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EFFICIENT APPROACHES FOR THE ACOUSTIC MODELLING OF AUTOMOTIVE EXHAUST DEVICES. APPLICATION TO CONFIGURATIONS INCORPORATING GRANULAR MATERIALS AND MONOLITHS

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The acoustic modelling of automotive exhaust devices, such as silencers and catalytic converters, usually requires the use of multidimensional numerical techniques. The presence of three-dimensional waves can be considered through the finite element method (FEM). With a view to reduce the computational effort of the FEM, efficient modelling techniques based on the numerical version of the mode matching method are presented in the current work to speed up transmission loss calculations in exhaust devices incorporating features such as granular materials and monoliths. Absorbent granular materials are studied here as a potential alternative to the traditional fibrous materials used in dissipative silencers, while monolith capillary ducts are commonly used in catalytic converters. First, the transversal pressure modes are computed for devices with arbitrary but axially uniform cross sections. For silencers with granular material, a number of geometries with different filling configurations are considered taking into account the abrupt transition air/material. Then, the mode matching method is applied to the compatibility conditions of the acoustic fields at all the device interfaces to couple the modal expansions and to obtain the complete solution of the wave equation. As shown in earlier studies, granular materials can be modelled through equivalent complex and frequency dependent density and speed of sound, the porosity playing an important role; regarding the acoustic modelling of monoliths, these can be replaced by a four pole transfer matrix relating the acoustic fields at both sides of the monolithic region. The computational efficiency and accuracy of the results associated with the numerical modelling techniques presented here are assessed for silencers with granular material and catalytic converters with monolith. The numerical mode matching approach provides accurate predictions of the device attenuation performance and outperforms the computational expenditure of FE computations.

Keywords: Automotive exhaust device, numerical mode matching, finite element method, computational efficiency, granular material, monolith, duct acoustics.

1. Introduction

In the last decades, concern about emissions produced by automotive vehicles has considerably increased. This is due to the harmful effects that affect both the environment and human health. One of the main sources of pollutant and noise emissions in vehicles is the internal combustion engine, the exhaust system playing an important role in their control. Thus, silencers and catalytic converters have a special relevance with a view to reduce the noise and pollution [1-5] to the levels allowed by the legislation in force.

In the first case, new configurations have been explored as a potential alternative to the traditional perforated dissipative silencers to avoid, on the one hand, the degradation of the absorbent material properties over time, as well as the dragging of the fibres produced by the exhaust gases. Some authors have proposed sintered and microperforated [4, 5] surfaces as possible alternatives to dissipative fibres. The former present a good acoustical behaviour at low frequencies, while the latter allow the reduction of sound emissions mainly in the mid and high frequency range [5]. In this work, granular materials are presented also as a potential alternative to fibrous materials. This kind of material allows obtaining variable geometric configurations by means of an in situ filling/emptying process of the chamber. From an acoustical point of view, granular materials can be modelled through their equivalent complex density and bulk modulus [6, 7].

On the other hand, after-treatment devices are usually characterized by using a ceramic monolith, formed by a great amount of capillary tubes with a very small cross section [3, 8]. In addition, these exhaust devices require acoustic models for the expansion chambers and the inlet/outlet ducts, placed at both sides of the monolith [8].

In general, multidimensional numerical techniques have been widely used in the literature to study the acoustic behaviour of this kind of devices. However, these techniques can be computationally expensive [5, 9-11]. For this reason, some authors have taken benefit of the fact that silencers and catalytic converters can be divided into subcomponents having arbitrary, but axially uniform, cross section. This geometric property allows the possibility of reducing the computational cost of a full FE formulation. This can be carried out by combining the FEM with the numerical version of the mode matching method or the point collocation technique. However, the former has been proved to be more efficient and accurate [8, 12, 13]. Therefore, in this work the mode matching method has been used to enforce the compatibility conditions of the acoustic fields (velocity and acoustic pressure) at the geometric discontinuities of the exhaust devices. A previous step for silencers with granular material consists of solving the quadratic eigenvalue problem associated with the cross section and obtaining its associated wavenumbers (eigenvalues) and pressure modes (eigenvectors). In the case of catalytic converters, the consideration of rigid walls greatly simplifies the computations. Finally, the acoustic attenuation performance can be assessed through the transmission loss.

