Proceedings of The Institute of Acoustics

# NONLINEAR ULTRASONIC FIELDS: THEORY AND EXPERIMENT

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## INTRODUCTION

The acoustic pulses radiated by diagnostic ultrasound equipment into water are often distorted due to nonlinear propagation [1], because the high acoustic amplitudes involved give rise to the production of a shock front as the wave progresses. Such pulses show an increase in the amplitude of the positive half-cycles of the pressure waveform compared with those of negative acoustic pressure. This asymmetry in the waveform arises from the interaction of the diffraction and nonlinear distortion processes but, although the feature is well-established both from observation and theoretical prediction [1-4], the effect has yet to be studied systematically from either viewpoint. Experimental determinations of the shape of the distorted waveforms have encountered difficulties associated with the limited bandwidth and impulse response of the detection systems, which give rise to errors in the determination of the peak-positive acoustic pressure [2]. It is extremely difficult to quantify these errors because considerable knowledge of the acoustic waveform is required, and there is a requirement for a careful comparison between calculated and measured waveforms in order to determine the accuracy of the measurements.

This paper presents a theoretical model of nonlinear propagation that can be applied to high-amplitude focussed ultrasonic fields; it takes account of the effects of nonlinear distortion and diffraction, together with small-signal attenuation and dispersion. It models a field with a Gaussian radial profile; this introduces a considerable simplification into the treatment of diffraction [1,5]. The theoretical predictions are compared directly with experiment for the case of an unfocussed transducer radiating into water. To validate the model for the focussed case, calculations are made for the conditions pertaining to a series of measurements previously reported in the literature [2]. Finally, calculations are performed to predict the waveform that would be produced by the focussed transducer radiating into a medium having acoustic properties similar to those of human liver. It is thus possible to draw certain conclusions regarding the relationship between the output waveform of ultrasonic diagnostic equipment when radiating into water and the corresponding waveform when the device is radiating into living tissue; in particular, the negative acoustic pressure in tissue may be significantly greater than that derived from measurements in water.

## THEORY

The basic equations used to describe the propagation have been given elsewhere, both for the case of an unfocussed [5] and a focussed [2] transducer. It is similar to that derived by Fenlon [6] in his treatment of parametric arrays and is obtained by writing the three-dimensional field as a sum of Gauss-Laguerre normal modes for each harmonic frequency. If the source transducer shading function can be modelled by the Gaussian function then, provided the losses due to nonlinear distortion do not become too great, only the mode of lowest order need be considered at each harmonic frequency. In the previously published derivations of the Gaussian field method [2,5,6] the phase variations occurring due to diffraction within the near field region of the transducer were neglected, but in the present study this effect is included. Details of this method are to be published shortly [7]. A numerical approach is adopted to solve the equations which performs the calculations in the frequency domain [8] and permits the introduction of attenuation and distortion into the calculations in a straightforward manner. Another physical process that can introduce phase variations into the propagation is dispersion; however, in this case, the propagation behaviour is different because the phase change is a function of frequency. Nonlinear propagation with dispersion has been accounted for previously for the plane wave case [9] but not in the presence of diffraction. Dispersion would be expected to affect the propagation in two ways, namely by altering the diffractive behaviour of the harmonic components and by interacting with the process of distortion. The first effect is expected to be small for the situations considered in this paper, because the phase velocity of the thirtieth harmonic differs by only 1% from that of the fundamental, and dispersion is allowed for in an ad hoc manner which only takes into account the interaction with the nonlinear distortion.

The calculations were performed using an IBM PC microcomputer with 8087 mathematics coprocessor, programmed in Pascal and using double precision. The execution time increases as the square of the number of harmonics retained in the calculation; a typical calculation time for forty harmonics was five minutes.

## MEASUREMENTS

Measurements were made in deionised water on the acoustic axis of a transducer that had been designed to have a Gaussian shading function [10] and an operating frequency of 2 MHz. The axial beam profile was determined as a check on the performance of the transducer, and a least-squares fitting procedure used to calculate the optimum value for the Rayleigh length, which was 160 mm. In the far field region of the transducer the experimental points were within 10% of the fitted curve, but in the near field region deviations of up to 40% were observed. The transducer field did not, therefore, conform very well with the expected profile and this was probably due to the relatively strong coupling of the material used for the active element (PZT) to lateral wave modes [11].

