Proceedings of the Institute of Acoustics

IDENTIFICATION OF STRANGE ATTRACTORS IN ACOUSTIC SIGNALS

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1. INTRODUCTION

The analysis of acoustic signals often involves some assumptions on the underlying sources, usually related to mechanical systems of unknown characteristics. A meaningful description of the source dynamics could be gained by observing just one scalar magnitude and studying both its amplitude probability distribution, Kapitaniak [1] and its time derivatives, Ruelle [2]. In this case the observed magnitude is the sound pressure.

It is also known that the transition to turbulence in fluids can give rise to complex sounds which resemble the response of nonlinear oscillators but could not be easily understood through frequency or statistical analysis. Signals of this type are generated by fluid flows interacting with solids, giving rise to self sustained oscillations in both the fluid and the solids. Under some conditions, the phase portraits of these signals show bifurcations and attractors that could be described as strange or chaotic, Collados [3].

The signals from chaotic states, however, involve broadband noise which could be confused with experimental noise, hence the need for further numerical analysis. From the amplitude probability distribution a quasi-periodic signal could be identified. From the reconstructed phase space some characteristic exponents and the approximate number of embedded dimensions could be obtained.

2. EXPERIMENTAL SIGNALS

A series of signals were obtained from an air edgetone under free field conditions. For increasing jet velocity and decreasing edge distance these tones exhibit several frequency doubling transitions followed by chaotic behaviour near the last transition.

Waveforms at four states are shown in Fig.1:

(a) just before transition
(b) just after transition
(c) chaotic state
(d) quasi-random state

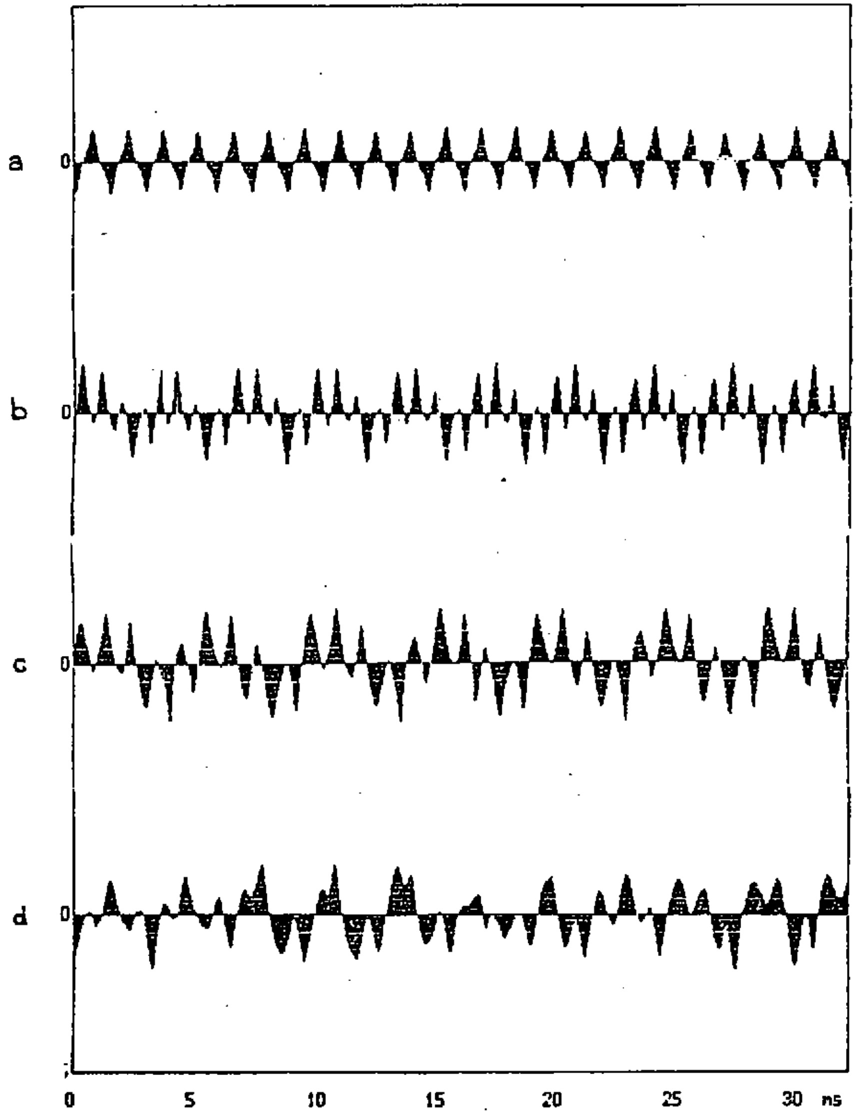

Fig.1 Typical waveforms at states (a), (b), (c) and (d).

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Signals of type (b) are quasi-periodic with waveforms changing continuously due to harmonics close to but not exact multiples of the fundamental frequency. Their spectra are stable, but their main components exhibit a slowly shifting phase.

