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# THE ANALYSIS OF THE INFLUENCE OF PLATFORM INSTABILITIES ON A CORRELATION LOG.

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## 1. INTRODUCTION.

Acoustic Correlation logs offer an alternative method of bottom referenced velocity measurement to Doppler Logs. The effects of platform instabilities on the latter have been well documented. Scattering theories developed by Smith and Atkins [1,2] are developed to predict the performance of a simple correlation log subjected to such conditions, paying particular attention to the effects of pitch and heave.

## 2. QUALITATIVE ANALYSIS OF THE EFFECTS OF PITCH AND HEAVE.

The main aim is to analyse the theoretical performance of a correlation log which is constantly pitched at an angle p. In order to achieve this, a quick summary of the principles of the operation of a simple correlation log follows.

The log consists of a transmitting transducer, T, and two receiving transducers R1 and R2, as shown in figure 1. The output from the transmitting transducer produces a random pressure field in the vicinity of the vessel due to scattering from the rough sea-bed. When the transmitting transducer moves forward with velocity $\underline{v}$, the pressure field remains constant in form, moving backwards with velocity $-\underline{v}$. The relative velocity of the pressure field with respect to the vessel is hence $-2\underline{v}$.

The delayed signal on R1 is correlated with the signal on R2 with respect to time delay to give a temporal correlation function. As the pressure field moves with a velocity $-2\underline{v}$ relative to the receiving transducers, which are separated by a distance $\underline{d}$, the maximum value of this temporal correlation function will be gained when the time delay is equal to $\underline{d}/2\underline{v}$ (this delay will be called the correlation time). Hence using this equation, the velocity may be estimated by measuring the correlation time, t.

$$\underline{v} = \underline{d} \,/\, 2t \qquad (1)$$

Another way of looking at this is to consider the spatial correlation function of the pressure field. This function is a measure of how the pressures at two points are correlated with respect to the spatial separation between the points. At time t=0, the spatial correlation function with respect to the reference signal on R1 (taken at t=0) will have a maximum value at position R1. Now, because the form of the pressure field remains constant, moving backwards with a velocity $-2\underline{v}$, the peak of the spatial correlation function at time t will be at position ($\underline{R1}$ - $2\underline{v}t$). The form of the function will remain constant apart from the displacement -2$\underline{v}$t. At the correlation time, $\underline{d}/2\underline{v}$, the peak of the correlation function will lie exactly on R2. Hence, at this time, the value of the correlation function at R2 will be a maximum.

When the correlation log is pitched, as in figure 2, $\underline{v}$ and $\underline{d}$ are not aligned, and so the peak of the spatial correlation function does not pass directly through R2. In order to predict the performance of the log under these conditions, it is necessary to compute the form of the temporal correlation function above. This is acheived by computing the values of the spatial correlation function of the pressure field in the vicinity of the peak, and mapping these back to the temporal function, with respect to the vessel velocity, $\underline{v}$.

The form of spatial correlation functions of scattered pressure fields can be computed using the Kirchoff-Helmholtz integral. It is found from this analysis, which is dealt with in more detail later, that the function in the plane perpendicular to the direction of wave propagation has a width of the same order as the size of the transmitting transducer. In the plane parallel to this direction, the width of the function is considerably greater. The easiest way to visualise this function is to consider it in contour form. The contours will be elliptical, with the major axis of the ellipses being in the mean direction of wave propagation. This is shown in figure 3. This property of the correlation function was demonstrated using optics to simulate acoustical scattering [3].

In order to see how the direction of propagation will affect the accuracy of the pitched correlation log, two cases are investigated qualitatively. The first of these is when the scattered sound has a mean direction of return in the vertical direction. The second is when this direction is at an angle off the vertical.

