\omega B}{2\pi a_o r}\iint\left\{in\omega\rho_o a_o h + \frac{xT}{r} - \frac{D}{M}\right\}J_n\left(\frac{nMy}{r}\right)e^{-in\phi}R\,dR\,d\phi\,, \quad n = mB,\; M = \frac{\omega R}{a_o} \tag{2}$$

Here the notation is as in Ref. 2. The integral is over a single blade planform which has now been expressed in polar coordinates R, ϕ.

This result is essentially the Gutin expression extended to supersonic rotors. It shows that the radiated noise can be associated with three distinct sources; the blade thickness h, the thrust exerted on the air T, and the circumferential drag D. Their relative importance depends upon the direction of radiation, the rotational speed, and harmonic number. Equation (2) also shows that because the ϕ integral has the same form as a Fourier transform, it is the nth spatial harmonic of the source distribution which couples directly into the mth temporal harmonic of the acoustic field.

However, the most important single feature of equation (2) is the Bessel function $J_n(nMy/r)$. For large values of n, the function values are only appreciable when My/r is greater than unity. Consequently, since the directional factor y/r is always less than unity, this functional behaviour confirms that it is only the supersonic sections of the blades (where the rotational Mach number M exceeds unity) which can radiate efficiently. However, it also shows that even these supersonic sources only radiate strongly into one sector of the acoustic field; the appreciable radiation is confined to a region $y/r < 1/M$. Thus a supersonic rotor radiates most strongly to the side, and an effective 'quiet' cone exists centred on the rotor axis into which the strong steady sources do not radiate. In this region the neglect of the unsteady sources is inappropriate, and these must be returned to the analysis for a full description of the acoustic field.

Comparison with Experiment

There are very few experimental measurements of open supersonic rotor noise available for comparison purposes. The most useful is the work of Hubbard and Lassiter (Ref.3), because both the acoustic radiation and the corresponding aerodynamic performance parameters were recorded. Qualitatively, their experiments clearly confirm this tendency for the rotor to radiate sideways, as well as other overall features of the acoustic radiation that might be expected on the basis of equation (2). More detailed quantitative comparisons have also been made, and Fig. 1 shows a typical result. For this purpose, equation (2) was evaluated numerically. The main conclusion drawn from these comparisons is that the theory predicts well the overall level of the acoustic radiation, but not the detailed spectral distribution of the acoustic energy. In particular the comparisons suggest that it is the frequency scale which is incorrect; the theory gives typical frequencies much higher than those observed in the data. Thus it appears that the analysis described above is incomplete, and that some other mechanism must be at work which causes the discrepancy in spectral shape.

Improved Theory

The clue to the missing mechanism is found in the extremely high sound pressure levels associated with supersonic rotors. For example, in Ref. 3. a near field sound pressure level of 154dB is recorded, and this is quite typical of these rotors. However, at these high levels, sound waves no longer propagate at a constant speed a_o - an implicit assumption in the Lighthill theory. Instead they propagate non-linearly; high pressure regions travel faster than low pressure ones, and the waveform continually changes its shape. Thus in the far field the observed wave profile (and also its spectrum) is not the same as that predicted above, and this could possibly account for the discrepancy already noted.

To place this idea on a more precise footing, it is assumed that the initial generation of the sound is correctly described by the above analysis, but that non-linear effects during propagation are important and must be included. Fortunately a complete theory of non-linear acoustic propagation exists (due to Whitham, Ref. 4) and can be applied here without difficulty. This theory centres around the so called 'F' function, which is essentially the initial pressure profile as a function of time. In this application, it is most easily obtained by summing the Fourier series implicit in the harmonic representation equation(2). The non-linear distortion of the waveform can then be represented by a uniform shearing of this initial profile. When the wave has travelled to a distance r, its new profile can be obtained from the initial one by moving each point on it by a time proportional to F $\log(r/r_o)$. If this leads to a multivalued profile at some points, shock waves discontinuities must be introduced to cut out the overlapping sections of the profile; the shocks cut out exactly equal areas either side of them. Thus in principle it is possible using the Whitham theory to calculate the non-linear development of the wave profile, and hence determine the modification to the acoustic spectrum.

