CHAOTIC BEHAVIOUR OF THE EDGETONE

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INTRODUCTION

The generation of sound by the interaction between a rigid edge in a stationary fluid and an impinging plane jet has been given considerable attention for over one hundred years.[1] However, the instability that controls the oscillation has not yet been completely modelled so as to predict the acoustic signal emitted.[2][3] The resulting edgetones are characterized by discrete frequencies which, when coupled to a resonant cavity, make the acoustic source of whistles, organ pipes and some wind instruments.

In the absence of a resonator the signal generated follows a regular sequence of states along several stages. The aim of this work was to examine those signals through non-conventional methods suitable for qualitative analysis of periodicity and harmonic generation. In particular, a criterion for the distinction between quasi-periodical and chaotic signals from experimental data was pursued.

SIGNAL DESCRIPTION

The frequency of the edgetone is controlled by the jet velocity u and the edge stand-off distance h. For a subsonic jet-edge system in air Brown [4] proposes an empirical formula

$$f = 0.466\, j \left(u - 40 \right) \left(1/h - 0.07 \right)$$

where u is in cm/s and h in cm. The parameter j takes the values 1.0, 2.3, 3.8 and 5.4 for the four stages observed by Brown.

Holger et al.[3] derived a dependence for f as $u/h^{3/2}$ within each stage but the transitions between stages are not explained.

In our experiment the signals were generated by air supplied through a channel measuring 1.7 x 50.0 mm and a 30 degree coplanar wedge, with pressure adjustable up to 4 mbar. Strong tones are audible for distances from 5 mm up to several cm. The transitions between stages occur as hysteretic jumps as can be observed in fig.1 which shows our results for a pressure of 1.8 mbar.

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Free field conditions were provided in order to avoid acoustical feedback which bolsters some states. The sound pressure signal was picked up by a 1/2" condenser microphone and digitized at a 20,000 samples per second rate into time series of 12,500 samples.

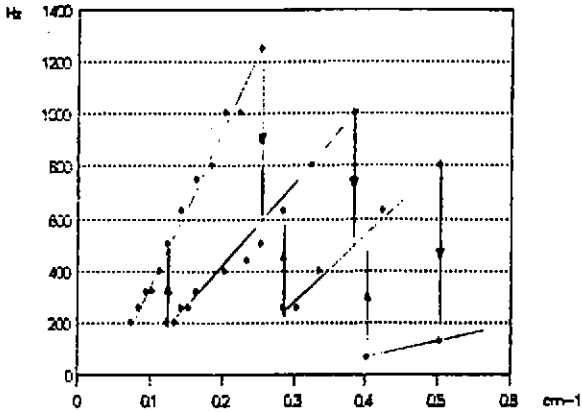

Fig.1 Edgetone frequency vs inverse distance

At most states the signal FFT shows a strong fundamental with a 3f and other minor harmonics. At some states, however, particularly near stage transitions, the signal exhibits a number of harmonics which are less stable in time.

Qualitative changes of the signal occur suddenly for very small variations of the wedge distance, and sometimes it spontaneously jumps back and forth from time to time.

SIGNAL ANALYSIS

The signals were analyzed through the phase space constructed from the pressure time series, its first and its second derivative.[5] The Poincare sections were also obtained for x=0 considering trajectories crossing the plane in both directions. Sections taken at locked phase were not attempted since in this case there is no forcing signal.

A sequence of states is presented in the following figures, from a first state of stable periodic motion to a last state of quasi-random motion, after undergoing several bifurcations. Phase portraits (left) of intermediate states exhibit closed orbits with period doubling and Poincare sections (right) stretching from discrete segments to a shape that suggests a torus. Details of a possible fractal substructure of the sections are obscured by noise.