The acoustic measurements were performed with a membrane hydrophone [12] which displayed a -3 dB bandwidth of over 100 MHz and was connected to a preamplifier which also had a -3 dB bandwidth of over 100 MHz. The signal was detected using a Tektronix transient digitiser, type 7912AD, with an analogue bandwidth of 200 MHz and an effective digitisation rate of 1 GHz. The digitised signal was transferred to a desk-top computer and analysed into its harmonic components using a Fast Fourier Transform routine. The amplitudes and phases of the harmonic components were corrected to compensate for the known performance characteristics of the preamplifier, and the results for up to forty-nine components were stored on magnetic tape for future reference. Various relevant parameters for the measurement were also stored; these included the propagation distance and the effective acoustic pressure amplitude at the transducer face, which had a maximum value of 1 MPa. This source amplitude was determined at low levels from the measured axial beam profile of the transducer, and at higher amplitudes it was derived by extrapolation, assuming that the transducer output was directly proportional to the drive voltage. The attenuation coefficient of the water was obtained from a standard formula [13], although there is some recent evidence that these values are a few percent higher than the true ones [14].

## RESULTS

Figure 1 gives the predicted and measured waveforms for a propagation distance equal to the Rayleigh length of the transducer. Neither of the profiles appears smooth; this is an artefact due to the Gibbs oscillations caused by truncating both the experimental and calculated data at the fiftieth harmonic. Qualitatively the two waveforms are similar, displaying similar asymmetry, and in both cases the peak-negative acoustic pressure occurs some 40 ns before the shock front. The main difference between the waveforms is in the shape immediately after the shock front, where a sharp peak is predicted theoretically, whereas a rounded peak is observed experimentally. This rounded peak profile is a consistent feature of the present set of measurements, but previous measurements with unfocussed transducers have indeed shown a sharp peak (see, for example, [12]). Thus, the rounding may be due to an imperfection in the transducer performance, such as a nonlinear drive characteristic, and further tests are required to examine this.

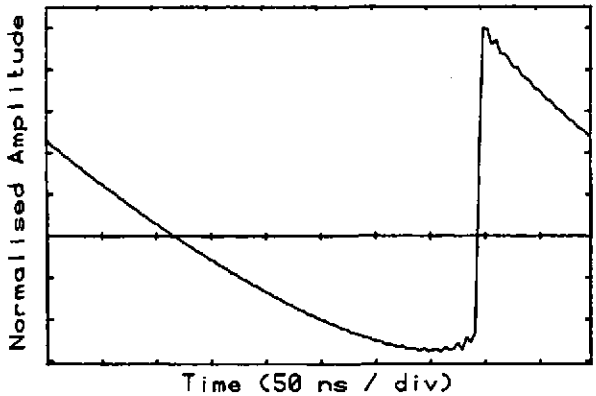
In Figure 2 the predicted and observed waveforms are compared for a propagation distance equal to five Rayleigh lengths. The asymmetry of both profiles is now less pronounced, presumably because the influence of diffraction is less, and the peak-negative acoustic pressure occurs immediately before the shock front, which is smoother due to the greater influence of small-signal attenuation at this larger propagation distance.

A typical calculated waveform for a focussed field is given in Figure 3, where the input parameters were chosen to model some previously reported measurements in water [2]. The transducer used for these measurements had a resonant frequency of 3.5 MHz, a focal length of 39 mm, and a diameter of 10 mm. It should be noted that, in Figures 3-5, the vertical scale factor is set so that a distance of ten divisions corresponds to the peak-to-peak amplitude expected if the propagation obeyed linear theory. The waveform in Figure 3 has much greater asymmetry than in the previous cases because the effect of diffraction is more pronounced, and the peak-positive acoustic pressure is typically 2.5 times greater than the peak-negative. The calculations also show that the peak-positive acoustic pressure can be as much as 80% greater than the value predicted by linear theory. The variation of peak-to-peak acoustic pressure with increasing source amplitude was determined, and was in agreement with the measured results [2]. Although the peak-to-peak pressure amplitude was generally greater than that predicted in the absence of nonlinear propagation, the root-mean-square (RMS) pressure was always lower than the corresponding value predicted by linear theory, showing that in this case there is no extra concentration of energy at the focus.

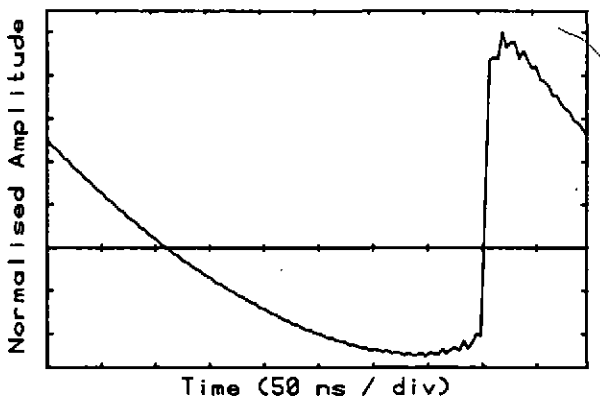
Having validated the theoretical model by comparison with experiment for propagation in water, calculations were then performed to predict the acoustic waveforms resulting from propagation in a tissue-like medium. The acoustical parameters used were those measured by Cobb for normal, fresh human liver [15], with the additional assumptions of a linear dependence of the attenuation coefficient on frequency and a value for the dispersion of 0.7% per decade of

