PROC. INST. ACOUSTICS VOL 7 PT 3 1985

MULTIPLY REVERBERATED SOUND FIELDS IN A SURFACE DUCT

E. Topuz (1) and L.B. Felsen (2)

(1) Technical University of Istanbul, Ayazaga, Turkey
(2) Polytechnic Institute of New York, Farmingdale, NY USA

ABSTRACT

Multiple reverberations of acoustic pressure fields in an underwater surface duct can be modeled by ray acoustics. When observed at long propagation ranges, these reverberations generate convergence zones which pile up near depths equal to that of the source location. This accumulation of caustics descriptive of a sound field having undergone many surface reflections renders conventional ray acoustic corrections for isolated caustics ineffective. The deficiency can be repaired by replacing the caustic forming rays with a group of surface ducted modal fields. Combining ray fields, where they are legitimate, with mode fields, where they are not, yields a self-contained hybrid formulation valid for all observation depths at long range. This is illustrated by numerical examples for propagation of a high frequency Gaussian pulse in a model environment. The hybrid ray-mode pressure field is compared with a reference solution obtained by modal summation. The comparisons confirm the accuracy of the hybrid scheme and its numerical efficiency at high frequencies. Moreover, the hybrid representation, which automatically emphasizes ray fields at early times and mode fields at later times, furnishes a clear physical interpretation of the waveform at the receiver.

INTRODUCTION

In an underwater surface duct, the propagation of sound signals over long distances involves multiple reverberations. When modeled by ray acoustics, these reverberations exhibit convergence zones near depths equal to that of the source location. The separation between the caustics associated with rays having undergone many surface reflections decreases with increasing range. This piling up of caustics around the source depth generates a transition region (see Fig. 1b) wherein a purely ray acoustic representation, even when augmented with corrections for isolated caustics, becomes inapplicable for some rays with many reverberations. Conventionally, one then resorts to the normal mode representation. However, the deficiency of the ray acoustic representation can be repaired by replacing the caustic-forming troublesome rays with a group of surface ducted modes while retaining all other troublefree rays intact. Implementation of this scheme is via the hybrid ray-mode representation, which provides a rigorous framework within which an arbitrary number of legitimate ray fields can be combined with a thereby uniquely specified number of modes plus remainder terms. The remainders, which account for the truncation of the ray or mode series, are conveniently computable. Thus, the hybrid representation of the acoustic pressure field becomes an attractive option in problems wherein both local and global features of the propagation process--sampled, respectively, by a few arrivals or the collective (i.e. modal) effect of many arrivals--are of significance.

For pulse propagation, the hybrid representation allows one to combine legitimate rays, which have undergone fewer reflections on the surface and contribute at early times, with the first few surface ducted modes that contribute at late times. This yields an automatic and self-consistent blending of the ray-like behavior of the received waveform at early times with the mode like behavior at late times, as demonstrated here in the model duct environment of Fig. 1a excited by a line source emitting a high-frequency Gaussian pulse. Using a reference solution obtained from summation over ducted modes, it is shown that the hybrid ray-mode representation is accurate and numerically efficient for this class of propagation problems.

FORMULATION

We consider the sound pressure field $\Phi(\underset{\sim}{\rho},\underset{\sim}{\rho}',t)$ generated in the surface duct by a line source emitting a high frequency Gaussian pulse p(t) of duration $2t_o$,

$$p(t) = \exp[-i\omega_o(t-t_o) - (t-t_o)^2/4\alpha^2], \quad 0 \le t \le 2t_o \tag{1}$$

where α is a constant and ω_o is the radian frequency modulating the pulse. This transient field is obtained from the time harmonic pressure field $\Phi(\underset{\sim}{\rho},\underset{\sim}{\rho}',\omega)$ via the Fourier inversion

$$\Phi(\underset{\sim}{\rho},\underset{\sim}{\rho}',t) = \frac{1}{2\pi}\int_{-\infty+i\xi}^{\infty+i\xi} e^{-i\omega t}\, P(\omega)\Phi(\underset{\sim}{\rho},\underset{\sim}{\rho}',\omega)d\omega \tag{2}$$

where $\underset{\sim}{\rho} = (x,z), \underset{\sim}{\rho}' = (x',z')$ refer to receiver and transmitter coordinates, respectively, $P(\omega)$ is the Fourier transform of the source function, and $\xi > 0$ is chosen to be sufficiently large so that the integration contour lies above all singularities of the integrand, thereby guaranteeing causal solutions [1].

