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ROBUST, HIGH RESOLUTION ARRAY BEAMFORMING.

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INTRODUCTION

Eigenvector methods are generally very effective at resolving closely spaced signals. Two the the most widely used techniques are the MUSIC algorithm [1] and a more recent method proposed by Kumaresan and Tufts [2]. The signal directions are extracted from such techniques by performing a multimodal search in a single dimension.

The majority of eigenvector methods rely on the asymptopic orthogonality of the signal direction vectors to the noise subspace eigenvectors. This paper will assume a more direct approach which relies on the properties of the signal subspace eigenvectors. The signal eigenvectors will be rotated, so that they more accurately represent signal direction vectors. This allows the resolution of closely separated signals [3] even in the presence of phase and amplitude errors to which eigenvector methods are very sensitive [4]. The technique may also be used for on-line sensor calibration.

One of the features of eigenvector methods which makes them particularly attractive is that they may be applied to arrays of arbitary geometry. This important property is also true for the rotation method described in this paper. Unlike eigenvector methods the proposed rotational algorithm retains a multidimensional search, but the rotation effectively makes each of the searches independent. This problem is simplified in practice by the exponential structure of a single direction vector.

The rotation technique is closely related to the Varimax [5] algorithm and its application to this particular problem is described. A method that allows online estimation of phase and amplitude errors is also presented. The effectiveness of the methods described in this paper are demonstrated by a number of simulations.

EIGENVECTOR ROTATION

For an array of p sensors with arbitary geometry we assume a complex data vector, $\underline{y}$:-

$$\underline{y} = \sum_{j=1}^{k} x_j \underline{h}_j + \underline{n}$$

where the k incident signals $\{x_j\}$ have power $\{\sigma j^2\}$ and are from directions $\{\underline{h}_j\}$, $\underline{n}$ is an uncorrelated sensor noise vector having power σ^2 on each sensor. Using the sensor data, $\underline{y}$, a (p×p) covariance matrix R may be estimated, which ideally should have the form

$$R = \sigma^2 + DD^{\dagger} \tag{1}$$

where D is a (p×k) direction matrix defined as

$$D = [\sigma_1 \underline{h}_1, \sigma_2 \underline{h}_2, \ldots, \sigma_k \underline{h}_k] = [\underline{d}_1, \underline{d}_2, \ldots, \underline{d}_k]$$

R may also be represented by an eigenvector decomposition

$$R = \sigma^2 I + \Lambda\Lambda \qquad (2)$$

where Λ is a $(p\times p)$ matrix of weighted eigenvectors

$$\Lambda = \{\sqrt{(\lambda_1-\sigma^2)}\underline{\xi}_1, \sqrt{(\lambda_2-\sigma^2)}\underline{\xi}_2, \ldots \sqrt{(\lambda_k-\sigma^2)}\underline{\xi}_k\}$$

The source eigenvectors are easily obtained but their information is difficult to interpret, unlike that contained in the direction matrix. Generally the direction matrix D will be related to the eigenvector matrix Λ, by a square $(k\times k)$ matrix, B

$$D = \Lambda B \qquad (3)$$

For the case k=2 it is shown in [4] that B must take the following form

$$B = \begin{bmatrix} \cos\theta, & -\sin e^{i\delta} \\ \sin\theta e^{-i\delta}, & \cos\theta \end{bmatrix} \qquad (4)$$

Two eigenvectors may now be rotated for particular values of θ and δ. Obviously those values of θ and δ which generate the correct direction matrix D are unknown. In order to evaluate θ and δ consider minimising the quantity ϕ, defined by:-

$$\phi = \sum_{i=1}^{k} \sum_{j=1}^{p} \left(|d_{ji}|^2 - \bar{d}_i\right)^2; \quad \bar{d}_i = \frac{1}{p} \sum_{j=1}^{p} |d_{ji}|^2 \qquad (5)$$

If the $\{\underline{d}_i\}$ are vectors of weighted complex exponentials then the $|d_{ji}|^2$ will equal $\bar{d}_i$, and ϕ will be zero. Since ϕ is a positive definite function this defines a unique set of $\underline{d}_i$. The quantity in (5) is the Varimax criterion proposed by Kaiser [5], except here it is only meaningful to minimise it. No analytic solution for the minimisation of ϕ exists, but a simple iterative solution may be implemented.