Figures 4.1 show the correlation surface where the direction of returns is normal to the sea-bed. In figure 4.1.a, the spatial correlation function is shown at t=0. As time passes, the correlation surface moves backwards relative to the log. In this case, it can be safely predicted that the maximum in correlation at R2 will occur at time t, when the peak of the correlation function is at its closest point of approach to R2. This prediction can be made because the major axis of the elliptical contours passes through the second transducer when the peak is at this point X. This is shown in figure 4.1.b. The corresponding correlation time is hence reduced by a factor (R1 N / d) (= cos(p) ), leading to an over-estimate in velocity, by a factor 1/cos(p).

For the case illustrated in figures 4.2, the direction of propagation is not normal to the sea-bed. This could be caused by two factors. Firstly, the effect of pitch will be to change the transducer directivity function. Secondly, a sloping sea-bed will alter the direction of mean return. The contours of the correlation function are shown for t=0 in figure 4.2.a. Figure 4.2.b shows the position of the correlation surface at the time of closest approach of the peak to R2. As can be seen, the major axis of the elliptical contours does not pass through the second transducer. As time passes, the surface moves further along the x axis. The value of the correlation function at R2 will continue to increase, and reach a peak at a later time, as shown in figure 4.2.c.

To summarise, the effect of changing the mean direction of propagation will be to change the correlation time, and hence a different velocity estimation will be gained from the log.

## 3. ANALYSIS OF THE CORRELATION FUNCTIONS.

In order to determine the values of the temporal correlation function for a pitched correlation log, the spatial correlation function of the pressure field is computed.

Figure 5 shows the positions of the two receiving transducers, between which values of the spatial correlation function of the field must be computed to obtain the values of the temporal correlation function. These positions are a function of the time delay, t.

The method used to compute the spatial correlation function follows that used by Smith and Atkins [1,2]. In this method, the Kirchoff-Helmholtz scattering integral is developed to compute the scattered pressure at two points separated horizontally by a small distance g. A spatial correlation of the two pressures is then formed.

In the derivation here, the correlation function is analysed for receiver separations in both the horizontal and vertical directions. Also a correction due to the pitched transducer directivity function is made.

As illustrated in figure 5, the equivalent positions of the two receivers R1 and R2, as a function of time, will be at the points (-g/2,s-f/2) and (g/2, s+f/2) respectively, in the x-z co-ordinate system. In this figure, s is the z-co-ordinate of the transmitting transducer, g is given by (d cosp - 2vt), and f by (d sinp). The origin is in the plane of the scattering surface, directly below the transmitting transducer. The scattering surface is denoted by S, comprising of elemental areas dS.

The surface is defined by a local height function h(x,y). The form this function is briefly described in section 4.

From the Kirchoff-Helmholtz scattering integral, the pressures $P_{1,2}$ will be:

$$p_{1,2} = \frac{p_0\Gamma}{4\pi}\int_s \frac{\partial}{\partial n}\left(\frac{D_t\exp(-jk(r+r_{1,2}))}{rr_1}\right)dS \qquad (2)$$

Where: $\Gamma$ is the reflection co-efficient at the boundary. As an approximation this is assumed to be a constant.

$p_0$ is the pressure related to the power of the transmitting transducer.

$k$ is the wavenumber of the transmitted radiation.

$r_1$ and $r_2$ are the distances from the receivers to the scattering surface as shown in figure 5.

Making the same approximations as [1], the scattering integral reduces to:

$$p_{1,2} = \frac{-jk\Gamma p_0}{2\pi}\int_s \frac{D\exp(-jkA_{1,2})}{r_0^2}\left(\frac{\partial h}{\partial y}\sin\phi\cos\theta + \frac{\partial h}{\partial y}\sin\phi\sin\theta + \cos\phi\right)dx\,dy \qquad (3.a)$$

$A_{1,2}$ is the propagation distance from T to dS to R1/R2. From the geometry in figure 5:

$$A_{1,2} = 2r_0 - 2h\cos\phi \pm \frac{g}{2}\sin\phi\cos\theta \pm \frac{f}{2}\cos\phi \qquad (3.b)$$

In comparison with the expression for $A_{1,2}$ in [1], the expression here contains the additional phase term $f\cos\phi/2$, which accounts for the vertical separation of the receiving transducers.