2. Mathematical approach

In section 2.1, the mathematical development associated with the cross section of the silencer chamber containing two different propagation media (air and granular material) will be provided. It should be noticed that the equations for the rigid ducts containing a single propagation medium (air) are straightforward [12-15].

2.1 Quadratic eigenvalue problem

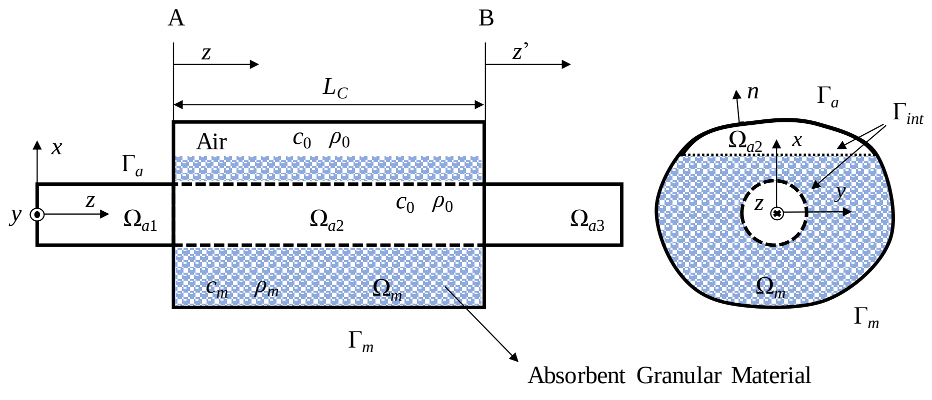


Figure 1: Geometry of the dissipative silencer containing granular material.

A silencer containing granular material within a chamber of length L_C is shown in Fig. 1. The chamber of the silencer has an arbitrary, but uniform, cross section. Air and granular material subdomains are denoted by Ω_a (different numbers are used to distinguish the air regions) and Ω_m, respectively, their boundary being Γ_a and Γ_m, while Γ_{int} represents the contour where the transition air/material is applied. The air can be characterized by means of its density ρ_0 and speed of sound c_0, while ρ_m and c_m are considered for the granular material (in this latter case, the "equivalent" properties are complex and frequency dependent). The inlet/outlet ducts (subscripts 1 and 3) are identical and have uniform cross section.

For the central chamber, the sound propagation in the air, assuming harmonic behaviour, is governed by the well-known Helmholtz equation that can be written as [14]

$$\nabla^2 P_a + k_0^2 P_a = 0, \tag{1}$$

where ∇^2 is the Laplacian operator, P_a is the amplitude of the acoustic pressure and k_0, the wavenumber, defined as the ratio between the angular frequency ω and the speed of sound c_0. Similarly, the wave equation for the granular material subdomain is [5]

$$\nabla^2 P_m + k_m^2 P_m = 0, \tag{2}$$

P_m being the amplitude of the acoustic pressure and k_m the wavenumber related to the granular material. Then, taking into account that the silencer cross section is axially uniform and applying separation of variables, the pressure amplitude results in

$$P(x,y,z) = \Psi^{xy}(x,y)e^{-jk_z z} \qquad \Rightarrow \qquad \Psi^{xy}(x,y) = \begin{cases} \Psi_a^{xy}(x,y), \ (x,y) \in \Omega_a \\ \Psi_m^{xy}(x,y), \ (x,y) \in \Omega_m \end{cases}, \tag{3}$$

where Ψ^{xy} is the transversal acoustic pressure and k_z is the axial wavenumber. Now, combining Eq. (3) with Eqs. (1) and (2), the following expressions can be obtained

$$\nabla^2 \Psi_a^{xy} + \left(k_0^2 - k_z^2\right)\Psi_a^{xy} = 0, \qquad \nabla^2 \Psi_m^{xy} + \left(k_m^2 - k_z^2\right)\Psi_m^{xy} = 0. \tag{4)-(5}$$

Subscript 2, related to the central duct air subdomain, has been omitted in Eq. (4) for simplicity.