The time-harmonic pressure field in a surface duct with exponential velocity profile along the depth coordinate z has been investigated in some detail in earlier publications [2,3]. Here we shall merely list relevant results and refer the reader to cited references for explicit expressions and derivations of the various field constituents. The desired hybrid ray-mode representation is given by [2],

$$\Phi(\underset{\sim}{\rho},\underset{\sim}{\rho}',\omega) = \sum_{j=1}^{4}\left\{\sum_{n=N_{1j}}^{N_{2j}} R_j(n,\omega) - \frac{1}{2}\,[R_j(N_{1j},\omega) + R_j(N_{2j},\omega)] \right.$$
$$\left. + \sum_{m=1}^{M_j(\omega)} G_j(m,\omega) + \sum_{m=\bar{M}_j(\omega)}^{M(\omega)} G_j(m,\omega) + B_j(N_{1j},\omega) + B_j(N_{2j},\omega)\right\} \tag{3}$$

where each R_j and G_j stands for a ray acoustic and partial mode contribution associated with the species $j (j = 1$ to $4)$, respectively, and each B_j represents a remainder term which, together with a (-1/2) · (ray contribution), accounts for the truncation of the ray and mode series. Partial modes are obtained by decomposing the standing wave type (Bessel function) z dependence of a normal mode $G(m,\omega)$ in this model environment into travelling wave (Hankel function) constituents (see [2]),

$$G(m,\omega) = \sum_{j=1}^{4} G_j(m,\omega) \tag{4}$$

As depicted in Fig. 1a, a ficitious perfectly absorbing boundary is introduced at depth $z = z_d$ in our model, with the effect that for given transmitter-receiver locations ρ',ρ, the minimum number of surface reflections, which a ray may experience and still remain completely in the duct, has a lower limit N_{1j}, since the turning depth of the ray of species j with n reverberations $z_{nt}(j)$ must satisfy

$$z_{nt}(j) < z_d, \quad n \ge N_{1j} \tag{5}$$

On the other hand, except for the special case when both the transmitter and the receiver are located on the surface $(z = z' = 0)$, the number of possible ray paths between the transmitter and the receiver remains finite. Hence, there is a maximum number $\bar{N}_{2j}$ of surface reflections a ray may undergo. The formal ray acoustic representation for the pressure field then reads

$$\Phi_R(\underset{\sim}{\rho},\underset{\sim}{\rho}',\omega) = \sum_{j=1}^{4} \sum_{n=N_{1j}}^{\bar{N}_{2j}} R_j(n,\omega), \quad z_{tN_{1j}} < z_d \tag{6}$$

The discrete set of normal modes of the surface duct represents pressure fields that are oscillatory above the depth of their respective modal caustics $z_m(\omega)$ but decay rapidly below this depth. The modal caustics are frequency dependent and move to greater depths with increasing mode index m and decreasing frequency [2],

$$z_i(\omega) < z_j(\omega), \quad j > i$$

$$z_i(\omega_2) < z_i(\omega_1), \quad \omega_1 < \omega_2, \quad i,j = 1,2.. \tag{7}$$

At any frequency, we construct our reference solution Φ_m as the sum over all ducted modes whose modal caustics are located within the surface duct,

$$\Phi_m(\underset{\sim}{\rho},\underset{\sim}{\rho}',\omega) = \sum_{m=1}^{M(\omega)} G(m,\omega)$$

$$z_m(\omega) < z_d, \quad m = 1,2..M(\omega) \tag{8}$$

In the hybrid representation (3) of the acoustic pressure field, the upper limit of the ray series $N_{2j} \leq \bar{N}_{2j}$ is chosen in such a way that rays having undergone N_{2j} surface reflections can still be resolved as legitimate ray fields, augmented when necessary with <u>isolated</u> caustic corrections. With the choice of N_{2j}, the hybrid presentation becomes uniquely determined at any frequency since $M_j(\omega)$ and $\bar{M}_j(\omega)$ in (3) are specified by