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Fig.2 Phase portraits, Poincare sections and waveform before bifurcation

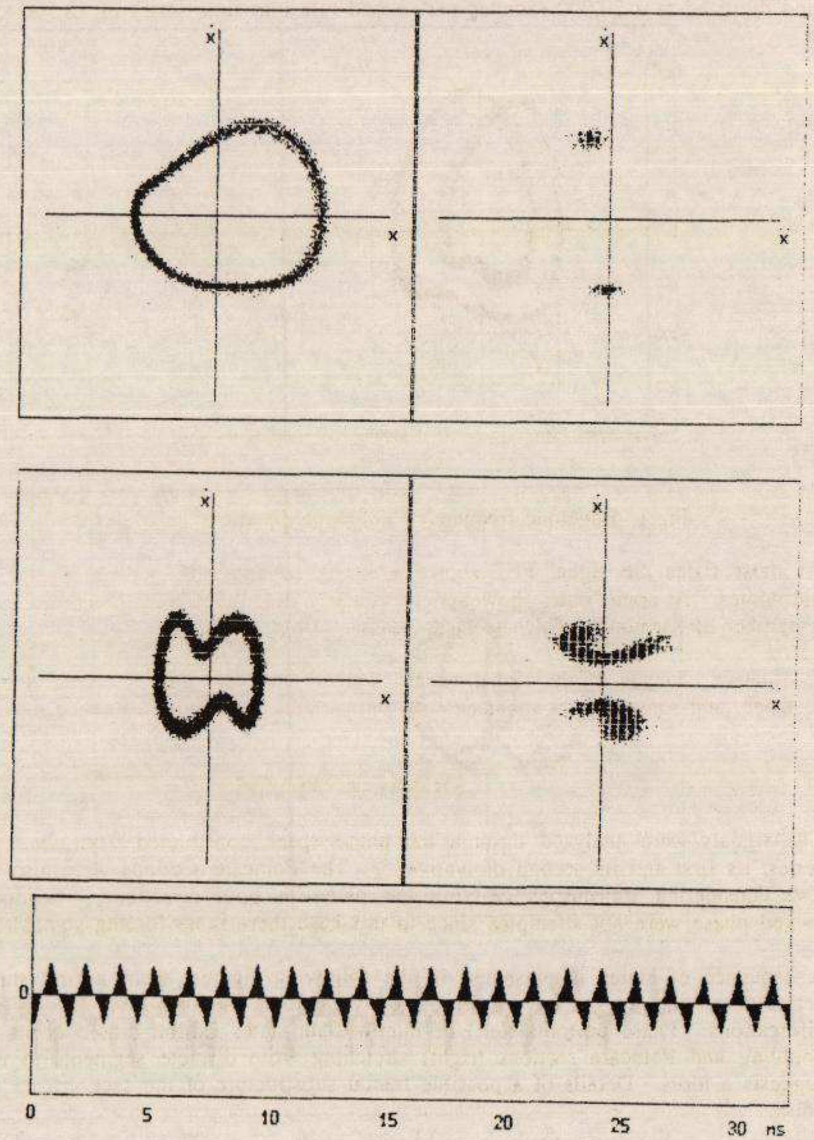

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Fig.3 Phase portraits, Poincare sections and waveform after bifurcations