Signals of type (c) will be shown to be chaotic, although their apparent characteristics are similar to those of type (b) in the presence of noise.

A diagram of a typical transition in Fig.2 illustrates the occurrence of different types of signals for an edgetone with Reynolds number around 5000.

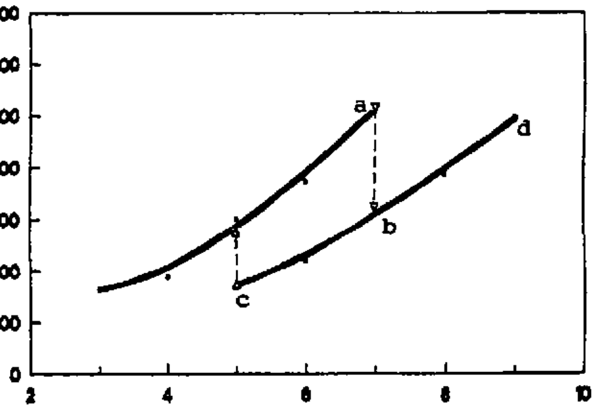

Fig.2 Schematic diagram of tone frequency vs. air jet velocity illustrating transitions and states.

3. AMPLITUDE PROBABILITY ANALYSIS

The shape of the amplitude probability distribution provides the first criteria for identifying quasi-periodic and chaotic signals, since frequency doubling bifurcations yield multiple maxima in the probability density function, Kapitaniak [1].

Signals of type (b) exhibit a function with two to four maxima, stable in time for each particular state sampled during 200 ms. Functions from signals type (c), however, present multiple maxima but they are not stable in time, so their shape would vary from sample to sample. Furthermore, the cumulative distribution for a series of samples of up to 3000 ms would not converge to a regular distribution, but to another multi-maxima function.

This fact was analyzed by plotting the probability density P(0,t) for x = 0 at time t vs. t+N, for both types of signals, as shown in Fig.3. The patterns of quasi-periodic and chaotic character are clearly identified.

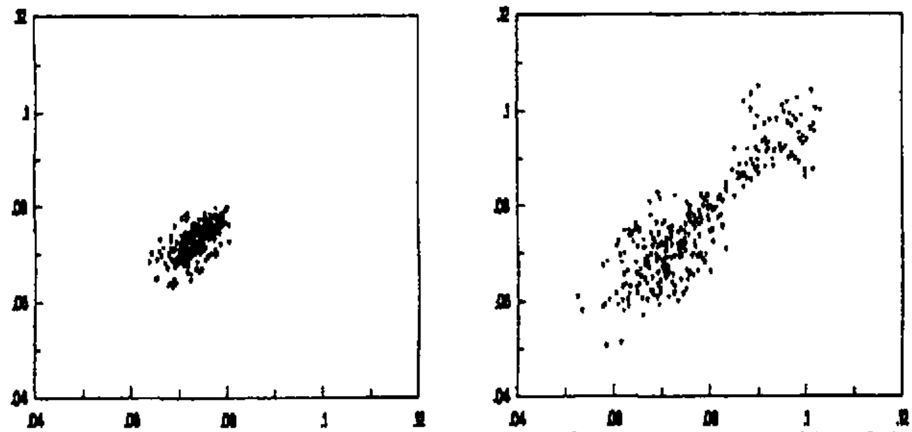

Fig.3 Probability density P(0,t) vs. P(0,t+N) for signals types (b) and (c).

4. PHASE SPACE ANALYSIS

The signals were first represented by the two-dimensional projection of their trajectories in the reconstructed phase space.

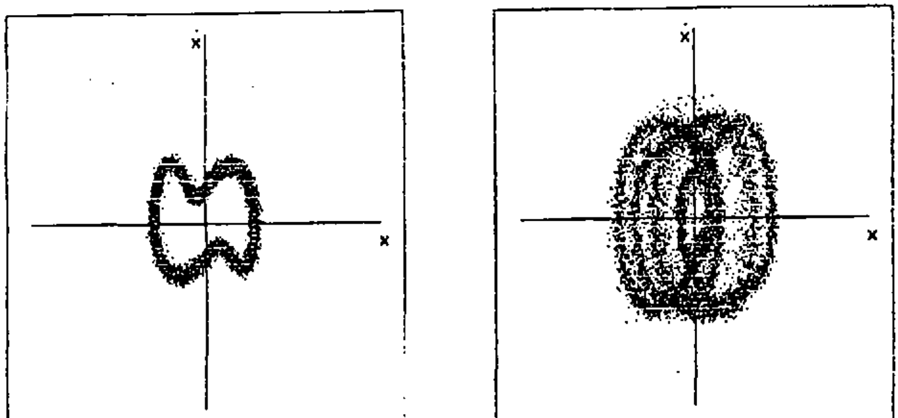

Fig.4 Trajectories from 4000 points (200 ms) of signals types (a) and (b).