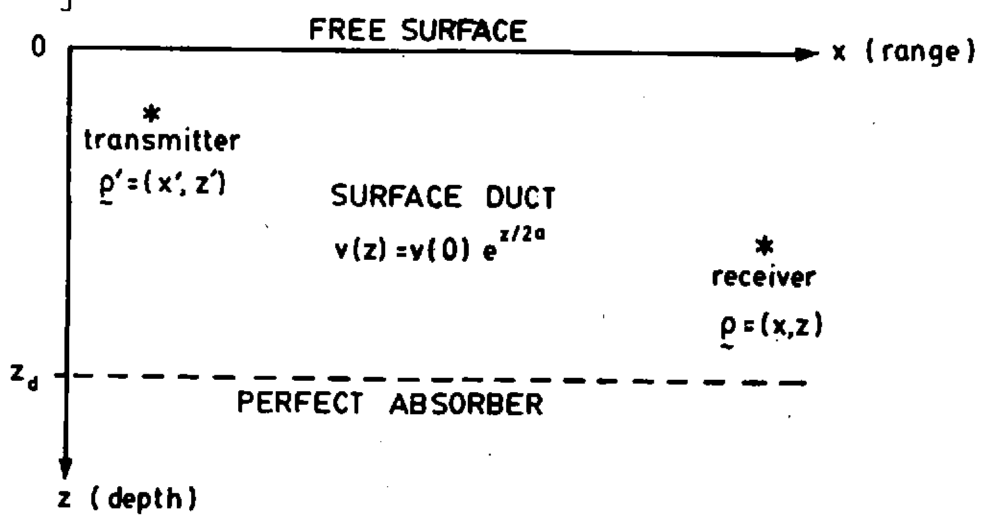


FIGURE 1a

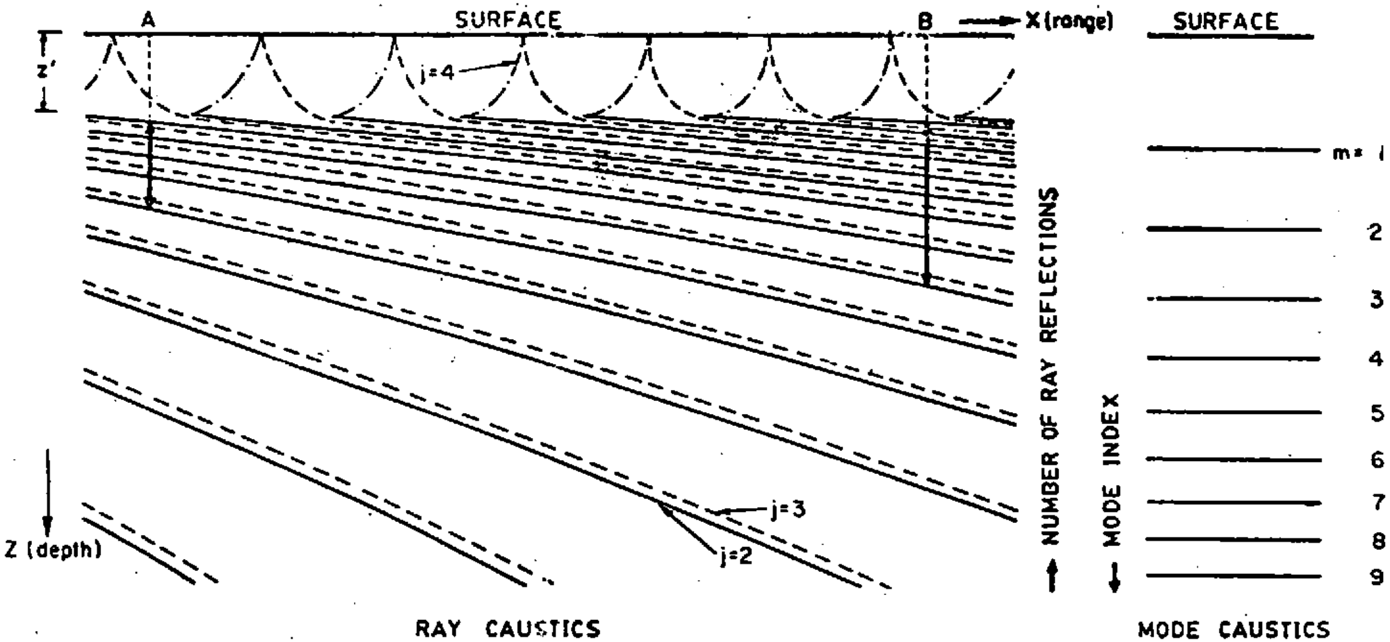

A
SURFACE
B
X (range)
SURFACE
z'
j=4
m= 1
2
3
4
5
6
7
8
9
NUMBER OF RAY REFLECTIONS
MODE INDEX
j=3
j=2
Z (depth)
RAY CAUSTICS
MODE CAUSTICS