A spatial correlation $< p_1 p_2^* >(g,f)$ is now formed. The resulting integral is then simplified to give the final result:

$$<p_1p_2^*>(g,f) \propto \int_0^{\pi/2} R(\phi)\left[\int_0^{2\pi} D^2(\phi,\theta)\exp(jk(g\sin\phi\cos\theta + f\cos\phi))d\theta\right]d\phi \qquad (4.a)$$

$$R(\phi) = \chi_H(\phi)\cos^3\phi\sin\phi + \chi_{HS}\cos\phi\sin^3\phi \qquad (4.b)$$

When the transmitting transducer is pitched, the directivity function, originally defined by the $\phi$ co-ordinate alone, becomes more complicated. Using trigonometry, this function is transformed from a function about an axis normal to the plane of the transducer to a function of co-ordinates $\theta, \phi$ and p:

$$D(\theta,\phi,p) = \frac{J1(kb\sin\Psi)}{(kb\sin\Psi)} \qquad (4.c)$$

$$\Psi = \cos^{-1}(\cos p\cos\phi + \sin\phi\sin p\cos\theta) \qquad (4.d)$$

The function $R(\phi)$, from now on called the response function, can be considered as a spectral filter multiplied by a geometrical weighting factor. The weighting factor takes into account the area of surface defined in the region $\phi \rightarrow \phi + d\phi$ as $\phi$ varies, and the increase in path length as $\phi$ increases. $\chi_H$ and $\chi_{HS}$ are Hankel transforms of the bottom height and slope related statistics [1,2].

The spectral filter characteristics are determined by the statistics of the scattering surface, and represent the effects of the scattering process [1,2].

## 4. Numerical Computation of the Correlation Functions.

The task of computing the results was split into two sections. A "MATHCAD" program was used to compute the response function. The output from this was then used to compute the integral in equation (4.b) using a program written in C.

The response function was computed for different surface parameters. The surfaces were assumed, as in [1], to have height functions h(x,y). These functions were assumed to be Gaussian with average value 0, and R.M.S value h. The functions were also assumed to have Gaussian Spatial Correlation functions of width a, and associated slope functions related to the gradient of h(x,y) with R.M.S. values equal to s.

The function was computed for 200 values of $\phi$, over the range $0 \rightarrow \pi/2$. Three curves of $R(\phi)$ are shown in figure 6 for different surface parameters. As expected, the widths of the curves increase as the ratio h/a increases. This corresponds to the scattering becoming more omnidirectional as surface roughness increases.

Figure 7 shows the computed spatial correlation functions for a correlation log without pitch. The functions are shown for two different transmitting transducers of different radii. For the computation here, the surface parameters used were for those for surface 2, figure 6. Because the Response function is broader than the directivity functions of the two transmitting transducers, the computed widths of the correlation functions are equal to the transducer dimensions.

The effect of scattering from a surface with a narrower response function is shown with the comparison of figures 7 and 8. In figure 8, the response function used is that for surface 1 (see figure 6). Here, this surface has the effect of broadening the correlation function for the 1cm transducer. This broadening was predicted in [1].

Figure 9 shows a contour plot of the computed correlation function in 2 dimensions. The surface used here was surface 2. The contours of the function are shown to be elliptical as predicted. The significance of this to the operation of a correlation log is that the accuracy will not be impaired in the presence of vertical motions. This was also predicted in [4].

Finally, curve 1 on figure 11 shows the spatial correlation function between R1 and a point along the line XX′ ( see figure 10), as a function of the position of this point. Curve 2 shows the same function computed along the line YY′. It is seen that along YY′, there is a reduction in the peak value of the correlation function compared to along XX′. Also the position of the peak is not at the closest point of approach, N, but at point N′. The positions of the x-co-ordinates, N and N′ in relation to the correlation log are shown in figures 4.2.