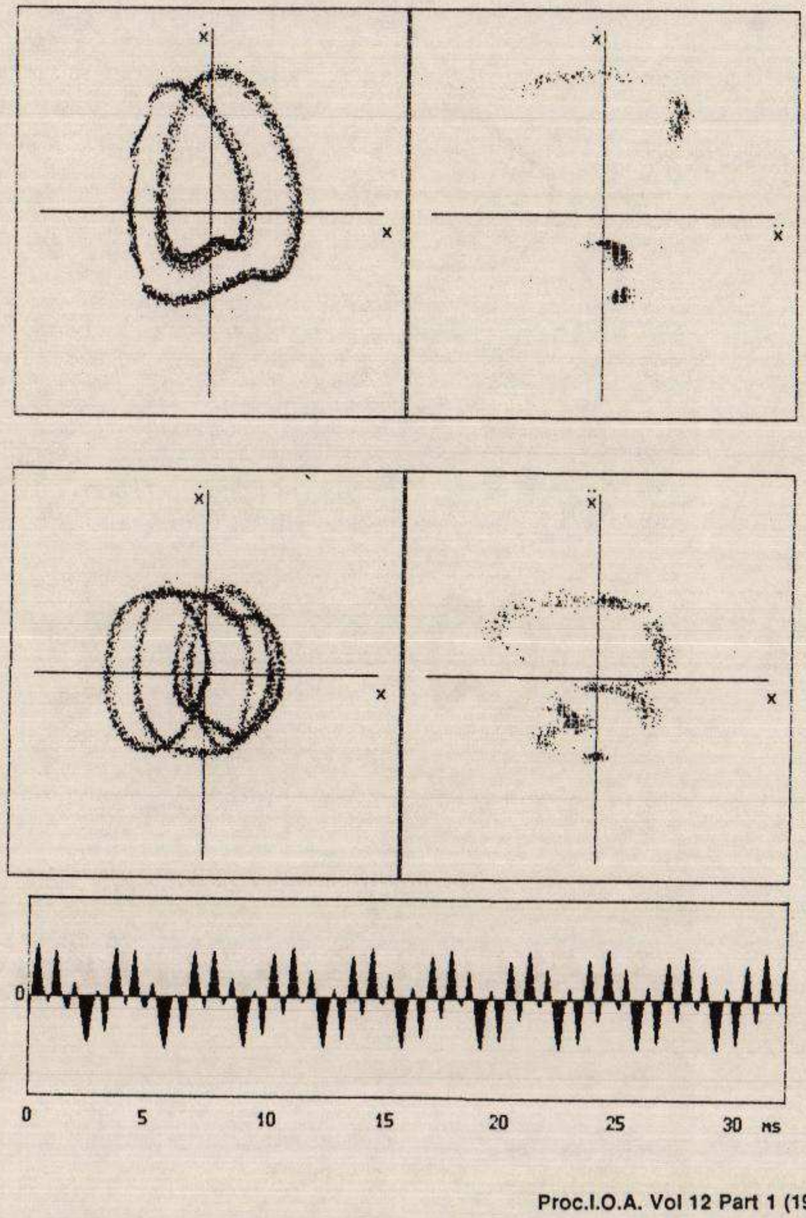

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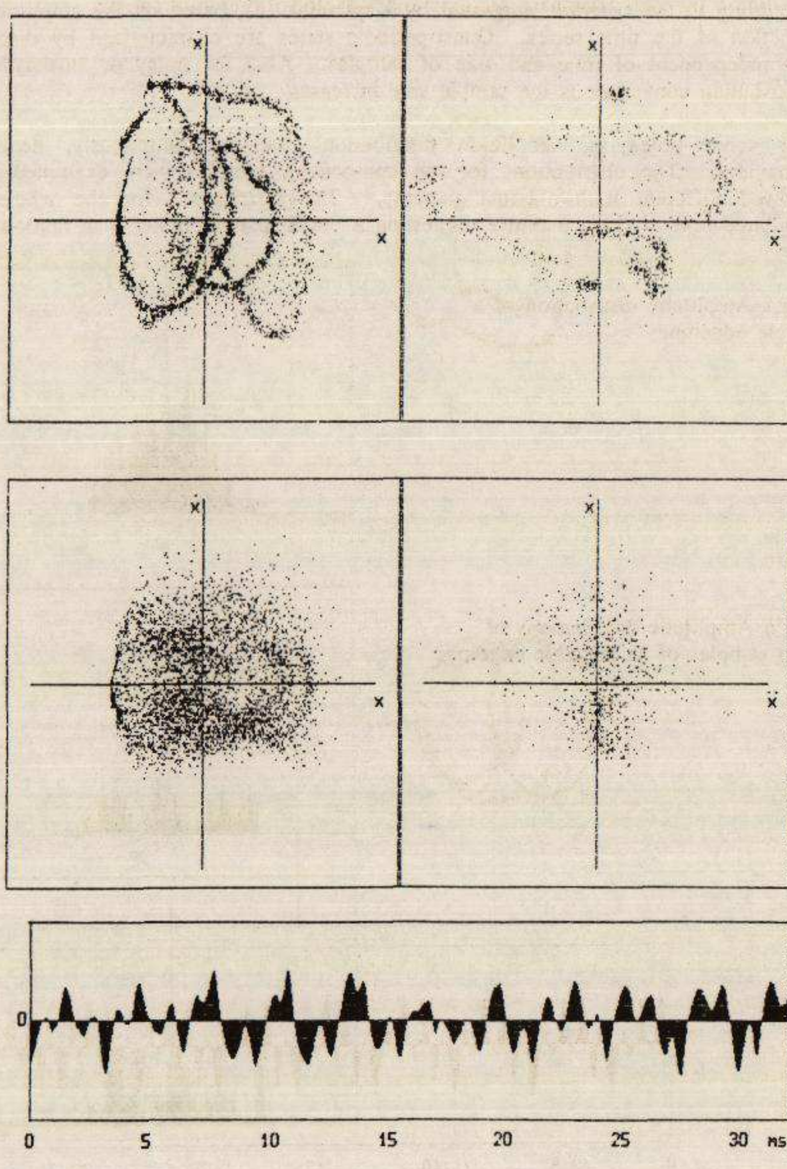
Fig.4 Phase portraits, Poincare sections and waveform for random edgetone