FIGURE 1b

$$z_{M_j}(\omega) < z_{tN_{2j}}, \quad z_d < z_{\bar{M}_j}(\omega) < z_{tN_{1j}} \tag{9}$$

It is to be noted that because the ray turning depths z_t are dependent on the transmitter and receiver locations but not on frequency, whereas the modal duct depths z_m depend on the frequency but not on transmitter and receiver locations, the hybrid representation becomes a function of both transmitter-receiver configuration and frequency.

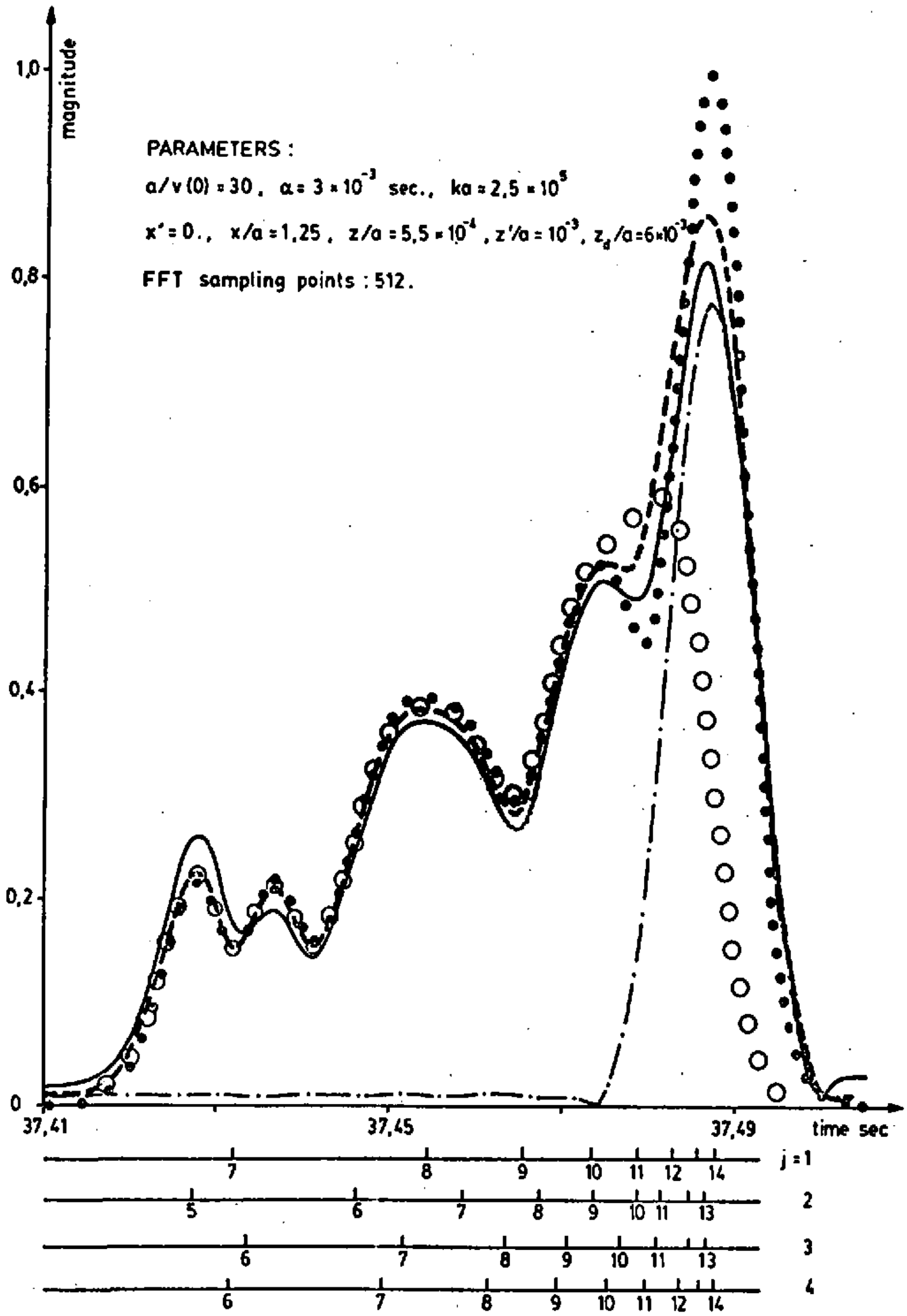
The hybrid form (3) is asymptotically equivalent to the normal mode reference solution (8) and reduces to it completely when no ray constituents are retained. On the other hand, if all the rays to the observer are legitimate, one may choose $N_{2j} = \bar{N}_{2j}$ in (3), thereby reducing the hybrid representation to the ray acoustic pressure field (6), apart from a correction term which, at each frequency, accounts for the spectral void left between $z_{t\bar{N}_{2j}}$ and $z_{M(\omega)}$. This correction remains small provided that the transmitter and receiver locations are not too close to the fictitious duct boundary z_d; in that event, one may conclude that all three representations are asymptotically equivalent whenever all rays are legitimate. However, at long ranges, a transition region builds up around the source depth and renders ray acoustics inapplicable there for some rays with many reverberations. The width of this transition region can be estimated in a numerical scheme that compares the results obtained from the hybrid representation (3) with those from the normal mode reference solution (8) for various parameter values. Any disagreement between these results is a direct indication that non-legitimate ray fields have been included in (3), and that N_{2j} must therefore be reduced. We have found that the width of the transition region around the source depth increases with increasing horizontal transmitter-receiver separation and with decreasing frequency. Our numerical calculations allowed us to furnish a semi-quantitative description of the dependence of the transition region upon these parameters. In turns out that the width of this region above the transmitter depth can be estimated as that of the nearest two or three modal ducts, whereas below the transmitter depth, the transition region extends to a depth, at which the separation between the caustics for N and N + 1 times reflected rays of the same species is at least as large as the separation between two modal caustics at the same depth. Using this criterion as a rule of thumb, one can determine the upper limit of N_{2j} of the ray series in (3) for given transmitter-receiver configurations and for given source frequency spectra, whereupon all remaining field constituents in the hybrid representation become uniquely specified.