This computed result here clearly supports the qualitative argument given in section 2. That is, when operating under conditions of constant pitch, the amount by which the velocity reading of a correlation log is affected will be dependent on the mean direction of acoustic returns from the sea-bed.

## 5. ACKNOWLEDGEMENTS.

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## 6. REFERENCES.

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# 6. Figures.

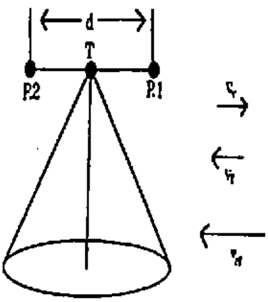



Figure 1. Diagram showing operation of a correlation log.
$V_v$ = vessel velocity.
$V_f$ = "velocity" of pressure field.
$V_{rf}$= relative "velocity" of pressure
field relative to vessel.

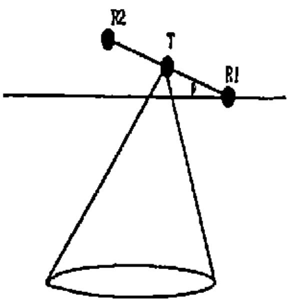



Figure 2. Diagram showing pitched correlation log.
Peak of correlation function starts at position R1, and moves along x axis. It will not pass through R2.

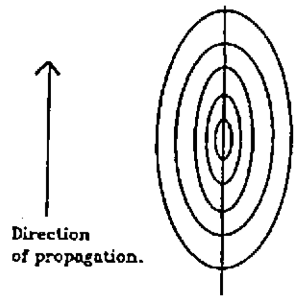



Figure 3. Contour plot of Spatial correlation function of pressure field shown with respect to the direction of wave propagation.

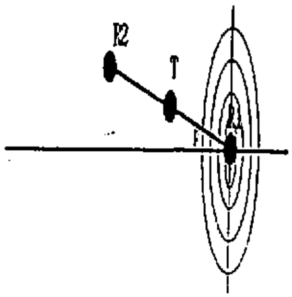



Figure 4.1.a. Spatial Correlation Function for pitched correlation log at time t=0. The main direction of the return signal is assumed to be in vertical direction.

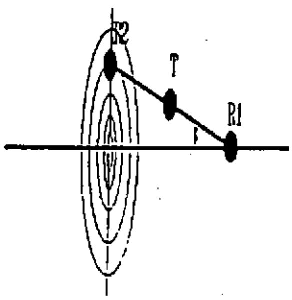



Figure 4.1.b. Position of correlation function at t = d cosp / 2v.

CORRELATION LOG

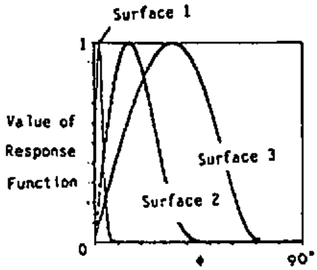



Figure 6. Graph of the function $R(\phi)$ shown for three scattering surfaces. Surface 1 has parameters, a=0.5m, h=0.1m, s=0.1; surface 2, a=1.0m, h=0.05m; surface 3 a=0.5m, h=0.3m, s=0.3.

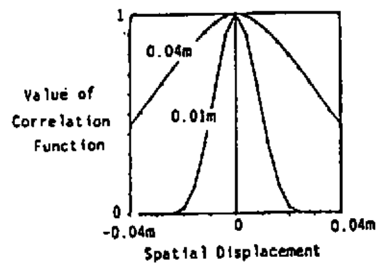



Figure 7. Graph showing the computed spatial correlation function along x-axis for transmitting transducers of radius 1cm and 4cm, using surface 2 parameters (see fig 6).

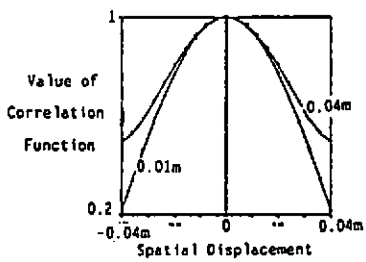



Figure 8. Effect of changing the surface parameters on the spatial correlation function of the pressure field.