In order to evaluate Λ the maximum likelihood estimate for σ^2 should be chosen [6]. This can be shown [3] to be given by:-

$$\hat{\sigma}^2 = \frac{1}{p-k} \sum_{i=k+1}^{p} \mu_i$$

where μ_i are the eigenvectors of a covariance matrix sampled from R. Having evaluated the rotated vectors $\{\underline{d}_i\}$ it is necessary to interpret them. This may be accomplished by conventional beamforming of the vectors $\{d_i\}$ or by phase unwrapping. The later technique will be explained in more detail in the next section.

Practically of course it is likely there will be more than two eigenvectors of interest. The techique can be easily extended to higher orders as is shown in [4]. When for example k=3, there are six unknown parameters, and using numerical techniques they can be easily evaluated. This is one approach, but an alternative solution may be obtained by rotating the eigenvectors two at a time in a cyclic fashion until satisfactory convergence is obtained. This is the approach which is widely used in Varimax rotation [7].

A number of assumptions are made during the design of array processors. In most situations some of these assumptions are false, and it is important that techniques should tolerate these defects. Some of the most common assumptions are listed below:

a. Sensor positions are known precisely.

b. Sensors are calibrated so there are no phase or amplitude errors across the array.

c. Sensors have an ommidirectional response.

d. Wave fronts are planar or have a simple geometric property.

High resolution eigenvector techniques tend to be very sensitive to these conditions, particularly to phase and amplitude errors. If they are not satisfied, especially when more than one signal is present the performance of such methods is impaired.

The major advantage of eigenvector rotation is the lack of prior information required. The only assumption made is that the signal directions are expected to be complex exponentials, and none of the properties in (a-d) need to be satisfied. Phase effects have precisely zero effect on the rotation criterion in (5) and amplitude effects have a small but unimportant outcome.

The vectors $\{\underline{d}_i\}$ may be found without making any assumptions about array geometry. It is possible, given strong enough signals, that these vectors could be used to identify sensor positions.

SENSOR CALIBRATION

Generally sensors are liable to be corrupted by random amplitude and phase perturbations [7]. The lack of prior information required allows the technique given in this paper to provide a simple but effective method for on-line estimation of amplitude and phase errors.

If random calibration errors are present they may be represented by modelling a perturbed covariance matrix, R_p, given by

$$R_p = PRP^{\dagger} \tag{6}$$

where $P = \text{diag}\ ((1+a_1)e^{i\psi_1}, \ \ldots\ (1+a_p)e^{i\psi_p})$,

a_j and ψ_j are amplitude and phase perturbations respectively

Consider first the case where $\{a_j\}$ are zero. It is then easily shown that the perturbed eigenvectors $\{\underline{\xi}_p\}$ are related to the eigenvectors $\{\underline{\xi}\}$ of R by the following expression

$$\underline{\xi}_p = P\underline{\xi}$$

Consequently the eigenvector matrix Λ becomes $P\Lambda$ and the direction vector matrix, D, becomes PD.

For simplicity we now consider a simple example where the array is linear and equispaced so that $\underline{h}_j$ is written as,

$$\underline{h}_j^{\dagger} = \{1, e^{-i\alpha_j}, \ldots, e^{-i(p-1)\alpha_j}\}$$

Taking natural logarithms of the k^{th} element of $P\underline{d}_j$ gives,

$$\ln(Pd_{kj}) = i(\psi_k + (k-1)\alpha_j) + \ln(C_k) \qquad (7)$$

where C_k is a real constant. If the phase term is unwrapped and plotted against k, a straight line results with slope α_j, if the ψ_k are zero. Using a least squares fit to estimate α_j and calculating the individual errors from this straight line provides simple estimates for the ψ_k.

In the presence of amplitude errors, the previous analysis no longer applies. We can now only equate the physical model of equations (1) and (6) with the eigenvector decomposition of equation (2), giving

$$PDD^{\dagger}P = \Lambda\Lambda^{\dagger}$$

Equation (3) is now replaced by the following equation

$$PD = \Lambda B \qquad (8)$$

Equation (8) now applies for phase perturbations, amplitude perturbations or both.

The real values of C_k in equation (7) are now given by

$$C_k = \sigma_i\ (1 + a_k)$$

The parameters a_k can now be estimated by calculating a sample mean, where

$$a_k = \frac{C_k - \overline{C}}{\overline{C}} \quad , \quad \overline{C} = \frac{1}{p}\sum_{k=1}^{p} C_k$$

In this manner both phase and amplitude errors may be estimated. The techniques do not exactly cancel these errors, but they are substantially reduced. For phase errors it is possible to cancel all except an irrelevant constant term, and a gradient term which causes a small offset in terms of wavenumber (causing or bearing error).