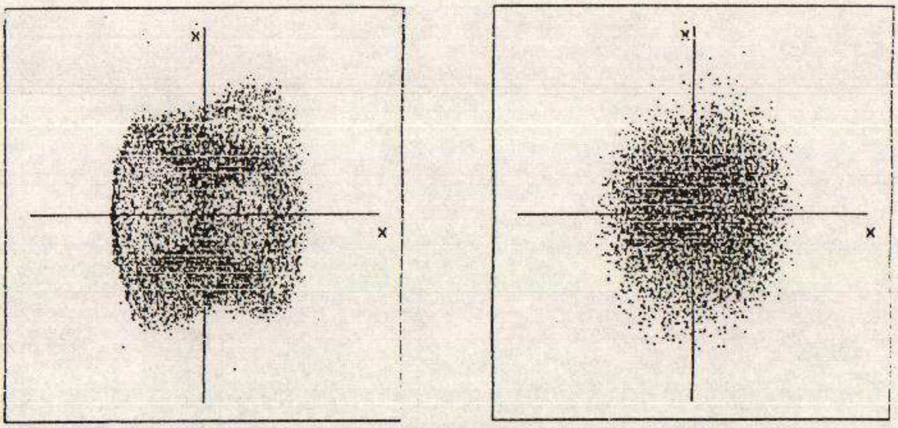

Fig.5 Trajectories from 4000 points (200 ms) of signals types (c) and (d).

Poincare sections characteristic of each type of signal were also generated from the time derivatives at zero-crossing points. The beginnings of the transition are shown on Fig.6 (left) where the attractor is expanding from its equilibrium state. After transition (Fig. 6 right) the attractor represents a closed three-dimensional surface folding over.

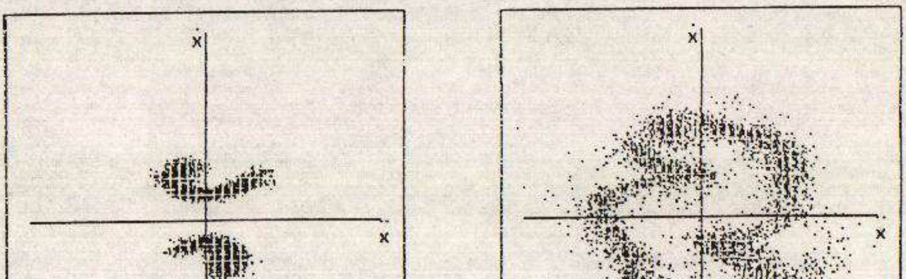

Fig.6 Poincare sections at x = 0 extracted from signals (a) and (b).

The chaotic state yields a Poincare section crossing itself, (Fig.7 left), suggesting a higher dimension. The section in Fig.7 (right) shows no structure but has a similar spectral content.

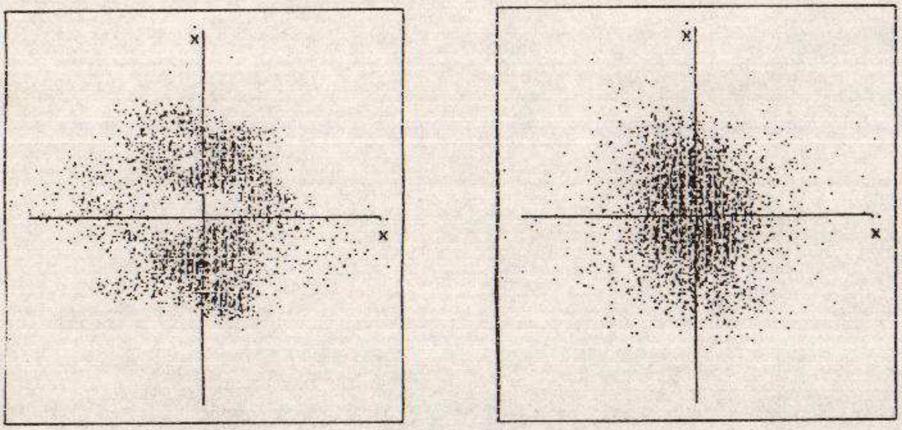

Fig.7 Poincare sections at x = 0 extracted from signals (c) and (d).

Following the procedure proposed by Eckmann et al. [4], the Liapunov exponents were estimated. They produced consistently values $\mu = 0.03 \pm 0.04$ for states type (b) and $\mu = 0.23 \pm 0.11$ for states type (c) when considering a three-dimensional orbit. For embedding dimensions 4 and 5 no significant variations were found, thus validating that low dimensional descriptions are appropriate for sounds originated by turbulence.

5. CONCLUSION

Methods using amplitude statistics and phase portraits have proven useful for the analysis of steady acoustic signals with shifting phase and the presence of noise. Different states could be identified where classical analysis was useless. Some of the states produced by turbulence exhibit broadband noise but should not necessarily be assigned to a stochastic source.

6. REFERENCES

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