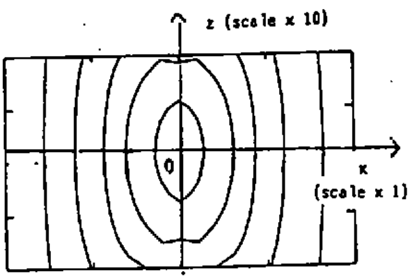



Figure 9. Contour plot of the Spatial Correlation Function in two dimensions.

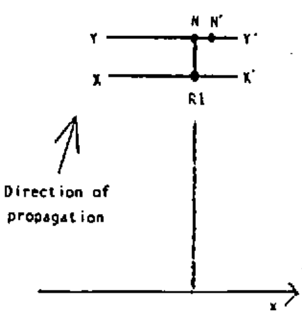



Figure 10. Figure showing the axes along which the curves in figure 11 are computed.

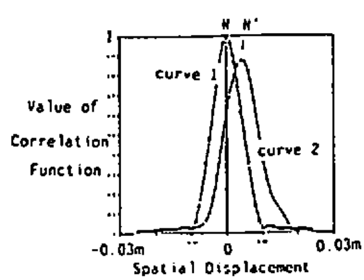
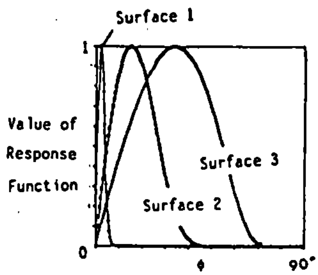



Figure 11. The effect of pitch on the Spatial Correlation Function.

CORRELATION LOG

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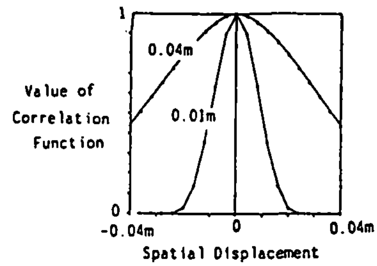



Figure 7. Graph showing the computed spatial correlation function along x-axis for transmitting transducers of radius 1cm and 4cm, using surface 2 parameters (see fig 6).

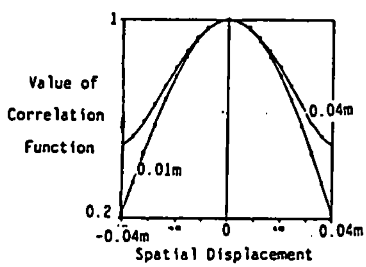
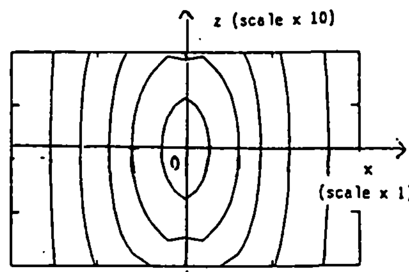



Figure 8. Effect of changing the surface parameters on the spatial correlation function of the pressure field.

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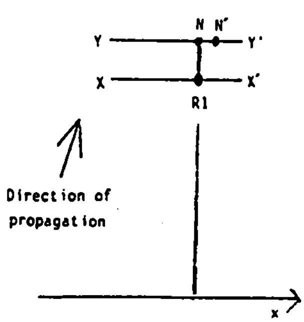


Figure 10. Figure showing the axes along which the curves in figure 11 are computed.

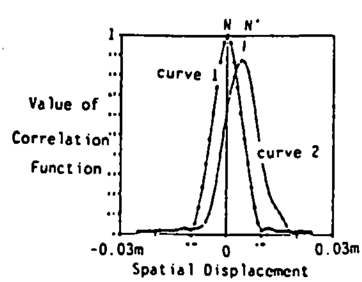


Figure 11. The effect of pitch on the Spatial Correlation Function.