The acoustic pressure within the chamber, considering a 2D FE discretization, can be approximated by means of shape functions as

$$\Psi_a^{xy}(x,y) = \mathbf{N}_a^T \mathbf{\Psi}_a \qquad (x,y) \in \Omega_a, \qquad \Psi_m^{xy}(x,y) = \mathbf{N}_m^T \mathbf{\Psi}_m \qquad (x,y) \in \Omega_m, \tag{6)-(7}$$

where $\mathbf{N}_a$ and $\mathbf{N}_m$ are vectors that contain the nodal shape functions of each subdomain. Then, the weighting residuals method, together with Green's theorem and Galerkin's approach is applied to Eqs. (4) and (5). Assuming that the wall of the chamber is rigid and impervious, the procedure leads to

$$\int_{\Omega_{a2}} \nabla^T \mathbf{N}_a \nabla \mathbf{N}_a d\Omega \{\mathbf{\Psi}_a\} + \int_{\Omega_{a2}} \left(k_z^2 - k_0^2\right) \mathbf{N}_a^T \mathbf{N}_a d\Omega \{\mathbf{\Psi}_a\} = \int_{\Gamma_{int}} \mathbf{N}^T \frac{\partial \Psi_a}{\partial n} d\Gamma, \tag{8}$$

$$\int_{\Omega_m} \nabla^T \mathbf{N}_m \nabla \mathbf{N}_m d\Omega \{\mathbf{\Psi}_m\} + \int_{\Omega_m} \left(k_z^2 - k_m^2\right) \mathbf{N}_m^T \mathbf{N}_m d\Omega \{\mathbf{\Psi}_m\} = \int_{\Gamma_{int}} \mathbf{N}^T \frac{\partial \Psi_m}{\partial n} d\Gamma. \tag{9}$$

The coupling conditions at interface Γ_{int} are related to continuity of pressure and normal acoustic velocity, the latter being written as

$$\frac{1}{\rho_0}\frac{\partial \Psi_a}{\partial n} = -\frac{\phi}{\rho_m}\frac{\partial \Psi_m}{\partial n}. \tag{10}$$

Eq. (9) is multiplied by $\rho_0\ \phi/\rho_m$, ϕ being the porosity of the granular material. Note that the load vectors associated with the terms on the right side of Eqs. (8) and (9) are cancelled when combining these equations, leading to the following FE expressions

$$\int_{\Omega_{a2}} \nabla^T \mathbf{N}_a \nabla \mathbf{N}_a d\Omega \{\mathbf{\Psi}_a\} + \int_{\Omega_{a2}} \left(k_z^2 - k_0^2\right) \mathbf{N}_a^T \mathbf{N}_a d\Omega \{\mathbf{\Psi}_a\}$$
$$+ \frac{\rho_0 \phi}{\rho_m} \int_{\Omega_m} \nabla^T \mathbf{N}_m \nabla \mathbf{N}_m d\Omega \{\mathbf{\Psi}_m\} + \frac{\rho_0 \phi}{\rho_m} \int_{\Omega_m} \left(k_z^2 - k_m^2\right) \mathbf{N}_m^T \mathbf{N}_m d\Omega \{\mathbf{\Psi}_m\} = \{0\} . \quad (11)$$

Finally, solving the assembly system from Eq. (11), the axial wavenumbers and pressure modes associated with the cross section of the chamber can be obtained.