The various measurements made in factor analysis may have widely different scaling values. These variations especially if large can degrade the performance of the Varimax technique and Kaiser proposed a modification to the basic method using the concept of communality to account for this phenomena.

In the absence of any amplitude errors, we may define communalities, Z_j, which are equal for all values of j:-

$$Z_j^2 = \sum_{i=1}^{k} |d_{ji}|^2 \quad , \quad j = 1,\ldots,p$$

When there are amplitude errors $\{a_j\}$ present, the communalities are unequal and given by,

$$z_j^2 = (1 + a_j)^2 \sum_{i=1}^{k} \sigma_i^2 \ , \qquad j = 1,\ldots,p$$

However, the communalities are unchanged by rotation [7] and so

$$z_j^2 = \sum_{i=1}^{k} (\lambda_i - \sigma^2)|\xi_{ji}|^2 \qquad , \ j = 1,\ldots,p \tag{9}$$

The values d_{ji} are now normalised by dividing with z_j calculated from equation (9). This removes the bias in the minimum of ϕ, which is caused by having unequal communalities. The amplitude errors, a_j, are normally much less than 1, hence the effect is only a small one. Hence it is not usually necessary to use this modified procedure.

RESULTS

To illustrate the effectiveness of the technique, consider a ten element equispaced line array with omnidirectional sensors, at a frequency corresponding to an element separation of $\lambda/5$. Two plane wave signals are assumed to be present with additive uncorrelated sensor noise. The signals have wavenumbers of 0.12 and 0.16, relative to a Nyquist wavenumber of 0.5. Although the array is equispaced no Toeplitz assumptions have been made and so the result should also by typical for arrays with similar geometries. Each sampled data set is assumed to have been obtained from 100 sets of random data.

Figures 1-3 examine the resolution properties of the present method and compare them to the MUSIC and KT technique. Each signal has a SNR of -3dB, and there are no phase or amplitude errors present. The 10 sampled MUSIC spectra shown in Figure 1 are not able to resolve the two signals. Figure 2, for the same 10 samples shows the KT spectra. It is clear that the two signals have been resolved, although in some cases the maxima may be difficult to locate. Some of the spectra present rather large spurious peaks, which is clearly undesirable.

One hundred sampled wavenumber estimates after rotation of the signal eigenvectors, phase unwrapping and linear least squares are given in Figure 3. It is clear from these discrete estimates that two signals are present and in approximately the correct directions.

If there are phase and amplitude errors present, signal direction estimation is obviously more difficult, as presented in Figures 4-6. Each signal is again assumed to have an SNR of -3dB, with phase and amplitude errors, created by using Gaussian random variables with variance of 0.1. It is clearly impossible to discern two signals from the 10 sampled MUSIC spectra shown in Figure 4. The performance of the 10 KT spectra shown in Figure 5 has also been degraded by the introduction of phase and amplitude errors. It is now difficult to infer anything meaningful from the information presented in Figure 5. Figure 6 gives the 100 wavenumber estimates after rotation phase unwrapping and linear least squares. The results as expected are worse than those shown in Figure 3, but it is still possible to see two distinct signals. For the results shown in Figures 4-6, each result has been obtained from the identical set of phase and amplitude errors. This is the cause of the slight wavenumber bias which can be seen in Figure 6. It is easily seen that the rotation method is insensitive to phase and amplitude errors, compared to the two eigenvector techniques.

To demonstrate the effect of phase and amplitude errors on systems performance, we consider two examples using an exact covariance matrix. Gaussian random variables with variance 0.1 are used to create phase only and amplitude only perturbations. Two signals are present, having wavenumbers 0.12 and 0.16 each with an SNR of 10dB. Figure 7 shows the unwrapped phase after eigenvector rotation for the case when only phase errors are present. A least squares fit is also shown for both direction vectors. The two gradients give estimates for the two signal directions as 0.125 and 0.165. When there are only amplitude errors, the results are given in Figure 8. It is clear that the amplitude errors have no affect on wavenumber estimates, and the gradients given perfect estimates for the signal directions.

The individual errors from the least squares fit in Figure 7 may be used to obtain estimates for the phase errors. These errors are given in Table 1, together with the true phase errors. Because of the small wavenumber bias, the phase errors are not cancelled exactly, but it is seen that the errors lie on a straight line. This is caused by the incorrect slopes of the two curves. The same information for the amplitude only case is presented in Table 2.