A particularly appealing simplification of the hybrid representation results when both the transmitter and the receiver are well above the duct boundary z_d. Under these conditions, the small effect of the truncation of the ray series at the low reflection end is further smoothed out when synthesizing the pulse response, so that one may simplify (3) to

$$\Phi(\underset{\sim}{\rho},\underset{\sim}{\rho}',\omega) \cong \sum_{j=1}^{4} \left\{ \sum_{n=N_{1j}}^{N_{2j}} R_j(n,\omega) - \frac{1}{2} R_j(N_{2j},\omega) + \sum_{m=1}^{M_j(\omega)} G_j(m,\omega) + B_j(N_{2j},\omega) \right\} \tag{10}$$

Substituting the time harmonic pressure field from (6), (8) and (3) [or (10)] into (2), one obtains the pulse response in terms of ray acoustic, normal mode and hybrid representations, respectively. For ordinary ray fields, the Fourier inversion in (2) may be performed asymptotically and explicitly via the saddle point technique, whereas for all other field constituents (uniform ray fields, normal mode fields and the remainder contributions), we have employed the FFT routine for numerical evaluation.

FIGURE 2



NUMERICAL RESULTS

The pulse pressure field in the surface duct has been computed from the full hybrid representation (3) and from the normal mode reference solution (8) for a number of test cases. In all cases, the hybrid representation has been found to be in excellent agreement with the modal sum. Both representations required comparable computation times. In the full hybrid representation, most of the computer time is consumed in the search for the modal eigenvalues at each frequency sampling point. It is therefore to be expected that a reduction in the number of required modes will lead to increased computational efficiency. This is indeed the case when both transmitter and receiver are well above the duct boundary z_d since the approximate form of the hybrid representation in (10), which involves the much smaller number $M_j(\omega)$ of modal eigenvalues rather than the total number $M(\omega)$, may then be employed without appreciable deterioration of accuracy.

