

## TVG FUNCTIONS FOR SHORT RANGE ACOUSTIC MEASUREMENT

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### INTRODUCTION

The time-varied-gain (TVG) function of an active sonar describes how the receiver gain varies with time after the transmission pulse. The TVG is supposed to compensate for the range dependence of the echo strength. Two kinds of TVG are commonly used, according to whether a single target or randomly distributed scatterers are detected. These are normally described respectively as "40 Log R" and "20 Log R" time-varied-gain, terms which express the spreading loss in dB for two way transmission to range R. The additional "2 $\alpha$ R" term may also be included to take account of absorption losses. The substitution  $R=ct/2$ , where  $c$  is the velocity of sound in the medium, is implicit when the TVG is given as a function of  $R$ . The origin of the time  $t$  is usually taken to be the beginning of the transmit pulse.

The need for more accurate measurements of echo strength, particularly in fisheries applications [1], has made it necessary to investigate errors in the TVG function. The effect of changes in the absorption coefficient has been discussed by Foote and others [2]. The equipment itself has become more sophisticated, for example through the use of microprocessors to control the receiver gain [3]. It has been assumed in these investigations that the "40/20 Log R" function exactly compensates for the range dependence. This is a reasonable assumption when the transmitted pulse length is very small compared to  $R$ , a condition which is likely to be satisfied by most fish targets observed during an acoustic biomass survey. Significant errors may occur, however, with targets close to the transducer, when the receiver gain may change significantly during one pulse length. It is particularly important to allow for such errors when using a calibration target at close range. In general, when targets may be close to the transducer, one should use the exact TVG function, or at least a function that is more exact than the 40/20 Log R formula.

### THEORY

#### Echo formation

Consider a sonar which insonifies a single target at range  $R$  on the acoustic axis. The receiver output signal is

$$V_o(R, t) = \Phi(t) \{ \exp(-\alpha R) / R^2 \} \int_{-\infty}^{\infty} g(\omega) f(\omega) H(\omega) \exp[i\omega(t-2R/c)] d\omega \quad (1)$$

where  $g(\omega)$  is the Fourier transform of the transmitter pulse and  $\omega$  is the angular frequency.  $H(\omega)$  is the transfer function of the main receiver amplifier and includes the transducer frequency response in both transmit and receive modes.  $\Phi(t)$  is the TVG function. The receiver gain changes in proportion to  $\Phi(t)$ . For the present, consider  $\Phi(t)$  to be a property of the echosounder, having no specific functional dependence.  $\alpha$  is the acoustic absorption coefficient.  $f(\omega)$  is the target form function.

The effective cross section of the target,  $\sigma_t$ , is defined by an integral over the frequency response of the sonar.  $S$  is the sensitivity factor, and

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$$\sigma_1 = \int_{-\infty}^{\infty} |f(\omega)g(\omega)H(\omega)|^2 d\omega/S \quad (2)$$

$$S = \int_{-\infty}^{\infty} |g(\omega)H(\omega)|^2 d\omega \quad (3)$$

### Echo integration

The exact TVG function depends upon what feature of the echo is required to be independent of range. Usually, but not necessarily, this is the energy in the received signal, which is proportional to the output of an echo-integrator. Consider first the case of a single target on the acoustic axis. The echo-integrator output resulting from one transmission is

$$E_1 = \int_0^{\infty} |V_o(R,t)|^2 dt \quad (4)$$

The range dependent TVG function  $\Phi(R)$  is defined as follows

$$\Phi^2(R) = \int_0^{\infty} |V_o(R,t)|^2 dt / \int_0^{\infty} |V_o(R,t) / \Phi(t)|^2 dt \quad (5)$$

$\Phi(R)$  depends primarily upon the time dependent function  $\Phi(t)$  over the duration of the echo from the target at range  $R$ . It is necessary to distinguish carefully between the range and time dependent functions.  $\Phi(t)$  is a property of the echo sounder electronics.  $\Phi(R)$  is a weighted average of  $\Phi(t)$  which takes account of pulse length, bandwidth and electronic delays in the echo sounder. It is a property of the echo sounder and target together.