CONCULSIONS

An eigenvector rotation method has been presented which uses source rather than noise eigenvectors. It has been demonstrated that this method enables good resolution to be achieved in situations which are totally unsuitable for eigenvector techniques such as the MUSIC and KT algorithms. The method may also be used to simultaneously estimate sensor calibration errors.

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FIG. 1 MUSIC spectra

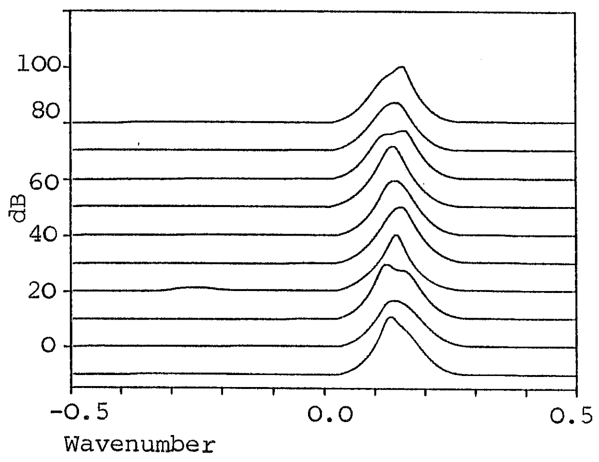

FIG. 4 MUSIC spectra with errors

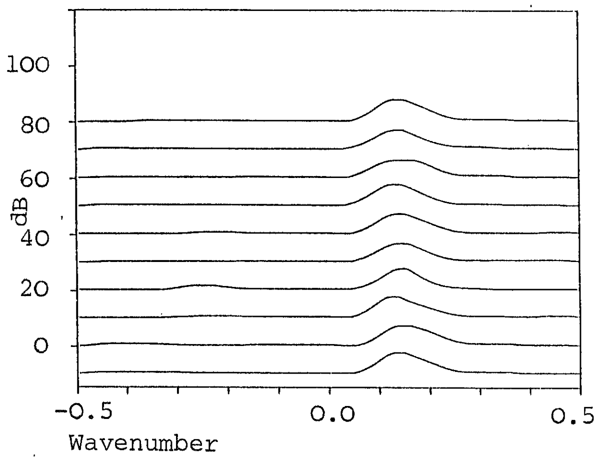

FIG. 2 KT spectra

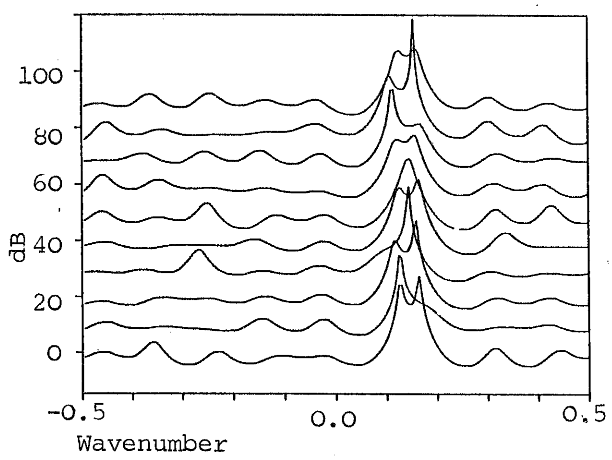

FIG. 5 KT spectra with errors

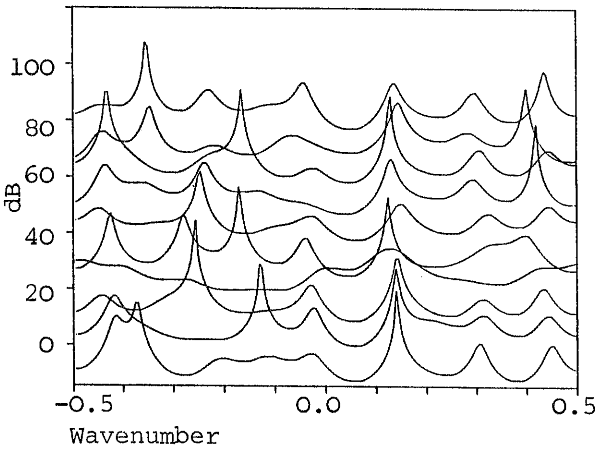

FIG. 3 Rotated estimates

