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Air Coupling Between the Front and Back Plates of a Violin

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Introduction

The most critical range in a violin's response is that between about 200 and 800 Hz., primarily because of the system's low modal density in this region. Many attempts to optimize the spacing of the lowest two resonances- the front plate's lowest mode or front mode and the Helmholtz air mode- have been made and it is generally accepted that these should occur at about 440 and 290 Hz. respectively (Hutchins, 1962). This has the additional advantage that for inputs with a fundamental of 220 Hz. the second harmonic is enhanced.

By altering the back plate it may be possible to increase its response and add another useful resonance. Understanding the coupling between the plates is necessary to determine the feasibility of this.

Theory- Physical Properties

For frequencies below 800 Hz. the front and back plates are excited primarily in their lowest mode, similar to that of a rectangular plate simply supported at its edges, and may be represented as pistons sharing a common air cavity. The mass of air enclosed in the f-holes may also be treated as a piston and the resulting mechanical model is shown in figure 1.

The following are generalized values for mass, stiffness, and Q-value, typical of a good violin:

$M_f=8.6\times10^{-3}$ kg $\quad M_b=1.83\times10^{-2}$

$K_f=6.3\times10^{2}$ Nm^{-1} $\quad K_b=1.9\times10^{3}$ Nm^{-1}

$Q_f=14.5$ $\quad Q_b=$

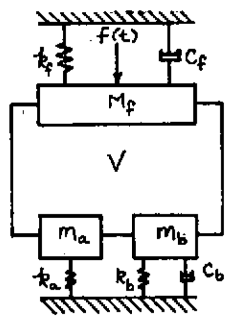

Figure 1: Mechanical model of a violin. The subscripts a,b, and f refer to air, back plate and front plate respectively.

For the air mode

$M_a=4.1\times10^{-5}$kg $\quad S_a=1.2\times10^{-3}.m^2 \quad V=1.65\times10^{-3}\ m^3$

where S_a is the area of both f-holes and two end corrections have been added to the air mass.

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The pressure in the enclosure will act on the front and back plates and so an equivalent piston area S_o must be defined by integrating the equation

$$S_o = \int_s f(x,y)\, dS$$

where the subscript o refers to the Helmholtz or zero mode and f(x,y) is the normalized displacement of the plate at (x,y), as determined from the iso-amplitude lines of a time-averaged vibration hologram. The piston areas are calculated to be

$$S_{fo} = 8.8\times10^{-3}\ m^2 \qquad S_{bo} = 9.7\times10^{-3}\ m^2 \quad .$$

In addition, at about 510 Hz. the air cavity develops a resonance similar to the (1,0,0) mode of a rectangular room. Because the f-holes are located at a pressure minimum little sound is radiated however the effect on the front and back plates is significant and so another equivalent area must be defined. Assuming a normalized pressure distribution of $\sin(2\pi x/l_x)$ within the cavity, the areas are

$$S_1 = \int_s f(x,y)\sin(2\pi x/l_x)\, dS$$

and

$$S_{f1} = 1.9\times10^{-3}\ m^2 \qquad S_{b1} = 6.1\times10^{-3}\ m^2 \quad .$$

Impedance Equations

Referring back to figure 1, the impedance equations may be written as:

$$\begin{aligned} Z_f &= Z_{mf} + Z_{cf} + Z_{kf} + S_{fo}P_o + S_{f1}P_1 \\ Z_{fa} &= (Z_f)(Z_{ma} + Z_{ca})/(S_{ao}P_o) \\ Z_{fb} &= (Z_f)(Z_{mb} + Z_{cb} + Z_{kb})/(S_{ao}P_o + S_{b1}P_1) \end{aligned} \qquad (1)$$

where P_o and P_1 represent the magnitude of the pressure for the zero and (1,0,0) modes. It is now necessary to establish equations for P_o and P_1.

Cavity Pressure

Standing waves are developed in the air cavity which are similar to the zero and (1,0,0) modes of a parallelepiped. By using a Green's Function approach, the expression for cavity pressure when excited by a point source of area dS_n at (x_o,y_o,z_o) is

$$p(x,y,z) = -jw\rho_o u \sum_n \frac{-\psi_n(x,y,z)\int_s \psi_n(x_o,y_o,z_o)\, dS_n}{V\Lambda_n(k^2 - K_n^2)} \quad . \qquad (2)$$

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where
$\Psi_n(x_o,y_o,z_o)$ and $\Psi_n(x,y,z)$ are the eigenfunctions and
K_n are eigenvalues such that $\nabla^2\Psi_n=-K_n^2\Psi_n$
$\Lambda_n=(1/V)\int_V\Psi_n^2(x,y,z)\,dV$
u=source velocity

If the four walls normal to the x and y axes (corresponding to the violin's ribs) are assumed to be perfectly rigid then the boundary conditions are

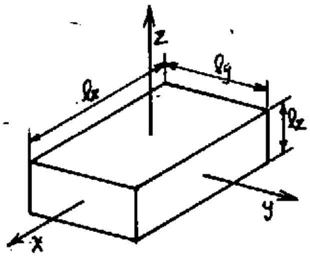

Figure 2: coordinate system on parallelepiped used as a model for standing waves in the violin air cavity.

$$\left.\frac{\partial\Psi}{\partial x}\right|_{\pm\frac{l_x}{2}}=\left.\frac{\partial\Psi}{\partial y}\right|_{\pm\frac{l_y}{2}}=0,$$

$$\left.\frac{\partial\Psi}{\partial z}\right|_{\frac{+l_z}{2}}=-j\Psi\beta_a\rho_o w,$$

$$\left.\frac{\partial\Psi}{\partial z}\right|_{\frac{-l_z}{2}}=+j\Psi\beta_b\rho_o w.$$

where β_n is the admittance S_n/Z_n.

Applying the boundary conditions and noting that

$$\int_S\Psi_n(x_o,y_o,z_o)f(x_o,y_o)dS_n=S_n \quad ,$$

equation (2) becomes

$$(3)\qquad p(x,y,z)=\frac{-j\rho_o wu}{V}\left\{\frac{S_{fo}}{\left[k^2-(\pi/l_z)^2q_{zo}^2\right]}+\frac{2S_{f1}\sin(2x\pi/l_x)}{\left[k^2-(\pi/l_x)^2-(\pi/l_z)^2q_{z1}^2\right]}\right\}$$

where $\quad q_{zo}^2=\frac{j\rho_o l_z^2 w}{V\pi^2}(S_{bo}\beta_b+S_a\beta_a) \qquad q_{z1}^2=j\rho_o l_z^2 wS_{b1}\beta_{b1}/(V\pi^2)$.

The terms for P_o and P_1 in equation (1) may then be written as

$$P_o=\frac{-j\rho_o wS_{fo}/V}{\left[k^2-(\pi/l_z)^2q_{zo}^2\right]} \qquad P_1=\frac{-2j\rho_o wS_{f1}/V}{\left[k^2-(\pi/l_x)^2-(\pi/l_z)^2q_{z1}^2\right]} \quad . \quad (4)$$

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Results and Conclusions

Using equations (1) and (4) one may determine the relative pressure amplitudes at one meter along the axis of each plate. Calculations are presently being made and results will be presented at the meeting.

Reference

C. M. Hutchins 1962 Scientific American, volume 207, number 5, 78-93. The Physics of Violin's.