Applying Percival's theorem to equation (1) and substituting (2) and (3) yields

$$\int_0^{\infty} |V_o(R,t) / \Phi(t)|^2 dt = \{\exp(-2\alpha R)/R^4\} \sigma_1 S/2 \quad (6)$$

Combining equation (4), (5) and (6) gives the following formula for the single target integral

$$E_1 = \Phi^2(R) \{\exp(-2\alpha R)/R^4\} \sigma_1 S/2 \quad (7)$$

Consider now the case of many randomly distributed targets in a thin shell of thickness  $\Delta r$  close to range  $R$ . The linearity principle [4] implies that the total echo integrator output  $E$  is equal to the sum of the outputs  $E_1$  which would have been produced by each target in isolation. If  $\sigma_v$  is the volume scattering coefficient, the cross section of the targets in the small volume bounded by the shell and the solid angle element  $\Delta\Omega$  is

$$\sigma = \sigma_v R^2 \Delta\Omega \Delta r \quad (8)$$

The echo-integral takes account of the transducer beam pattern, through the equivalent beam angle  $\Psi$ . Substituting (8) for  $\sigma_1$  in (7), integrating over the shell leads to

$$E = \Phi^2(R) \{\exp(-2\alpha R)/R^2\} (\Psi \sigma_v \Delta r S/2) \quad (9)$$

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### EXACT TVG FUNCTIONS

#### Single target echo

Consider first the echo produced by a single target in isolation. In this connection, a single target means a reflecting object whose dimensions are small compared with a wavelength.

The exact TVG function appropriate to a single target is one such that

$$\Phi^2(R) = R^4 \exp(2\alpha R) \quad (10)$$

Define the normalised echo waveform to be

$$V(t') = \int_{-\infty}^{\infty} g(\omega) f(\omega) H(\omega) \exp(i\omega t') d\omega \quad (11)$$

$V(t')$  describes the shape of the echo as determined by the frequency response of the echosounder and the target, ignoring the effect of TVG and propagation delay. If  $\tau = 2R/c$ , the time delay for two way transmission between the transducer and the target, then

$$V_o(R, t) = \Phi(t) [\exp(-\alpha R)/R^2] V(t - \tau) \quad (12)$$

Writing the transformed time  $t' = t - \tau$ , the exact TVG function, except for a constant factor, must satisfy the relationship

$$\int_{-\infty}^{\infty} \Phi^2(t' + \tau) |V(t')|^2 dt' = \int_{-\infty}^{\infty} |V(t')|^2 dt' = kc^4 \tau^4 \exp(\alpha \tau) \quad (13)$$

where  $k$  is an arbitrary constant, independent of  $t$ . The inclusion of  $k$  is necessary because, if  $\Phi(t)$  is exact, so also is  $\Phi(t)$  multiplied by any constant.

At long range, when  $\tau \gg T$ , where  $T$  is the transmitter pulse duration, (13) is satisfied by the asymptotic function

$$\Phi_o(t) = (ct)^2 \exp(\alpha ct/2) \quad (14)$$

which is the usual "40 Log R + 20αR" function. For the general case, consider the trial solution

$$\Phi^2(t) = \Phi_o^2(t) \left\{ 1 + \sum_{n=1}^4 a_n (T/t)^n \right\} \quad (15)$$

It can be shown that the following coefficient values satisfy (13).

$$\begin{aligned} a_1 &= -4 I_1 \\ a_2 &= -6 I_2 + 12 I_1^2 \\ a_3 &= -4 I_3 + 24 I_1 I_2 - 24 I_1^3 \\ a_4 &= -I_4 + 8 I_1 I_3 + 6 I_2^2 - 36 I_1^2 I_2 + 24 I_1^4 \end{aligned} \quad (16)$$

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Where, for  $m = 1 - 4$ , the signal moments  $I_m$  are defined as

$$I_m = \int_{-\infty}^{\infty} t^m \exp(\alpha t) |V(t)|^2 dt / T^m \int_{-\infty}^{\infty} \exp(\alpha t) |V(t)|^2 dt \quad (17)$$

$\Phi(t)$  exists only at times when the right hand side of (15) is positive. When this factor is negative, there is no exact TVG function. This is most likely to occur at close range, when  $t$  is small.

