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UNDERSTANDING THE DYNAMIC BEHAVIOR OF COMPLEX VIBRATORS

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The dynamic behavior of vibrators can be described by progressive wave fields, or by the contributions of its natural modes. The first method leads to the mean value theory, which predicts the mean line through the logarithmically recorded frequency response of the vibrator from the first resonance on to very high frequencies, whereas the second method leads to a simple two mode circuit that describes the vibrator by its resonance peaks and antiresonance minima. When a vibrator is excited at an interior point by a point force, a progressive wave field V_c propagates towards its boundaries. The reflections return with different phases, and when averaged over a frequency interval of several resonances, they do not contribute to the geometric mean between the resonance maxima and the antiresonance minima. Thus, the mean line through the logarithmically recorded frequency response curve is identical with the amplitude of wave field V_c that is generated by the driver and propagates away from it as if the vibrator were infinitely large or had ideally absorbent boundaries. This wave field is the solution of the differential equation for a point force acting on an infinitely extended but otherwise similar vibrator. It is defined as the characteristic velocity, V_c, of the vibrator and the admittance as the characteristic admittance, Y_c, in analogy to the characteristic admittance of a telephone line or a chain of filters. The characteristic admittance is obtained by computing the solution of the differential equation for a point force for the infinitely extended vibrator, or in a still simpler manner, by adding up the modal contributions.

Because for computing Y_c the vibrator is assumed to be very large, and the density of the resonances is high, the summation can be replaced by an integration. The real part of the resulting integral has a pole at the driving frequency and yields the contribution of the resonance range of the vibrator:

$$\mathrm{Re}(Y_c) = \pi/2\varepsilon_\nu M_\nu \quad ,$$

where ε_ν is the average radial frequency different between successive mode resonances in the frequency range of the excitation, and M_ν are the point mode masses, defined by [1]

$$M_\nu = \frac{M\langle\xi\rangle_\nu^2}{\xi_\nu(A)\,\xi_\nu(F)} \quad ,$$

which are obtained by simple computations. They are usually equal to half or one-quarter of the mass of the vibrator. Here $\langle\xi_\nu^2\rangle$ is the vibrator average square displacement and $\xi_\nu(A)$ and $\xi_\nu(F)$ are the displacements at the point of interest "A" and at the driver at "F". M is the total mass of the vibrator. Because $\varepsilon_\nu M_\nu$ is asymptotically independent of the size of the vibrator, it usually can be easily computed. The imaginary part of Y_c ($\mathrm{Im}(Y_c)$) depends on the magnitude and variation of $\varepsilon_\nu M_\nu$ in the whole frequency range. It is zero if $\varepsilon_\nu M_\nu$ is constant or is approximately constant, and is small and it can be frequently neglected.

Because reflections do not contribute, ribs, inhomogeneities, loads and supports have no effect on the "geometric mean line response" of the vibrator, provided they are a few bending wave lengths away from the driver and receiver. If on the other hand such discontinuities are close to driver or receiver, their response has to be added to the amplitude of the characteristic wave that is generated at the driver. The predictions for the driving point velocity usually agree within ± 2 dB with the experimental results. The theory also applies to shells and more complex vibrators, where the displacement is described by a vector variable.

The dynamic behavior of a vibrator can also be described by a circuit of a finite or infinite number of simple series-resonant circuits in parallel, driven by the sum of the forces that are applied to it. The parameters of these circuits [1] are the point mode masses M_ν the point mode compliances $K_\nu = 1/\omega_\nu^2 M_\nu$ and the point mode resistances R_ν. If damping is great, negative elements must be included in some of the circuits. Network theory is applicable and all the theorems derived in circuit theory apply also to complex vibrators.

The behavior of a vibrator can frequently be approximated with a high degree of accuracy by considering only the contributions of a small number of modes. For instance if a vibrator is not heavily damped so that its resonance peaks are separated, a two mode circuit describes peaks and minima. The resonance peaks (ξ_ν)res then are determined by the contributions of single modes:

$$(\xi_\nu)_{res} = f/R_\nu \quad ,$$

where f is the total driving force,

$$R_\nu = \eta\omega_\nu{}^2 M_\nu/\omega = \eta\omega_\nu M_\nu$$

is the "point mode resistance and η is the loss factor. At the antiresonance minima ($Y = Y_{anti}$) the imaginary part of the modal contributions cancel, and the real parts survive, but their contributions decrease drastically with their frequency difference from the modal resonance frequencies. Thus, only the two modes whose resonance frequencies are next to the frequency of the force need to be considered.