2.2 Continuity of the pressure and the axial acoustic velocity

In this section, more details about the continuity of the acoustic fields will be provided in order to obtain the 3D complete pressure field. The continuity equations will be weighted and integrated at the geometric discontinuities of the silencer, that is, the expansion/contraction between the chamber and the inlet/outlet ducts. For the catalytic converter, the presence of multidimensional higher order modes in the chambers and the inlet/outlet ducts will be combined with one-dimensional wave propagation within the monolith. The orthogonality properties of the pressure modes will allow to speed up the computations [8].

2.2.1 Silencer containing granular absorbent material

The continuity of the acoustic pressure and axial velocity are weighted and integrated at geometric discontinuities determined by planes A and B, denoting inlet and outlet sections respectively (see Fig. 1). These acoustic fields can be written in terms of a modal expansion, containing both the incident and reflected waves. The continuity of the acoustic pressure can be written as

$$\int_{\Omega_{a1}} P_{a1} \Psi_{i1}^n(x,y) d\Omega_{a1} = \int_{\Omega_{a2'}} P_{a2'} \Psi_{i1}^n(x,y) d\Omega_{a2'} , \quad \int_{\Omega_{a2'}} P_{a2'} \Psi_{i1}^n(x,y) d\Omega_{a2'} = \int_{\Omega_{a3}} P_{a3} \Psi_{i1}^n(x,y) d\Omega_{a3} , \quad (12)\text{-}(13)$$

where Ψ_{i1}^n is the weighting function chosen, given by the eigenvalue associated with the incident wave at the inlet section [9, 14, 15], *a*2' makes reference to the area of section *a*2 coincident with those of sections *a*1 and *a*3 respectively. In addition, the kinematic condition associated with the axial acoustic velocity can be expressed as

$$\int_{\Omega_{a1}} U_{a1} \Psi_{ic}^n(x,y) d\Omega_{a1} = \int_{\Omega_{a2'}} U_{a2'} \Psi_{ic}^n(x,y) d\Omega_{a2'} , \quad \int_{\Omega_{a2'}} U_{a2'} \Psi_{ic}^n(x,y) d\Omega_{a2'} = \int_{\Omega_{a3}} U_{a3} \Psi_{ic}^n(x,y) d\Omega_{a3} , \quad (14)\text{-}(15)$$

Ψ_{ic}^n being the weighting function associated with the incident wave at the chamber [9, 15, 16]. Then, weighting integrals are numerically evaluated after truncating the number of unknown wave amplitudes at *n* modes. Equations are simultaneously solved to obtain the unknown modal amplitudes, considering a unity amplitude for the incident wave at the inlet section and an anechoic termination of the silencer [12, 15].

2.2.2 Catalytic converter

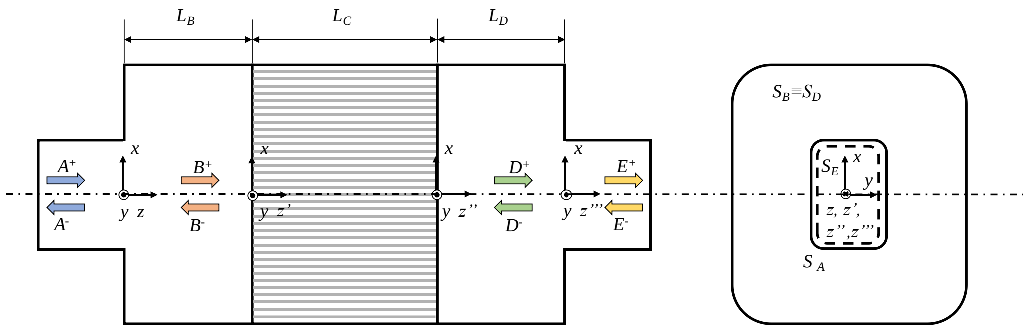


Figure 2: Scheme of a catalytic converter with monolith.