### Randomly distributed targets

The above theory will now be extended to the case of a large number of targets randomly distributed within a thin shell close to range  $R$ . From equation (9), the condition for exact TVG compensation is now

$$\Phi^2(R) = R^2 \exp(2\alpha R) \quad (18)$$

The time dependent part of  $V_0(R, t)$  is still proportional to  $\Phi(t) V(t-\tau)$ , as in the single target case. The exact TVG function must now satisfy the condition.

$$\int_{-\infty}^{\infty} \Phi^2(t'+\tau) |V(t')|^2 dt' / \int_{-\infty}^{\infty} |V(t')|^2 dt' = k(c\tau)^2 \exp(\alpha c\tau) \quad (19)$$

The asymptotic solution is

$$\Phi_1(t) = ct \exp(\alpha ct/2) \quad (20)$$

which is the usual "20 Log  $R + 2\alpha R$ " function. The trial solution is now

$$\Phi(t) = \Phi_1(t) \sqrt{\{1 + b_1(T/t) + b_2(T/t)^2\}} \quad (21)$$

and (19) is satisfied by

$$b_1 = -2I_1 \quad (22a)$$

$$b_2 = 2I_1^2 - I_2 \quad (22b)$$

It is convenient to rewrite (21) in the form

$$\Phi(t) = ct \exp(\alpha ct/2) \sqrt{\{(1-T_1/t)^2 - (T_2/t)^2\}} \quad (23)$$

where  $T_1 = I_1 T$  and  $T_2 = T \sqrt{(I_2 - I_1^2)}$  are time parameters which define the exact TVG function. The function is zero when  $t = T_1 + T_2$  and it does not exist at earlier times, when the right hand side of (23) is negative. The implication is that exact compensation is not possible for targets whose echoes are significant at times less than  $T_1 + T_2$ . This is not an important limitation since  $T_1$  and  $T_2$  are generally of the same order as the transmitter pulse duration  $T$ .

### Some examples

Analytic expressions for the exact TVG functions are practical only in the case of ideal hypothetical systems which have simple transfer functions. Suppose that the transmitter generates a sinusoidal pulse with a rectangle envelope. This pulse may be

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delayed and its frequency composition changed by the transducer and receiver electronics. In the "wideband ideal" case, the transmitter pulse is unaltered.  $|V(t)|$  is constant for  $0 < t < T$  and zero at other times. Ignoring the absorption term,  $T_1$  is then half the pulse length ( $T/2$ ), an intuitively reasonable result, and  $T_2 = T/\sqrt{2}$ .

Consider next the example of a receiver with one simple LCR bandpass filter, in an echo sounder which insonifies a target with a wideband (flat) frequency response. This will illustrate the relative significance of bandwidth and pulse length. If  $B$  is the bandwidth at 3dB down points, the normalised signal envelope is

$$|V(t)| = 1 - \exp(-\pi Bt), \text{ for } 0 < t < T \\ = \{\exp(\pi BT) - 1\} \exp(-\pi Bt), \text{ for } t \geq T$$

Figure 1 shows how  $T_1$  and  $T_2$  depend upon  $BT$ , the pulse length bandwidth product of the simple bandpass filter. At large  $BT$ , the exact TVG function tends to the wideband ideal, whilst at small  $BT$  it tends to the short pulse case for which  $T_1 = T_2 = 1/(2\pi B)$ .

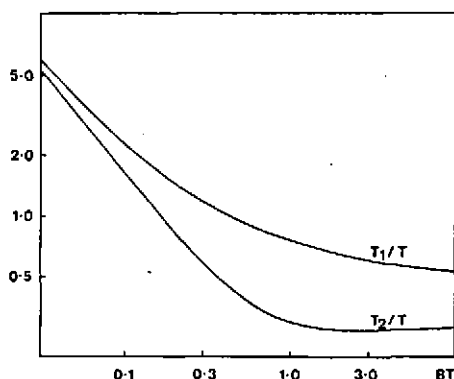


Figure 1

Exact time varied gain parameters of a receiver with one LCR bandpass filter.  $BT$  is the product of bandwidth (kHz) and pulse length (ms).

In typical practice, echo sounders and sonars are operated with pulse length bandwidth products in the range 1 to 3.

### PRACTICAL TVG FUNCTIONS

The time varied gain in present day sonars is produced by analogue or digital control of the receiver gain. The TVG function realised in practice may be quite different from the exact form represented by (23). Moreover, the exact TVG function depends upon the target form function. It is not possible for one TVG function to provide exact range compensation for different targets which do not have the same frequency dependence of scattering. It is nevertheless important to know the exact TVG function for any particular application, in order to assess the error due to the non-exact function generated by the equipment.