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The distinction between quasi-periodic states and chaotic states was also investigated according to the criterion suggested by Kapitaniak [6], based on the amplitude probability function of the time series. Quasi-periodic states are characterized by distributions that are independent of time and size of samples. Also for noisy or stochastic signals the distribution converges as the sample size increases.

For some states the amplitude distribution changes continuously, denoting chaotic behaviour. The distributions for ten consecutive samples were examined and they all showed different multi-maxima patterns. The distribution for the whole series also exhibited several sharp maxima suggesting a fractal pattern rather than random changes.

Fig.5 Amplitude distribution of a stable edgetone

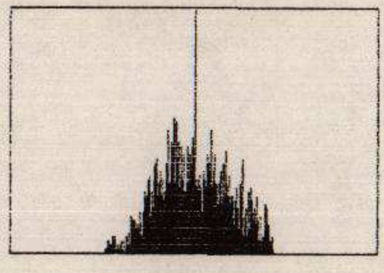

Fig.6 Amplitude distributions of two samples of an unstable edgetone

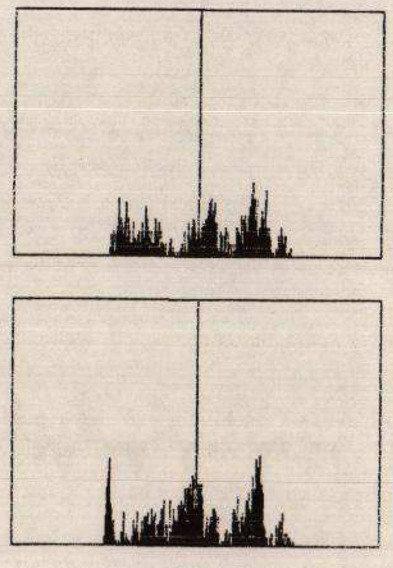

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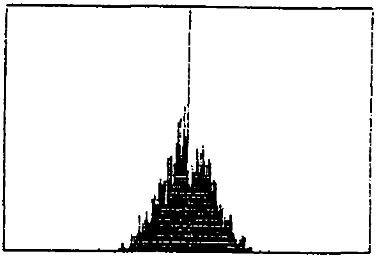
Fig.7 Amplitude distribution of a random edgetone



CONCLUSIONS

- A simple experiment is used to generate acoustic signals at different states displaying periodic, chaotic and random features.

- The analysis of the phase portrait constructed from the waveform time series reveal a sequence of bifurcations.

- The Poincare sections suggest the existence of chaotic states close to the transitions.

- The amplitude probability distribution for these states exhibit some sharp peaks which are dependent on the timing of the sample.

- Extending the length of the sample does not overrun the peaks but creates another pattern of peaks.

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