A typical example of our calculations is depicted in Fig. 2. For the set of parameters used in this computation, the formal ray series contains 30 elements and the number of ducted modes varies from 32 to 38 as the frequency increases from the lowest to the highest spectral component. We note that the purely ray acoustic field differs appreciably from the normal mode reference solution at late times, which correspond to arrival times of rays having undergone many reflections on the surface (see bottom scales of Fig. 2). Evidently, some terms in the formal ray series can no longer qualify as proper ray acoustic fields. In this example, the receiver depth is less than two modal duct widths above the transmitter depth at the center frequency, and hence is located within the transition region above the transmitter depth. Using the numerically derived criterion stated earlier, we have concluded that rays with more than 11 surface reflections are no longer legitimate. Thus, $N_{2j} = 11$ has been chosen in the hybrid formulations (3) and (10). With this choice, when calculated via the full hybrid formulation (3), the pulse response agrees with the reference solution to within the accuracy of the figure drawing. The computation from either the full hybrid formulation or the normal mode sum requires approximately 100 sec. of CPU time on a CDC 170-730 computer. On the other hand, when the pulse response is calculated via the approximate hybrid representation (10), the number of normal modes to be included reduces to ≤ 4, and the computation time decreases by nearly an order of magnitude. As seen from Fig. 2, the pulse response obtained from (10) is still in very good agreement with the reference solution. On the same figure, we have also plotted the partial ray and partial mode responses, which are obtained by putting $\bar{N}_{2j} = 11$ in (6) and $M(\omega) = 4$ in (8), respectively. These curves demonstrate strikingly the complementary roles of the ray and mode fields in determining early and late time responses, respectively. It is therefore not surprising that the hybrid representation, which combines rays and modes so as to take maximum advantage of both, results in improved computational efficiency for certain pulse propagation problems observed over long (early to late) time intervals.

CONCLUSIONS

High frequency sound pulse propagation over long ranges in an underwater surface duct with an exponential sound speed profile has been calculated from ray acoustic, normal mode and hybrid ray-mode representations. The ray field transition region around the receiver depth, wherein ray acoustics is inapplicable for rays with many surface reflections, is accommodated by the hybrid formulation, which substitutes modes for those field constituents that cannot qualify as legitimate ray fields while retaining all (or some) of the legitimate rays intact. Since rays and modes are naturally suited to describing early and late time responses, respectively, the rigorous framework combining them in the hybrid representation yields a computationally superior alternative to normal mode summation and grants physical insight into the relative roles played by rays and modes in structuring the received waveform.

ACKNOWLEDGEMENT

This research was sponsored by the Office of Naval Research under Contract No. N-00014-79-C-0013.

REFERENCES

[1] L.B. Felsen, ed., "Transient electromagnetic fields", Springer, pp. 19, (1976).

[2] E. Niver, S.H. Cho and L.B. Felsen, "Rays and modes in an acoustic channel with exponential velocity profile", Radio Science, Vol. 16, 6, pp. 963-970, (1981).

[3] E. Niver, A. Kamel and L.B. Felsen, "Modes to replace transitional asymptotic ray fields in a vertically inhomogeneous earth model", Geophys. J.R. Ast. Soc., Vol. 80, in press (1985).

FIGURE CAPTIONS

Fig. (1a) Model surface duct with an exponential velocity profile in depth having profile gradient "a". The transmitter is a y-directed line source emitting a high frequency Gaussian pulse; i.e., $a\omega/v(0)>>1$ for all relevant spectral components of p(t) in (1).

Fig. (1b) Ray and mode caustics. The width of the ray field transition region below the transmitter depth z' is indicated schematically by the arrows at A and B. The region increases with increasing range and extends to a depth where the separation between the caustics of N and (N+1) times reflected rays of species j is at least equal to the distance between modal caustics at the same depth. Above the transmitter depth, the transition region encompasses two or three modal duct widths. At any range, the ray turning depths are slightly greater than the corresponding caustic depths.

Fig. (2) Pulse reponse in the surface duct for the set of parameters given in the figure, where $k=\omega_o/v(0)$, α is the pulse width parameter in (1) and "a" defines the profile gradient (see Fig.1).

—— Calculated from normal mode reference solution (8) or from full hybrid formulation (3), with $N_{2j}=11$.

--- Calculated from approximate hybrid formulation (10), with $N_{2j}=11$.

●●● Calculated from formal ray series (6), with $N_{2j}\leq 14$.

○○○ Partial ray response. Calculated from (6), with $\bar{N}_{2j}=11$.

-·-· Partial mode response. Calculated from (8), with $M(\omega)=4$.

Ray arrival times are shown for all species ($j=1$ to 4) on the same time scale. The numerals identify the number of reflections experienced by the ray.