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### Calculated TVG functions

Exact TVG functions have been determined for two echo sounders used in fish surveys. The Simrad EK400 has digital gain control, while the older EK38 has analogue gain control. The centre frequency is 38kHz in both cases. Table I shows the TVG parameters for the case of an ideal point target, for which  $|f(\omega)| = 1$ . The two echo sounders have different bandpass characteristics. This is the main reason for the differences in the TVG parameters.

TABLE I

Time-varied-gain parameters for  $T = 1$  ms

Echo-sounder	EK400		EK38		Wideband
Nominal bandwidth	1 kHz	3 kHz	1 kHz	3 kHz	infinity
$T_1$ (ms)	1.213	0.950	1.155	0.845	0.500
$T_2$ (ms)	0.297	0.271	0.338	0.290	0.289

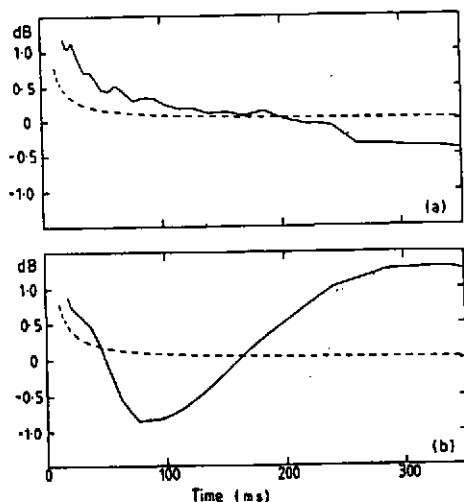


Figure 2

Comparison of TVG functions;  
(a) EK400; (b) EK38.  
Horizontal axis; time after  
start of transmit pulse.  
Vertical axis; ratio of TVG  
functions, arbitrary origin;  
—measured/exact;  
---nominal/exact;  
1 ms pulse length, 3 kHz  
bandwidth and an ideal point  
target.

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### Comparison with measured functions

The precise measurement of the TVG function is part of the echosounder calibration for which there are well established techniques [1]. Figure 2(a) shows an example of the measured gain of the EK400 when set to "20 Log R", compared with the calculated exact function. The nominal function given by (24) is included for comparison. The unevenness of the measured curve is due to the digital control which changes the gain in steps. The TVG ramp of the EK400 starts at  $t=0$ , coincident with the transmission. This explains the relatively large deviation from the exact function immediately after the transmission. However, the digital control provides a stable and near exact gain function over most of the range.

Figure 2(b) shows a similar curves for the EK38. The analogue gain control provides a smooth change with time, but the deviations are large. The analogue technique gives a poor representation of both the nominal and exact TVG functions.

### An approximate function

When the nominal function generated by the equipment starts from  $t=0$ , no account is taken of the electronic delays. This may be partly corrected by starting the same function after a delay  $T_0$ . Thus the nominal "20 Log R + 2  $\alpha$  R" function becomes

$$\phi_a(t) = c(t - T_0) \exp(\alpha ct/2) \quad (28)$$

This approximation allows the circuitry or program which generates the nominal function to be used without modification, apart from the delayed start time.

$T_0$  may be chosen to make  $\phi_a$  exact for targets at a given range, say  $R_0$ . It can be shown that

$$T_0 = T_1 + 2R_0/c - \sqrt{\{(2R_0/c)^2 - T_2^2\}} \quad (29)$$

$T_0$  lies between  $T_1$  and  $T_1 + T_2$ . In the limit of infinitely long range,  $T_0$  tends to  $T_1$ .

### CONCLUSIONS

The conventional 20/40 Log R time varied gain functions do not compensate precisely for the range dependence of sonar echoes. For exact compensation, it is necessary to take account of the finite pulse length and bandwidth of the sonar.

Exact TVG functions have been derived which completely remove the range dependence of the echo integral. Functions have been derived for the single target echo and the superimposed echoes of randomly distributed targets. In each case, the exact function is shown to be equal to the asymptotic 20 or 40 Log R function multiplied by a polynomial in  $T/t$ . The coefficients of the polynomials are calculated from moments of the echo waveform.

If the asymptotic TVG function is assumed to apply generally, the error resulting from the range dependence of the echo integral becomes more significant as the range decreases. The error may well be significant at the close range of the reference target

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used to calibrate fishery echo sounders. Exact TVG compensation is then necessary to avoid bias in the calibration.

The theory of exact TVG functions is not restricted to underwater acoustics. It is generally applicable to the measurement of reflected pulses in any medium, when the pulse length is a significant proportion of the target range.

### REFERENCES

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