A scheme of a catalytic converter is shown in Fig. 2, including a central monolith. The geometry is divided into different subdomains: regions *A* and *B* are associated with the inlet duct and the expansion chamber, while the contraction is formed by regions *D* and *E*. These ducts and chambers have an axially, but uniform, cross section, the air being the medium for sound propagation. As indicated previously, multidimensional propagation of the acoustic fields is considered in theses regions. On the other hand, the monolith placed in the central part of the device can be considered one-dimensional from an acoustical point of view. Thus, it is modelled by its four-pole transfer matrix in terms of complex and frequency-dependent equivalent properties, such as density ρ_m and bulk modulus K_m (see details in section 3) [8, 17].

A weighted integration (similar to the one considered for the silencer with granular material) is carried out for the compatibility equations, that is, the continuity of the acoustic pressure and axial velocity at geometric discontinuities. In this case, the weighting functions are the transversal modes of the ducts. Therefore, at the expansion, the continuity of the pressure field yields

$$\int_{\Omega_A} P_A(x,y,z=0)\Psi_{A,s}(x,y)d\Omega=\int_{\Omega_A} P_B(x,y,z'=-L_B)\Psi_{A,s}(x,y)d\Omega\,, \tag{16}$$

where s = 1, 2, …, N_a modes. Besides, the axial velocity condition can be written as

$$\int_{\Omega_A} U_A(x,y,z=0)\Psi_{B,s}(x,y)d\Omega=\int_{\Omega_B} U_B(x,y,z'=-L_B)\Psi_{B,s}(x,y)d\Omega\,, \tag{17}$$

that now is truncated at s = 1, 2, …, N_m modes. The expressions corresponding to the contraction can be easily obtained following a similar procedure with the appropriate modifications.

The acoustic coupling between fields at both sides of the monolith can be expressed through the four-pole matrix as

$$\begin{Bmatrix} P_B(x,y,z'=0) \\ U_B(x,y,z'=0) \end{Bmatrix}=\begin{bmatrix} T_{11}^m & T_{12}^m \\ T_{21}^m & T_{22}^m \end{bmatrix}\begin{Bmatrix} P_D(x,y,z''=0) \\ U_D(x,y,z''=0) \end{Bmatrix} \quad (x,y)\in\ \Omega_B\equiv\Omega_D\,, \tag{18}$$

T_{11}^m, T_{12}^m, T_{21}^m and T_{22}^m being the monolith four poles. Now, similarly to the approach followed for Eqs. (16) and (17), Eq. (18) will be multiplied by the transversal mode $\Psi_{B,s}(x, y) = \Psi_{D,s}(x, y)$ with s = 1, 2, …, N_m. Integrating over the cross section, taking advantage of the orthogonality relations and removing common factors, a very simple set of equations is obtained. As indicated in reference [8], neither integrations nor modal summations appear in these equations. In addition, they do not depend on the geometry of the transversal cross section (provided that this is axially uniform) and relate directly wave amplitudes with equal modal number, thus forming an uncoupled set of equations.

Finally, once the unknown modal amplitudes are obtained from solving the previous system of equations, the acoustic attenuation of the catalytic converter can be computed through the transmission loss [8].

3. Model for granular material

The granular material can be modelled by means of its equivalent properties, that is, complex density and bulk modulus [6, 7]. According to Umnova *et al.* [6] the equivalent density of the granular material can be expressed as

$$\rho_m=\rho_0 q\left(1-\frac{j\mu\sigma}{\omega\rho_0 c_0 q}\sqrt{1+\frac{\omega\rho_0 4q^2k_p^2}{-j\mu\Lambda^2\sigma^2}}\right), \tag{19}$$

ρ_0 and c_0 being the air density and speed of sound, μ the dynamic viscosity, σ the volumetric porosity (different from the surface porosity ϕ previously defined), q the tortuosity, k_p the steady state thermal permeability, and Λ the viscous characteristic length that can be written as $\Lambda=4(1-\Theta)\sigma q r_{part}/(9(1-\Theta))$, where r_{part} is the radius of the particle and Θ is the cell parameter radius defined as $\Theta = 3(1-\sigma)/(2^{1/2}\pi)$. Tortuosity is defined as $q=1+(1-\sigma)/(2\sigma)$, while permeability is represented by $k_p = \mu/R$, where R is the flow resistivity given by

$$R = \frac{9\mu(1-\sigma)}{2r_{part}\sigma}\frac{5(1-\Theta)}{5-9\sqrt[3]{\Theta}+5\Theta-\Theta^2}. \quad (20)$$