Figure 1

Waveforms for the unfocussed transducer and a propagation distance of one Rayleigh length. The vertical scale factor is set so that one division on the vertical axis corresponds to 0.2 times the peak-positive acoustic pressure.




a) Calculated waveform.



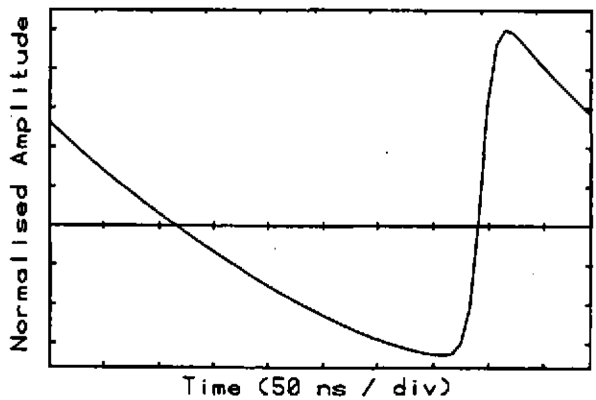

b) Measured waveform.

frequency [16]. Predictions were made for the case of the focussed transducer described above, assuming that the initial acoustic pressure amplitude was the same as that for water, and a typical waveform is given in Figure 4. As would be expected, the shock front is much smoother than that of Figure 3, due to the larger attenuation of the medium, but the most significant feature is the asymmetry, which has the opposite sense than for water with a peak-negative pressure up to 15% greater than the peak-positive pressure. Comparing results for propagation in water with those in tissue, the acoustic amplitude in tissue is always lower than the corresponding value for water. However, if the acoustic parameters are multiplied by a factor equal to the low-amplitude attenuation loss at the fundamental frequency (as is common practice), then the peak-negative acoustic pressure in tissue can be as much as 1.8 times higher

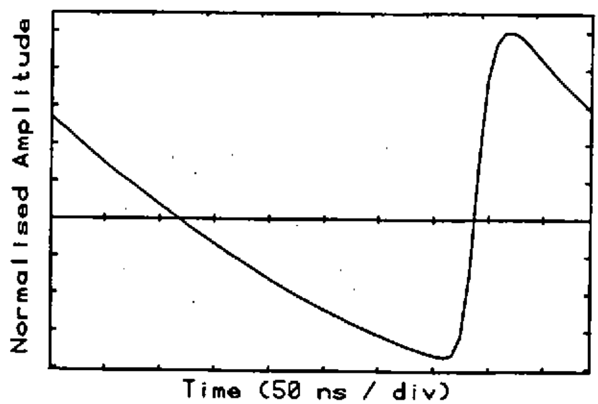
Figure 2

Waveforms for the unfocussed transducer and a propagation distance of five Rayleigh lengths. The vertical scale factor is derived as in Figure 1.

a) Calculated waveform.




b) Measured waveform.




than the value obtained in water, whilst the peak-positive pressure is up to 1.9 times lower.

A final calculation was performed to demonstrate the significance of dispersion in tissue by setting the dispersion equal to zero; Figure 5 gives the result, where the asymmetry has the same sense as in water, but the ratio of the two peak acoustic pressures is now 1.5. The harmonic content of the waveforms in Figures 4 and 5 is similar, demonstrating that the main effect of dispersion is to alter the shape rather than the energy content of the wave.

Figure 3

Calculated waveform for the 3.5 MHz focussed transducer radiating into water. The vertical scale factor is set so that one division on the vertical axis corresponds to 0.2 times the amplitude that would be predicted in the absence of nonlinear propagation.

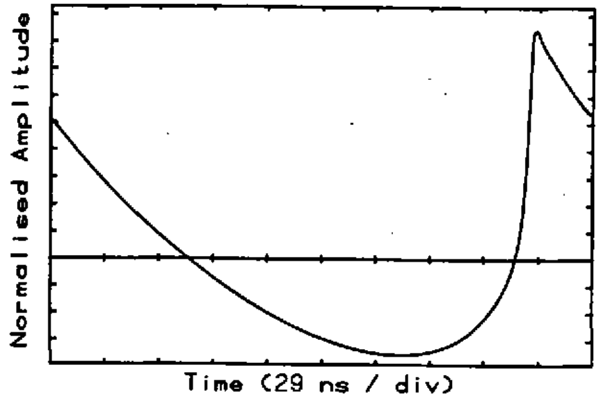



Figure 4

Calculated waveform for the 3.5 MHz focussed transducer radiating into liver tissue. The vertical scale factor is derived as in Figure 3.