The height of the resonance peaks (Y_{res}) and the magnitude of the antiresonance minima (Y_{anti}) can be obtained by standard computations; the result is

$$Y_{res} = 1/R_\nu = \beta Y_c = \frac{1}{\omega_B M_\nu} \quad , \quad Y_{anti} = Y_c/\beta = \frac{\pi}{2}\frac{\omega_B}{\epsilon_\nu{}^2 M_\nu} \quad ,$$

and $$(Y_{res}Y_{anti})^{1/2} = \sqrt{\frac{\pi}{2}}\,\frac{1}{\epsilon_\nu M_\nu} = \sqrt{\frac{2}{\pi}}\ \mathrm{Re}Y_c = 0.92\mathrm{Re}(Y_c) \doteq \mathrm{Re}(Y_c) \doteq Y_c$$

where

$$\beta = \frac{2\epsilon_\nu}{\pi\eta\omega_\nu} = \frac{2\epsilon_\nu}{\pi\omega_B}\ , \quad \omega_B = \eta\omega_\nu \ .$$

β is defined as the peak factor. If the density of the resonances is constant, and we add the contributions of all mode pairs, we obtain the exact solution

$$(Y_{res}Y_{anti})^{1/2} = \left[\frac{1}{\omega_B M_\nu}\ \frac{2\omega_B}{\epsilon_\nu^2 M\nu}\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right)\right]^{1/2} = \frac{\pi}{2\epsilon_\nu M_\nu} \ .$$

Thus, the second mode pair contributes only 5.5% to the characteristic admittance, the third mode pair only 3%.

The equivalent two-mode circuit gives insight into the variations of the response curve of a vibrator with frequency and it helps to design vibrators with a prescribed frequency response curve.

Increasing the damping beyond $\epsilon_\nu/\omega_B = 5$ decreases the resonance peaks and increases the minima. More modal contributions then have to be added up. If the frequency is sufficiently high so that $\omega/\epsilon_\nu >> 1$, then the summations for the resonance peaks and for the antiresonance minima leads to similar results as those given above, except for a decrease in the peak factor β. For instance, for $\epsilon_\nu/\omega_B = 2$, $\beta \to 1.52$, for $\epsilon_\nu/\omega_B = 1$, $\beta \to 1.09$.

The mean value theory can be extended also for transfer admittances(1), when the receiver is not coincident with the driver.

Vibrators with loads, inhomogeneities; ribs and other discontinuities are best described in terms of the mode functions of the unloaded, homogeneous vibrator. Each mode function of the discontinuous vibrator then corresponds to a sum of mode function of the homogeneous vibrators: the loads and discontinuities couple the mode functions of the corresponding unloaded vibrator. For instance, if a simple beam is suspended at its ends and excited at the center, each of its mode functions is made up by the mode functions of the free-free, supported -supported, and because of dissymetries of the free-supported beam. Coupling by the boundary condit-ions is particularly simple, and does not destroy or change the resonance frequencies of the three different types of modes; they all show up in the frequency response of the suspended beam by their resonance peaks.

A load generates a point force which always has a considerable effect on the driving point velocity if it is located at the driver. For instance, if a mass of 1/100 of the vibrator mass is at the center of a circular plate, the vibrator admittance may decrease by 100 dB as the frequency reaches a value of about 10 times the fundamental resonance frequency. But its effect is hardly noticeable if it is a few bending wave length away from the driver. The load increases slightly the damping because it excites modes that do not contribute to the driving point velocity. Suspensions and supports lead to particularly strong mode coupling. They may generate forces of the same magnitude as the driver. Of all supports tested, soft rubber foam was best suited for vibration measurements. Welds on shells reduce the vibration amplitude considerably, if they coincide with the driver. But if the weld is remote from the driver, reflections no longer contribute to the geometric mean and the weld has only a small effect on the "mean value " response.

The mean value theory is also excellently suited to deal with ribbed shells and more complex types of vibrators. The practical computations are always very simple, and agree well with the experimental result usually within ± 2 dB with the theory.

The transients of a coupled vibrator are dealt with by compounding the modal transients, and in simple cases, by adding them up with the aid of the Wattson transformation. In such computations, the rigid body velocity (the zero order mode) is usually of great importance.

For rods, transducers and similar one-dimensional vibrators, the transients repeat periodically with the fundamental period T of the vibrator, except for the effect of the rigid body motion and the damping. This periodicity with T sometimes leads to sharp minima in the displacement time curve, as if the system was strongly non-linear. At a resonance, energy is fed into the vibrator with optimum phase and the motion builds up to a large value, off resonance the vibrator impedance is dominantly reactive, and the transient amplitude is not much greater than the characteristic wave that is continuously generated at the driver and travels towards the boundaries of the vibrator; the duration of the transients then is not much greater than fundamental period of the vibrator.

When a vibrator is excited by a frequency modulated pulse and damping is small the switching on and switching off transients can be made to cancel each other, and the generated pulse becomes a replica of the applied forcing pulse. But if damping is increased, cancelling out is no longer possible, and the transient becomes much longer. Thus, contrary to the expectations increasing the damping may increase the transients.

In room acoustics, the reflections represent the transients. Parallel walls generate periodic sequences of reflections. In contrast, oblique walls broaden the reflection and spread them out. Because the ear also responds to the envelope of a sound phenomenon, parallel walls lead to a harsh sound impressions, and oblique walls usually lead to a smooth and agreeable room acoustics.

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