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A Simple Method of Correction for Refraction in Underwater Acoustic Navigational Systems

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Introduction

Figure 1 illustrates the problem. If an acoustic transponder is fixed on the sea bed, the travel time of an acoustic signal from a transceiver mounted on a ship can be measured. Knowing the depth of water and the acoustic velocity profile in the sea it is possible to deduce the slant range between ship and transponder. Measurements for a number of transponders at known positions will then enable the ship's position to be estimated.

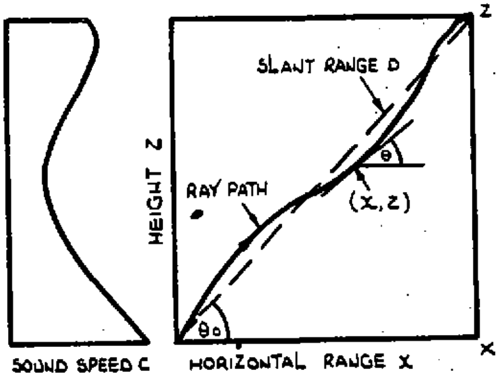

Figure 1. Geometry of Ray Path

Corrections for ship motion are comparatively straight - forward, and are not considered here. A major difficulty however, lies in the correction for refraction, due to the variation in sound velocity. Even with the usual simplifying assumption of horizontal stratification, standard ray-tracing methods are laborious, involving many calculations, the necessity for interpolation, and the storage of a large number of data defining the velocity profile. Real-time monitoring of position is thus impracticable unless a very large and fast computer is available.

In this paper a direct method is described which requires little computer storage or speed, and whose accuracy is good for all practical purposes (and probably as good as is justified by the accuracy of the input environmental data).

Theory

As usual, horizontal stratification is assumed (sound velocity a function of depth only). In the first instance direct acoustic paths only will be considered, i.e., no surface or bottom reflections. If x denote horizontal range and z vertical height of a point on the ray-path at which the grazing angle is θ (where $\tan\theta = dz/dx$); t the travel time to the ship at point (X, Z); C the sound velocity at height z; (figure 1); then the equations to be solved are

$$X = \int_0^Z \cot\theta \, dz \qquad (1)$$

$$t = \int_0^Z (\operatorname{cosec}\theta/c) \, dz \qquad (2)$$

where $\cos\theta/c$ = constant along ray path (Snell's Law) (3)

The method described here is based on the observation that both X and t are weighted averages of functions of c along the path, and thus that the assumption of a constant value of C equal to the mean velocity over the depth Z will be a good first approximation. Another way of looking at this is to note that since by Fermat's principle the actual path corresponds to a stationary value of travel time, the travel time computed along the straight-line path from source to receiver can be in error only by terms of second-order magnitude.

To apply these concepts define $\bar{C}$ to be the average sound speed, where

$$\bar{C} = (1/Z) \int_0^Z C dz$$

Writing $C = \bar{C}(1 - y)$, where for any real situation $|y| \ll 1$, then $\bar{y} = 0$ (by definition), and $\overline{y^2}$ and higher moments may be computed from the observed profile by integration.

A mean grazing angle $\bar{\theta}$ may now be defined by the equation

$$\cos\theta / c = \cos\bar{\theta}/\bar{C} \qquad (4)$$

$\bar{\theta}$ is always a real angle, since $\bar{C}$ must occur somewhere on the actual velocity profile.

Finally write $X/Z = \cot\theta_o$ (see figure 1).

Substituting from (4) into (1) and (2), expanding by the binomial theorem, retaining terms to the second order in y, and performing the integration, it is easily shown that

$$\cot\theta_o = \cot\bar{\theta}\,(1 + 3/2\,\overline{y^2}\operatorname{cosec}^2\bar{\theta}\cot^2\bar{\theta}) \qquad (5)$$

$$\bar{C}t/Z = \operatorname{cosec}\bar{\theta}\,[1 + \overline{y^2}(1 - \tfrac{1}{2}\cot^2\bar{\theta} + 3/2\cot^4\bar{\theta})] \qquad (6)$$

Inversion of equation (5) gives $\cot\bar{\theta}$ in terms of $\cot\theta_o$ and $\overline{y^2}$, and substitution into (6) leads to the equation

$$\bar{C}t = Z\operatorname{cosec}\theta_o\,[1 + \overline{y^2}(1 - \tfrac{1}{2}\cot^2\theta_o)] \qquad (7)$$

error occasioned by the use of equation (8) is shown in figure 3; this figure also shows the maximum possible error given by equation (11).

The maximum value of the term in $\overline{y^2}$ (equation (8)) is 68 m in 29,000 m (i.e., 0.23%), and so the maximum possible error at this range is ± 34 m, the actual error being only -9 m. It would be expected that, for any profile in which the values of sound speed are distributed between the extreme values, the actual error will be considerably less than the maximum possible, and order-of-magnitude arguments suggest that practical limits would be $\frac{1}{2}$ to $\frac{1}{3}$ of the maximum. This rule has been found to apply even to a much more extreme profile, with a total variation of 10% in sound speed.

Instrumental Accuracy

Equations (8) and (11) give a criterion for the allowable tolerance in instrumental error. By the preceding arguments, for typical profiles the maximum error is likely to be less than one part in 2,000, and so $\overline{C}$ must be known to something better than this, say one part in 5,000, i.e., about 0.3 m/sec. It is clear therefore that any systematic error in the measuring instrument should not exceed 0.3 m/sec.

On the other hand, it is not necessary that individual readings need be made to this accuracy, since $\overline{C}$ and $\overline{y^2}$, being averages, will have a precision of $1/\sqrt{n}$ that of a single reading, where n is the number of points taken. If, for example, n is 100, it is only necessary to read to the nearest 3 m/sec to estimate $\overline{C}$. A slightly more complex analysis shows that, to obtain $\overline{y^2}$ to sufficient accuracy, reading to the nearest 1.5 m/sec is adequate.

These accuracies are easily attainable with existing instruments, and are probably as good as is justified by the assumption of horizontal stratification.

Multiple Reflections

This situation is easily dealt with if required. It is obvious that there must be an equal number of surface and bottom reflections, and that the ray path will consist of an odd number of segments of bottom/surface travel, each having the same travel time, the same shape of ray path, and the same increment of horizontal range. If the number of segments is 2n + 1 then it is only necessary to compute Dn' by using equation (8) with time entered as $t/(2n + 1)$; the true slant range Dn is then given by

$$Dn^2 = (2n + 1)\, Dn'^2 - 4n(n + 1)Z^2$$

The error will increase with range. If ε' is the estimated error in computing Dn', then the error in Dn is given by

$$\varepsilon \sim (2n + 1)^2 (Dn'/Dn) \times \varepsilon'$$

which at the maximum limit for Dn' will become approximately

$\varepsilon \sim (2n + 1)\ \varepsilon'$, i.e., proportional to range.

Summary of Advantages

1. While the conventional approach requires the computation and storage of twenty or more values of slant range and time, and has an awkward interpolation to perform for each reading, the method described here involves only an initial computation of the mean and standard deviation of the velocity profile, and subsequently only the storage of these two parameters, with a simple computation for each observation - a task well within the real-time capability of a small computer.

2. With the conventional method rounding-off errors are a problem, and the final accuracy is difficult to assess; with the method described here rounding-off is not a problem, environmental data do not have to be entered with a large number of significant figures, and the final accuracy can be easily estimated.

3. In practical situations, for typical velocity profiles and assuming that reflected paths are not used, an accuracy of better than ± 15 m may be expected at all slant ranges. This is a figure of practical value.