In addition, the bulk modulus can be written as [18]

$$K_m = \gamma P\left(\gamma-(\gamma-1)\left(1+\frac{8\mu}{j\omega\rho_0 N_{Pr}\Lambda^2}\sqrt{1+\frac{j\omega\rho_0 N_{Pr}\Lambda'^2}{16\mu}}\right)^{-1}\right)^{-1}, \quad (21)$$

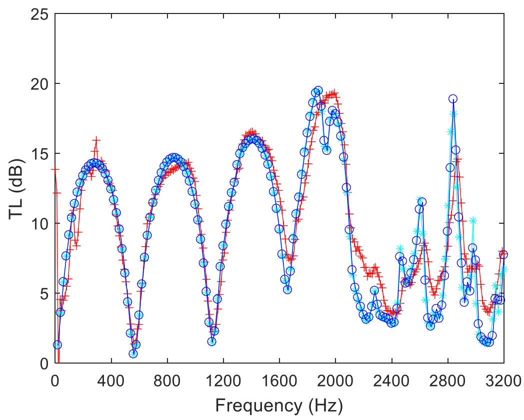
where γ is the specific heat capacity ratio, P_0 the atmospheric pressure, N_{Pr} the Prandtl number (N_{Pr} = 0.71 for the conditions considered here), ω the angular frequency, and Λ' is the thermal characteristic length defined as $\Lambda'=3\,\Lambda/(2q(1-\Theta))$. Once the complex density and bulk modulus are known, the characteristic impedance $Z_m=\sqrt{K_m\rho_m}$, wavenumber $k_m=\omega\sqrt{\rho_m/K_m}$ and the speed of sound $c_m=\sqrt{K_m/\rho_m}$ can be obtained.

4. Results

4.1 Silencer containing granular material

In order to validate the mode matching model previously developed for a silencer containing granular material, a circular configuration has been considered. The granular material is composed of spherical particles of 0.00595 m in diameter, its porosity being 39.9%. The geometry of the silencer is defined by the dimensions R_1 = R_3 = 0.0268 m (radii of de inlet/outlet ducts, respectively), R_C = 0.091875 m and L_C = 0.3 m (radius and length of the central chamber). The air properties are c_0 = 344.13 m/s and ρ_0 = 1.1979 kg/m^3, corresponding to a temperature of 21.4ºC. The following cases have been studied: (I) The chamber of the silencer has been filled up to approximately 0.02 m below the bottom of the inlet/outlet ducts (see Fig. 3a); (II) the chamber is filled until the spheres reach the bottom of the inlet/outlet ducts (see Fig. 4a).

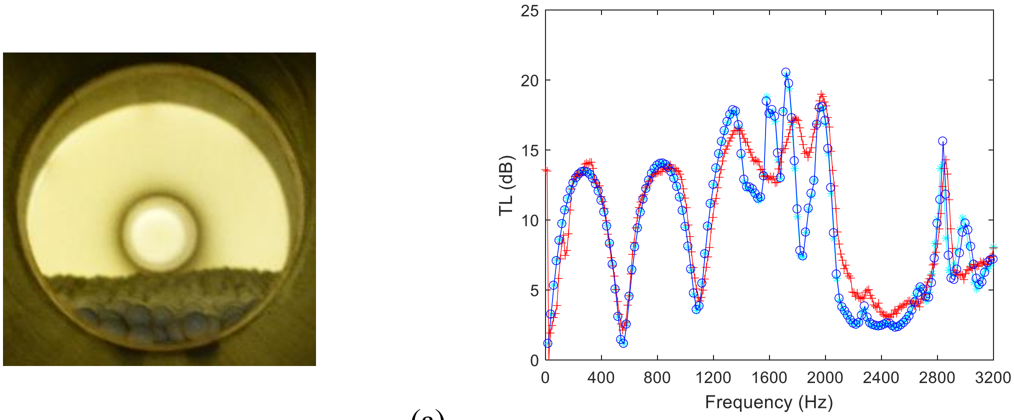
(a)



(b)

Figure 3: a) Picture of the prototype; b) TL of a silencer partially filled with granular material, case I: +++, experimental measurement; ***, mode matching method; ooo, Comsol Multiphysics®.