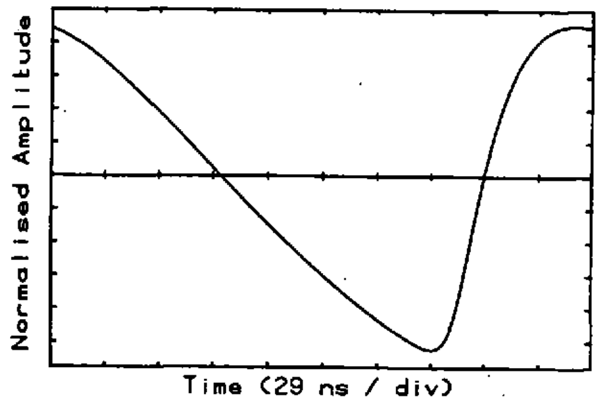
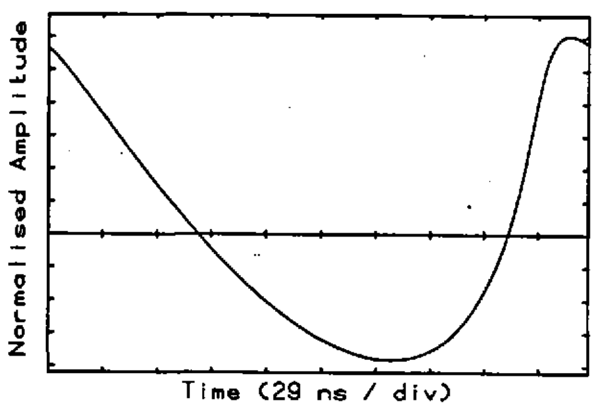



Figure 5

Calculated waveform for the 3.5 MHz focussed transducer, radiating into tissue but neglecting dispersion. The vertical scale factor is derived as in Figure 3.



## DISCUSSION

It is of interest to analyse the physical processes that determine the shape of the waveforms. For convenience, in the following discussion the term 'harmonic components' will be used to refer to all of the frequency components in the wave apart from the fundamental. To explain the profiles it should be noted that both diffraction and dispersion lead to an increase in the phase velocity of the wave when compared to the plane-wave or low-frequency value. However, at least for a Gaussian beam, diffraction introduces the same phase change at each of the harmonic frequencies, whereas the effect of dispersion increases with frequency. Thus, in the absence of dispersion, the harmonic components, which are produced during propagation, have a phase that is some kind of average of the phase of the wave during propagation. On the other hand, the fundamental component, which only loses energy as it travels, has a phase that is similar to that expected in the absence of distortion. Consequently, the contribution to the shock front from the harmonics will be delayed in time relative to the contribution from the fundamental, giving rise to a large positive peak in acoustic pressure.

A similar argument explains the result in the presence of dispersion, where the harmonics produced during propagation travel more quickly than the fundamental. Their contribution to the shock front is thus advanced in time compared with that of the fundamental, giving rise to an increase in the value of peak-negative acoustic pressure. In tissue, diffraction and dispersion are in competition with each other, and so one would expect the resulting acoustic profile to vary considerably, depending on the relative importance of the two effects.

## CONCLUSION

This paper has presented a theoretical model for predicting the effects of nonlinear propagation in the presence of diffraction, attenuation and dispersion, and in the fields of both focussed and unfocussed transducers. The model has been verified by comparison with experiment for propagation in water, and has been used to predict the waveforms that would be produced by propagation in liver tissue. Diagnostic ultrasound equipment is often characterised by making measurements in water and using those results to predict corresponding acoustic levels for propagation in tissue [17]; in this case the peak-negative acoustic pressure may be underestimated by a factor of 1.8 whilst the peak-positive acoustic pressure may be overestimated by a similar amount. This result is particularly significant if the possibility of biological effects due to cavitation is being considered [18], because the peak-negative acoustic pressure is likely to be an important predictive parameter for the presence of cavitation. Although this study has only considered propagation in one type of biological material, it nevertheless demonstrates a limitation in the use of water as the standard medium for the characterisation of diagnostic equipment. If water is used as the reference medium then the effects of nonlinear propagation in water and in tissue should be taken into account when deciding on the potential hazard of a particular commercial device.

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