This type of modification to the theoretically predicted acoustic field has been applied to the detailed comparisons cited above. In those examples, it is found that the intial acoustic waveform consists mainly of a sequence of short duration pulses whose shape is roughly an N wave. There is one pulse in each blade passing period, and the duration of the pulse is of order (chord/tip speed), which is much less than the periodic time. For such acoustic waveforms, the non-linear distortion simply converts it into a proper N wave which then slowly lengthens out and decreases in amplitude. The pulse length and amplitude are changed by the same factor, which increases logarithmically with distance travelled. For the examples considered above, this factor is about 2.

Thus, instead of the spectrum of the initial profile being observed in the far field, the modified theory suggests that the spectrum of an N wave whose length is about twice that of the initial pulse and half its amplitude is more appropriate. Revised comparisons on this basis do indeed give a better agreement between theory and experiment as can be seen from Fig. 1, and it can be concluded that non-linear propagation is an important aspect of open rotor noise.

Conclusion

A theoretical analysis of open supersonic rotor noise has been demonstrated. The theory is based on a combination of the Lighthill aerodynamic noise analysis to describe the generation of sound, allied to the Whitham theory describing its subsequent non-linear propagation. The former gives satisfactory predictions of trends and overall levels in the radiated field, but the non-linear propagation effects must be included if an accurate prediction of spectral content is required.

References

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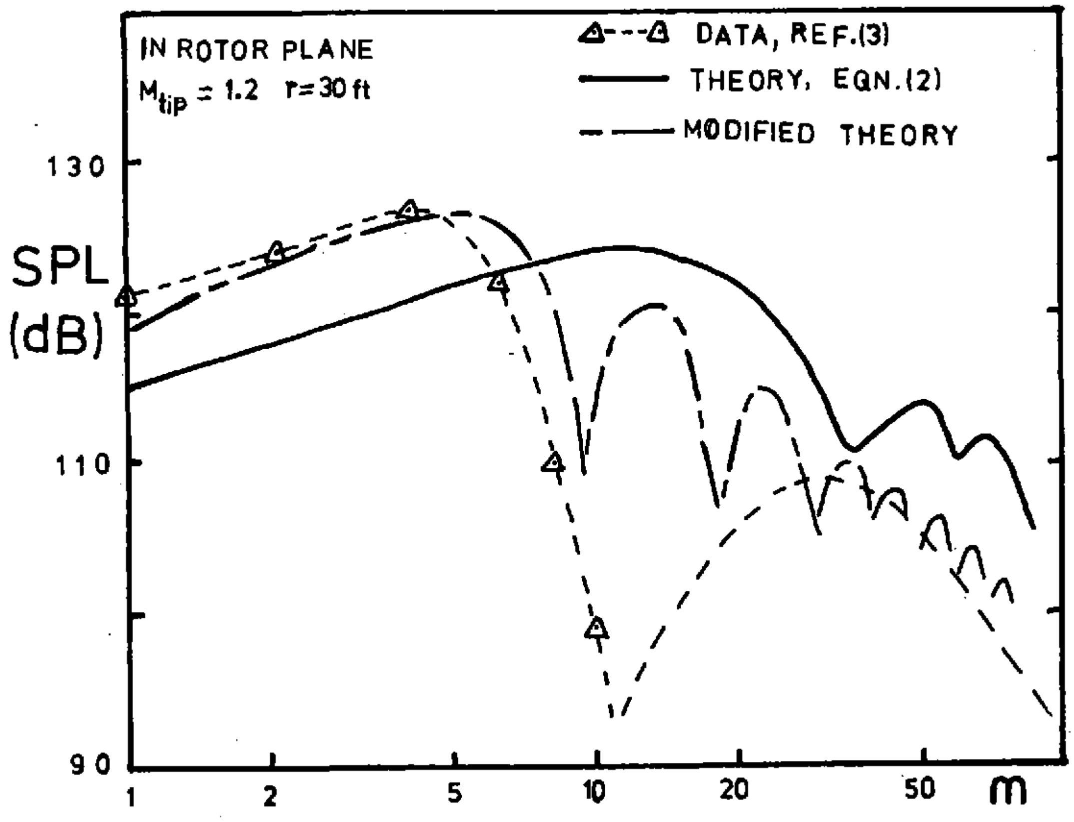

Figure 1 - Comparison of theoretical spectra with data of Ref. 3.