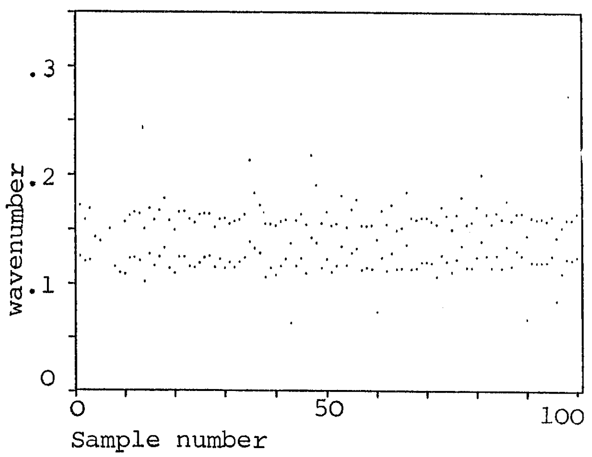

FIG. 6 Rotated estimates with errors

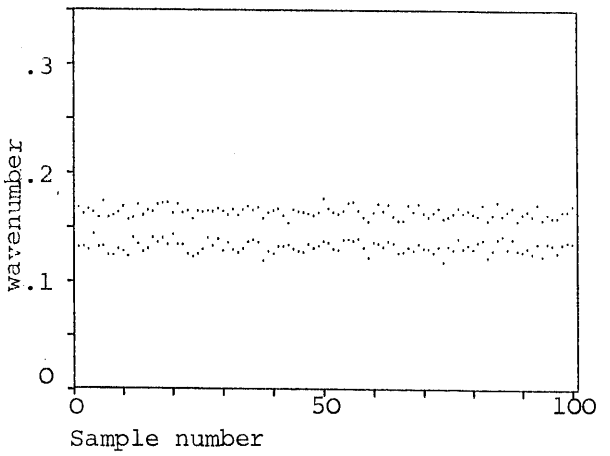

FIG. 7 Phase errors only

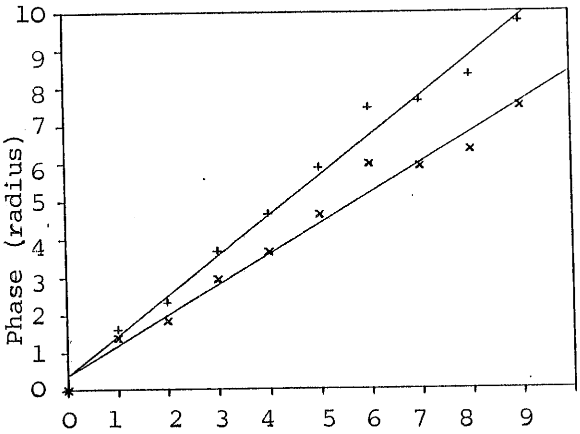

TABLE 1 Phase errors

Actual	Apparent	Ac-Ap
0.758	0.391	0.367
0.144	-0.174	0.318
0.442	0.173	0.269
0.092	-0.127	0.219
0.132	-0.038	0.170
-0.095	-0.215	0.120
-0.682	-0.752	0.070
0.140	0.119	0.021
0.460	0.488	-0.028
0.037	0.134	-0.077

FIG. 8 Amplitude errors only

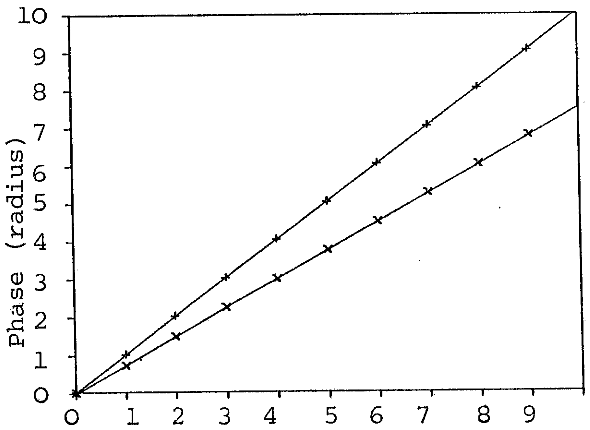

TABLE 2 Amplitude errors

Actual	Apparent	Ac-Ap
0.710	0.495	0.215
-0.035	-0.170	0.135
0.130	-0.049	0.152
0.407	0.217	0.190
0.595	0.385	0.210
0.053	-0.091	0.144
0.176	0.017	0.159
-0.258	-0.363	0.105
-0.017	-0.152	0.135
-0.170	-0.289	0.119

Ac-Ap = Actual - Apparent.