The results computed with the mode matching method (using 40 modes) for filling cases I and II are compared in Figs. 3b and 4b, respectively, showing a good agreement with those obtained from experimental measurements as well as an analysis carried out with the FE commercial software Comsol Multiphysics®. However, in filling case II (see Fig. 4b) the discrepancies between the experimental measurement and the numerical models are slightly higher, probably due to a stronger effect of the material model inaccuracy as the amount of spheres increases. The use of granular material increases the attenuation achieved by the silencer in the mid and high frequency range, as expected.



(a) (b)

Figure 4: a) Picture of the prototype; b) TL of a silencer partially filled with granular material, case II: +++, experimental measurement; ***, mode matching method; ooo, Comsol Multiphysics®.

4.2 Catalytic converter

A circular configuration has been studied, its main dimensions being $R_1 = R_3 = 0.0258$ m (radii of the inlet/outlet ducts, respectively), $R_2 = 0.1275$ m (radius of the chambers placed at both sides of the monolith) and $L_B = L_D = 0.1975$ m (see Fig. 2 for details). The properties of the air are $c_0 = 345.76$ m/s and $\rho_0 = 1.1866$ kg/m^3 (speed of sound and density, respectively, at 24.4 ºC). The acoustic model of the monolith is characterized by the following values: length $L_C = 0.075$ m, resistivity $R = 1500$ rayl/m, $\phi = 0.88$, geometric factor $\alpha_g = 1.14$, dynamic viscosity $\mu = 1.783 \cdot 10^{-5}$ Pa s, thermal conductivity $\kappa = 0.02534$ W/(m K) and specific heat at constant pressure $C_p = 1005$ J/(kg K).

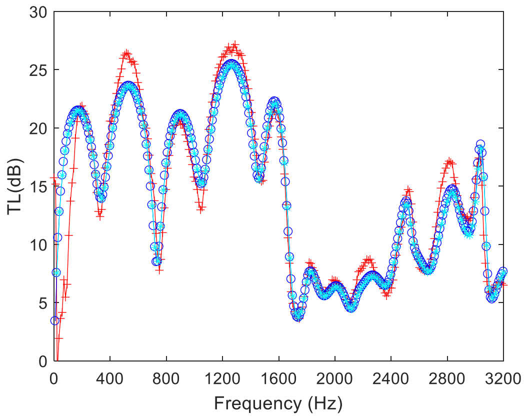


Figure 5: Transmission loss of a catalytic converter: +++, experimental measurement; ***, Mode matching method; ooo, Comsol Multiphysics®.

As it can be observed in Fig. 5, the experimental measurement shows good agreement with the numerical approaches, the main discrepancies appearing at the domes and in the low frequency range, where measurements are less accurate. It is also worth noting that the *TL* results obtained by using the mode matching method and Comsol Multiphysics® are practically undistinguishable.

5. Conclusions

Numerical models based on the mode matching technique have been presented in this work to assess the acoustic behaviour of two kinds of exhaust devices with arbitrary but axially uniform cross section: silencers containing granular material and catalytic converters incorporating a monolith. The approaches proposed in this work have been shown to provide accurate predictions of the attenuation performance while reducing the computational time of a FEM computation. It should be noticed that the models used to characterize the granular material and the monolith have proved to be accurate enough. In the first case, however, some discrepancies have been found when the amount of spheres within the chamber is higher. Further research is required on this particular subject.

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